Capacitance Standard based on Counting Electrons

By:
Zeinab Mousavi
Stephanie Teich-McGoldrick
Jaspreet Wadhwa

SI Electrical Base Units

Electrical Units: m, s, kg, A

- Fundamental Units: same for all times and places

  second: time taken by 9,192,631,770 cycles of radiation that comes from electrons moving between two energy levels of the caesium-133 atom

  meter: wavelength of radiation from a transition in KR atom

- Non-fundamental Units: not constant

  kilogram: mass of a metal cylinder kept in Paris

Ampere: current that when flowing in straight parallel wires of infinite length and negligible cross section, separated by a distance of one meter in free space, produces a force between the wires of 0.2 μ Newton per meter of length (1960)

volt: based on electrochemical re-actions within chemical cells

ohm: based on measuring a wire-wound standard resistor against the impedance of a capacitor at known frequency, drift range of -0.7 to 0.7μΩ/year

need higher-reproducibility fundamental standards based on quantum phenomena

1990 Standards

- Quantum Hall Effect → resistance standard

  Klaus von Klitzing, 1985 Nobel Prize

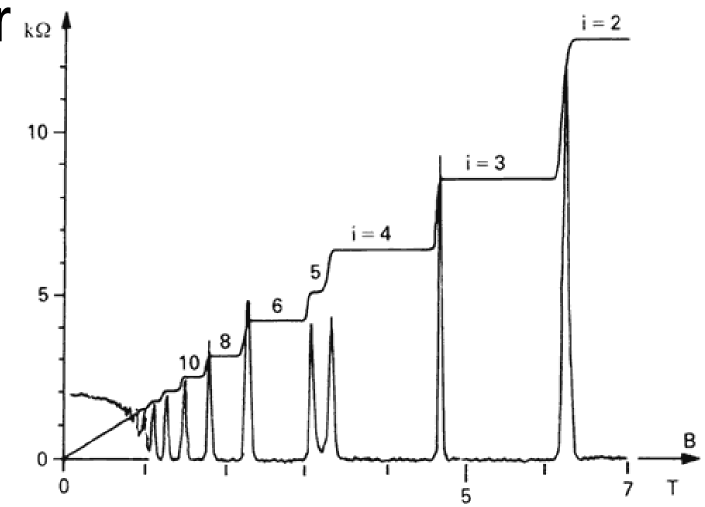
  2D Hall Effect, Low Temp

  resistance varies stepwise with magnetic field

  step size does not depend on material properties

  step size = $h/(i*e^2)$, i = step number

  1 Klitzing: hall resistance at 4th step



(Kosmos, 1986)

1990 Standards

- Josephson Effect → voltage standard

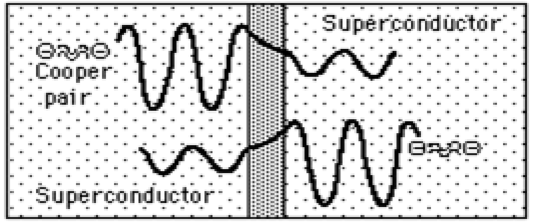
Brian Josephson, 1962

two superconductors separated by thin insulator

Cooper pairs tunnel through junction

applied DC voltage → oscillation of frequency $f_j = \frac{2e\Delta V}{h}$

1 Volt: voltage required to produce 483,597.9 GHz



Calculable Capacitor

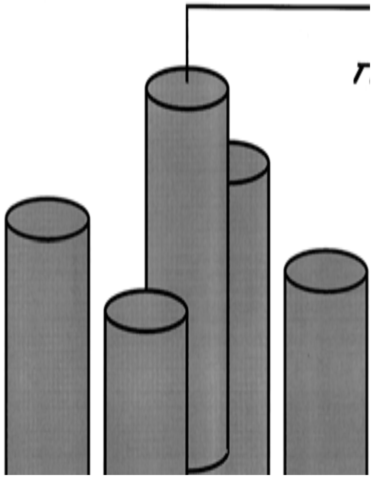
Parallel Plate Formula: $C = \frac{\varepsilon_0 A}{d}$ neglects fringing fields effect

Calculable Capacitor:

Special geometry rejects effects of fringing fields

capacitance depends on only one length

$$\frac{\Delta C}{\Delta L} = \varepsilon_0 Ln(\frac{2}{\pi})$$



- ✓ no uncertainty in relationship
- ✓ measurement of displacement
- ✓ can determine impedance

(Zimmerman, 1997)

Electron Counting Capacitance Standard (ECCS)

Capacitance: transfer of charge between two conductors creates potential difference $C = \frac{Q}{\Delta V}$

- Single Electron Transistor (SET) : detect single electron

$$C = \frac{Ne}{\Delta V}$$

→ Capacitance standard based on quantization of charge

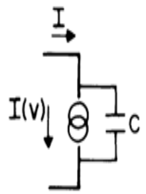
Use small current to charge a small capacitor to a large voltage in a short time

Three components of SET Capacitance Standard:

1. electrometer
2. 7-junction electron pump
3. cryogenic vacuum-gap capacitor

SET Device

Metal tunnel junction
capacitance C, current I(V), bias volt V
tunneling → $E_{change} = eQ/C – e^2/2C$
R_t: tunneling resistance



(Fulton, 1987)

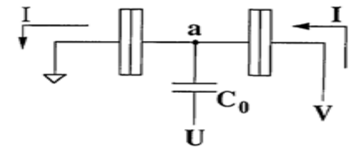
If R_t>>h/e^2 and kT<<$e^2/2C$ → Coulomb Blockade

no tunneling for $|V|<e/2C$

C ≈ 0.1fF, $e^2/2C$ ≈ 10K, Temp ≈ 0.1K (-460F)

SET effects dominate thermal fluctuation

SET Electrometer



(Williams, 1992)

two tunnel junctions:

each has capacitance C

electrode *a* : island electrically isolated from circuit

Input: potential U coupled to island *a* thru C_0

lower potential barrier across junction (through U)

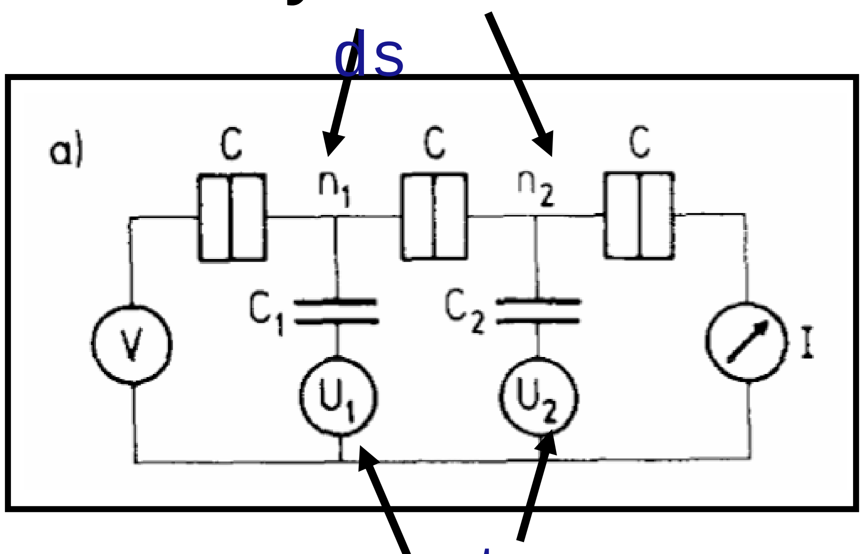
→ current passes through

→ device current linearly proportional to U

Accuracy of electron counting using a 7-junction electron pump

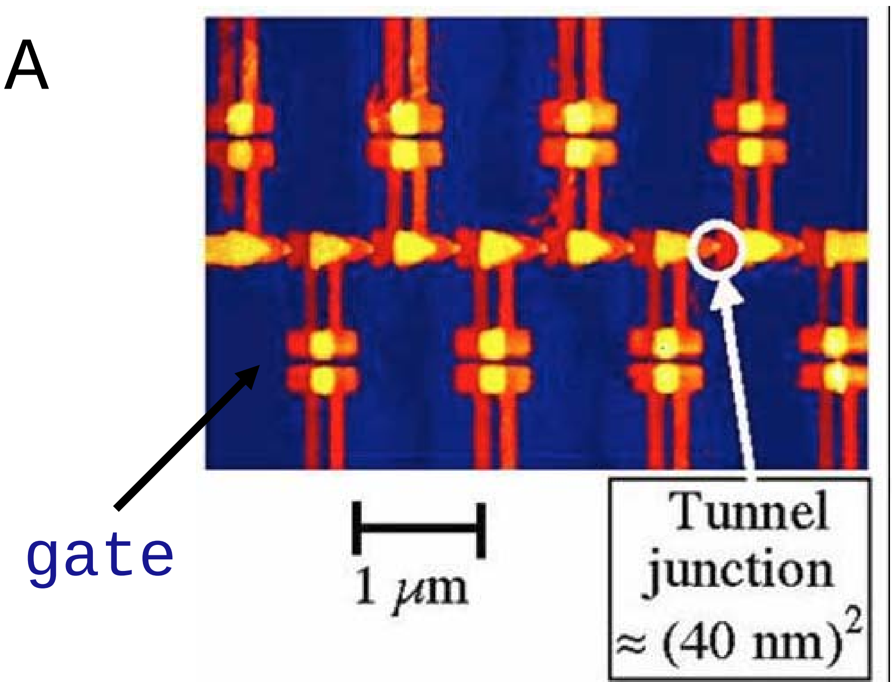
The electron pump

- Based on Columbic tunneling
- A series of metal islands separated by tunnel junctions
- A gate electrode coupled capacitively to each island



The electron pump

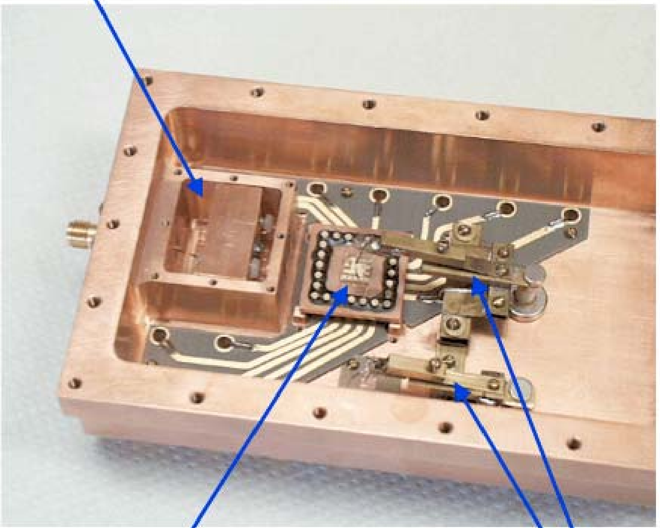
- Changing the gate voltages changes the Coulomb blockade at each junction
- Allows individual electrons to be sent down the chain of islands
- Current -- 1pA



Important as a standard for Capacitance

- A way to count e-
- As a capacitance standard
  - Use electron pump to put e- onto Cyro Cap
  - Stop pumping e- (hold mode)
  - Compare C_{CRYO} with another capacitor at room temp
  - Determine the value of conventional room temp capacitor in terms of e-
  - Room temp capacitor can now be used as a basis for calibrations

Cryogenic Capacitor



Chip with SET pump

switches

Zimmerman et al. Measurement Standards and Technology, 2003

Important as a standard for Capacitance

- Requirements
  - Ability to put 10^8 e- on 1pF capacitor
  - Uncertainty $^{+-}$1 e- (10 ppb)
  - Small leakage current when off

Design

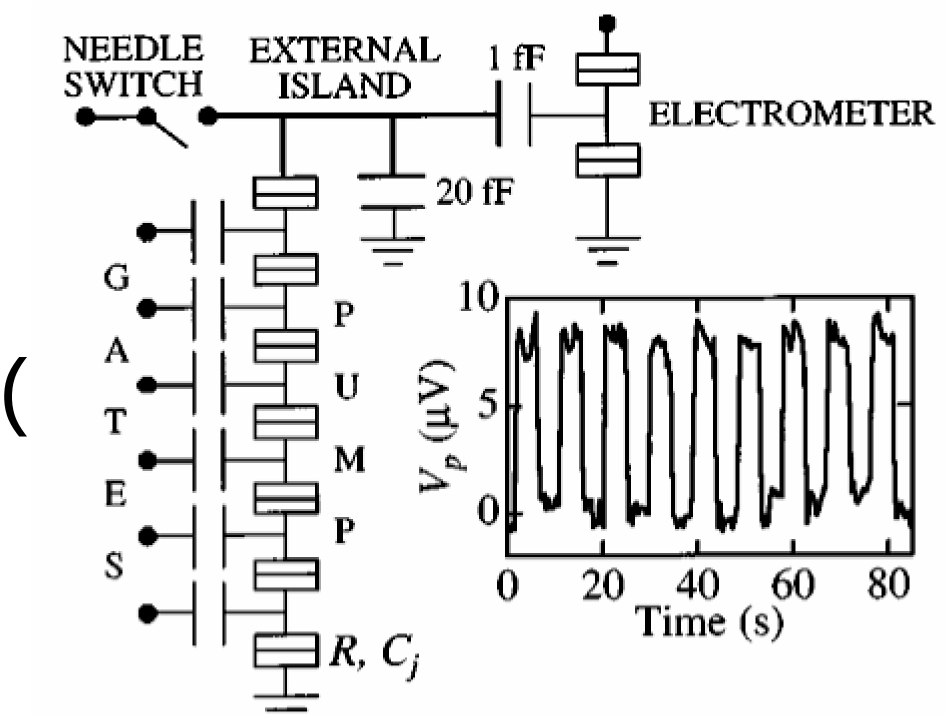
- Minimize error
- Maximize Coulomb Blockade
  - Small junctions to reduce junction capacitance
  - Small islands to reduce self-capacitance
  - Low k constant substrate to reduce stray capacitance
  - Minimize cross-capacitance

Fabrication

- Two-angle evaporation of Al
- PMMA bi-layer mask patterned with EBL

Circuit Design

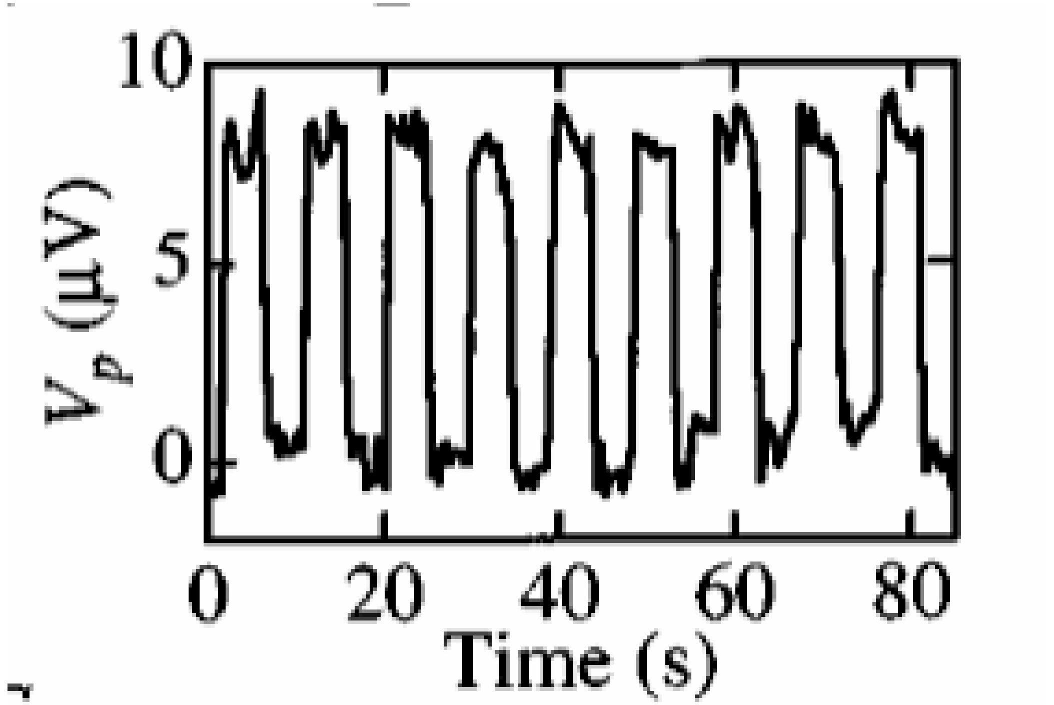
- Pumps connected to external island
- Electrometer (based on tunnel junction)
- Switch
  - Closed to obtain I/V curve of pump
  - Open to detect pumped e-



All components except the switch were fabricated on a single chip

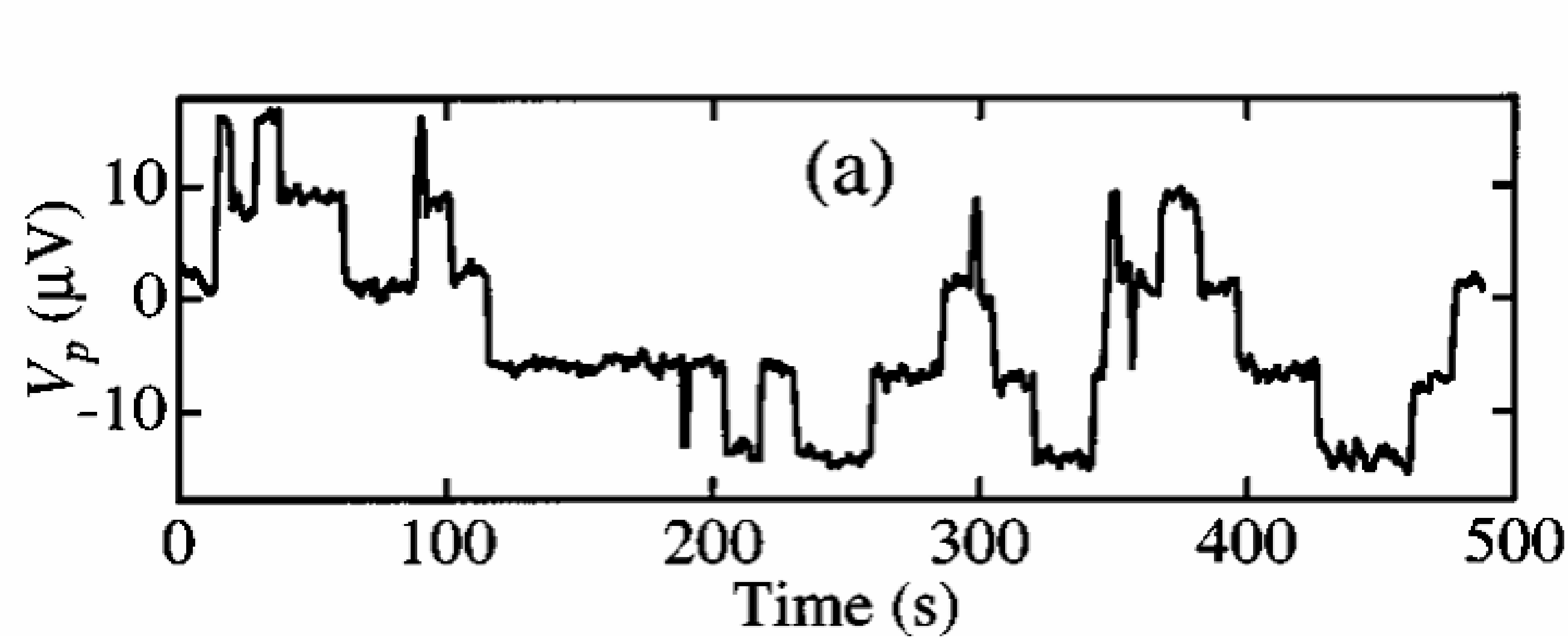
Circuit Design cont.

- Plot of voltage on external island (V_P) vs time
- 1 e- repeatedly pumped on/off island
- Jump of 7.6eV = 1e- on/off



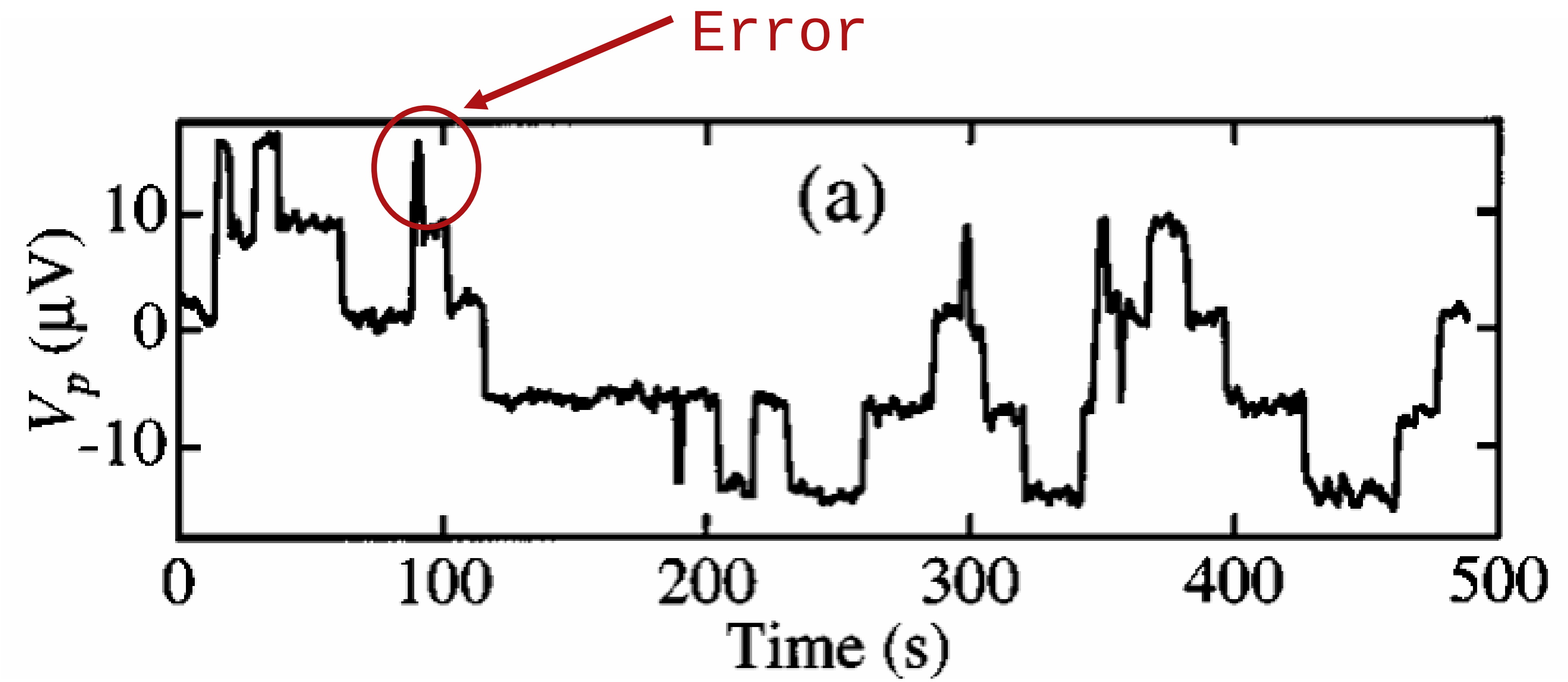
Accuracy of electron pump

- Pumping 1 e- on/off external island
- Measure Vp vs. time
- Pumping rate faster than can measure



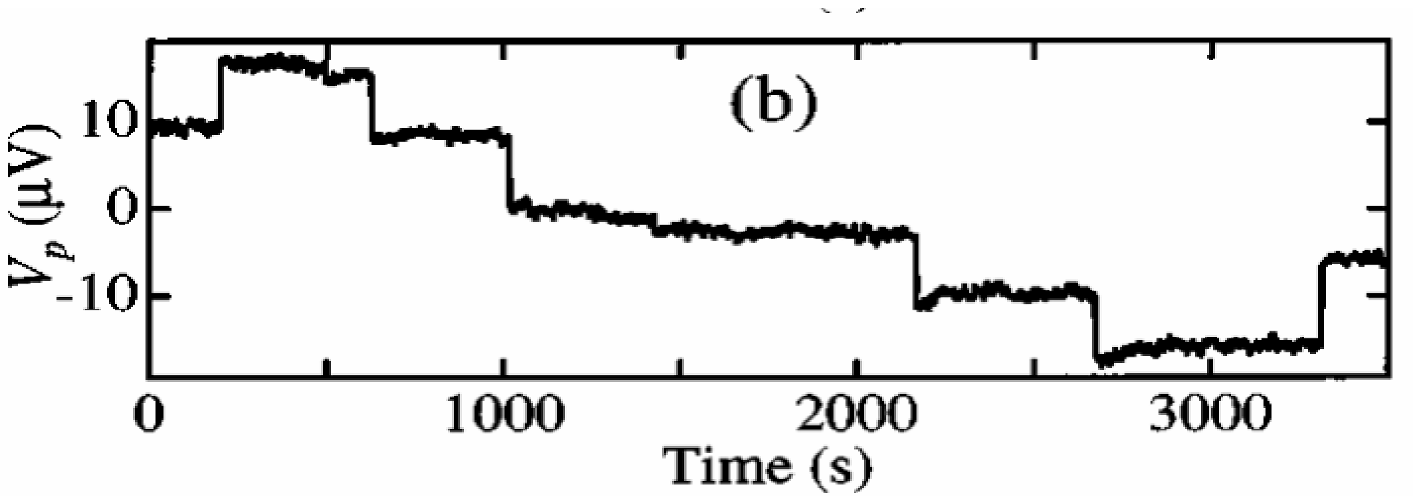
Accuracy of electron pump

- Pumping rate = 5.05MHz
- Avg error rate 1error/13s
- Error per pumped e-, *15ppb*



Leakage Rate

- Hold mode (e- pumped onto islands and gate pulses turned off)
- Each jump in figure is 1e- leaking through the pump



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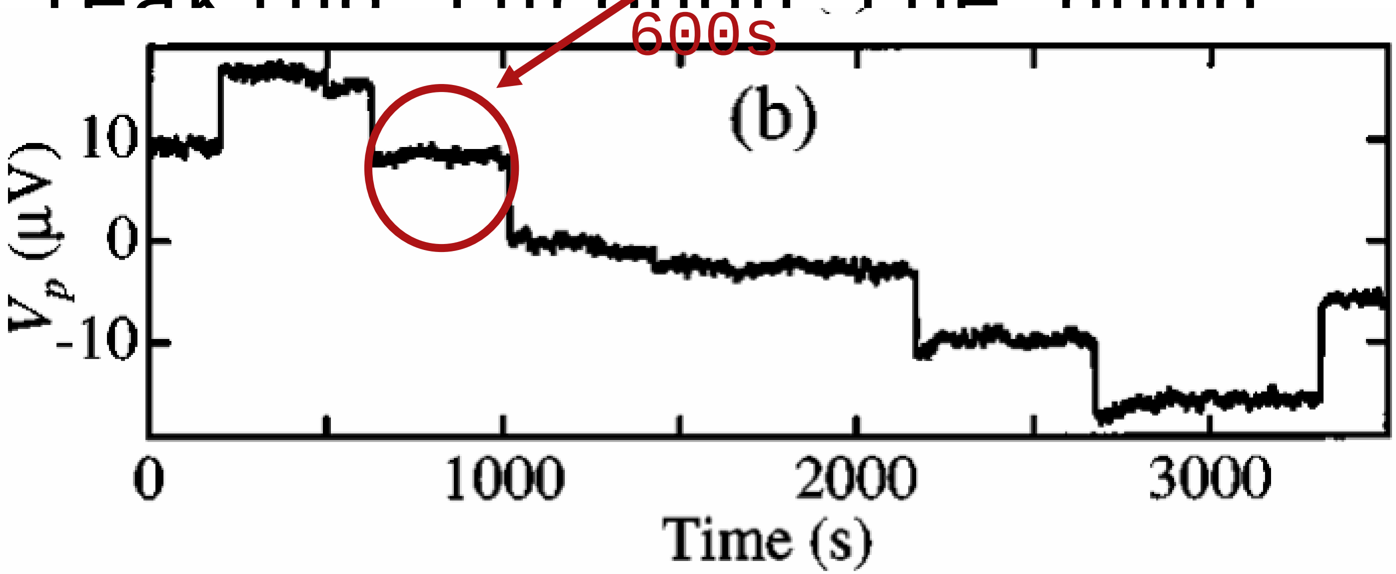
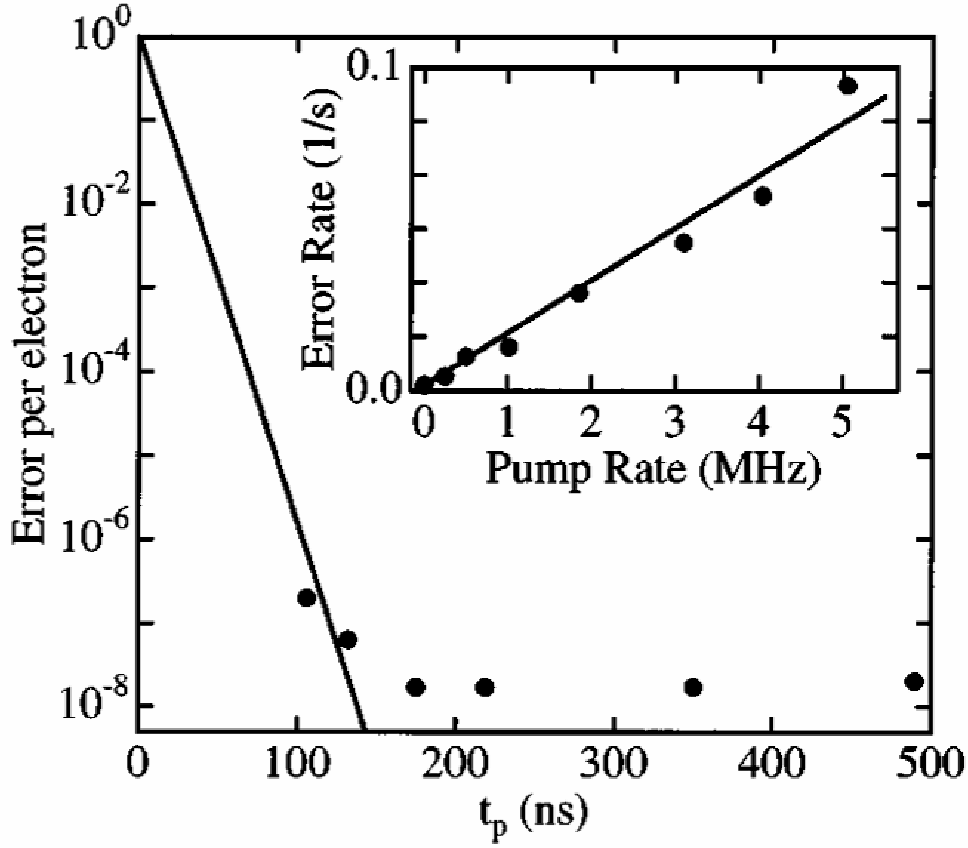


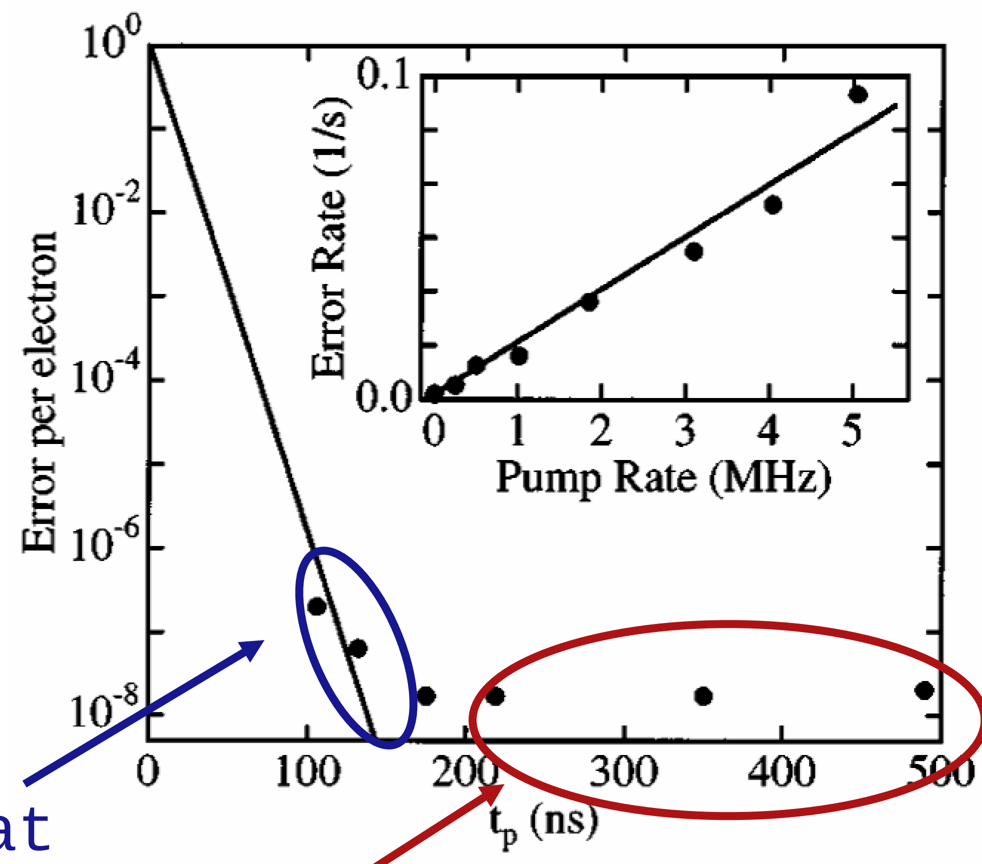
Pumping speed errors

- t_P = pump time
- t_W = wait time
- $1/(t_P + t_W)$ = overall pump time



Pumping speed errors

- Time each junction is activated must be long compared to RC to avoid errors due to missed tunneling events
- $\varepsilon_{th} = exp(-at_P/RC)$

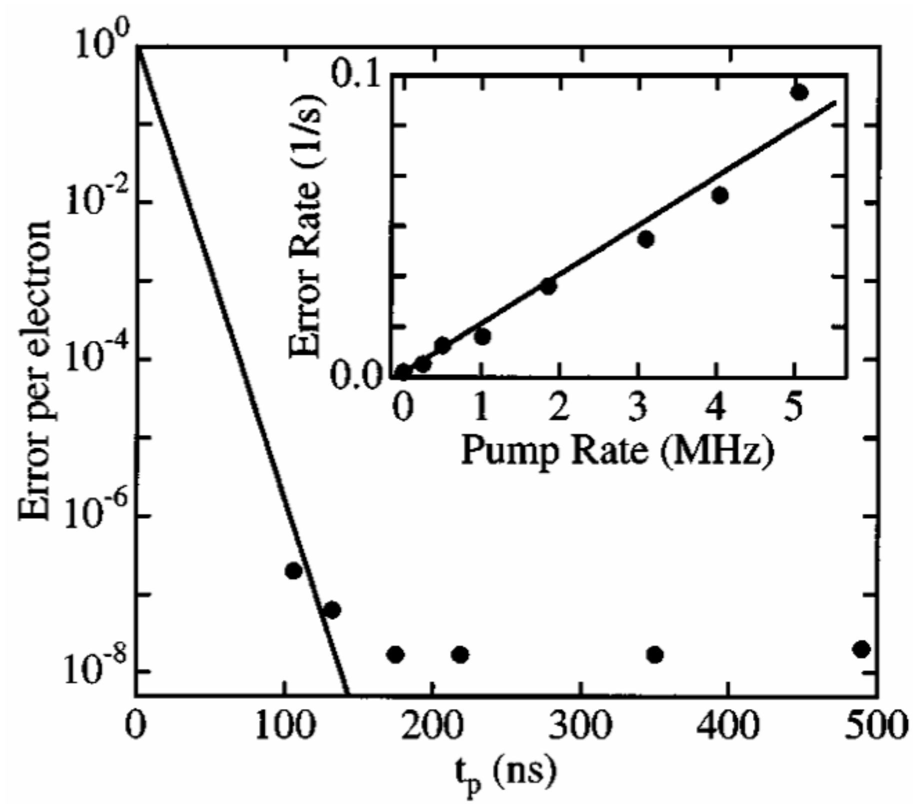


Increases at small t_P

Constant at large t_P

Pumping speed errors

- Inset graph
- Error rate as pump rate is varied holding t_P constant and varying t_W
- Overall pump rate can be adjusted without

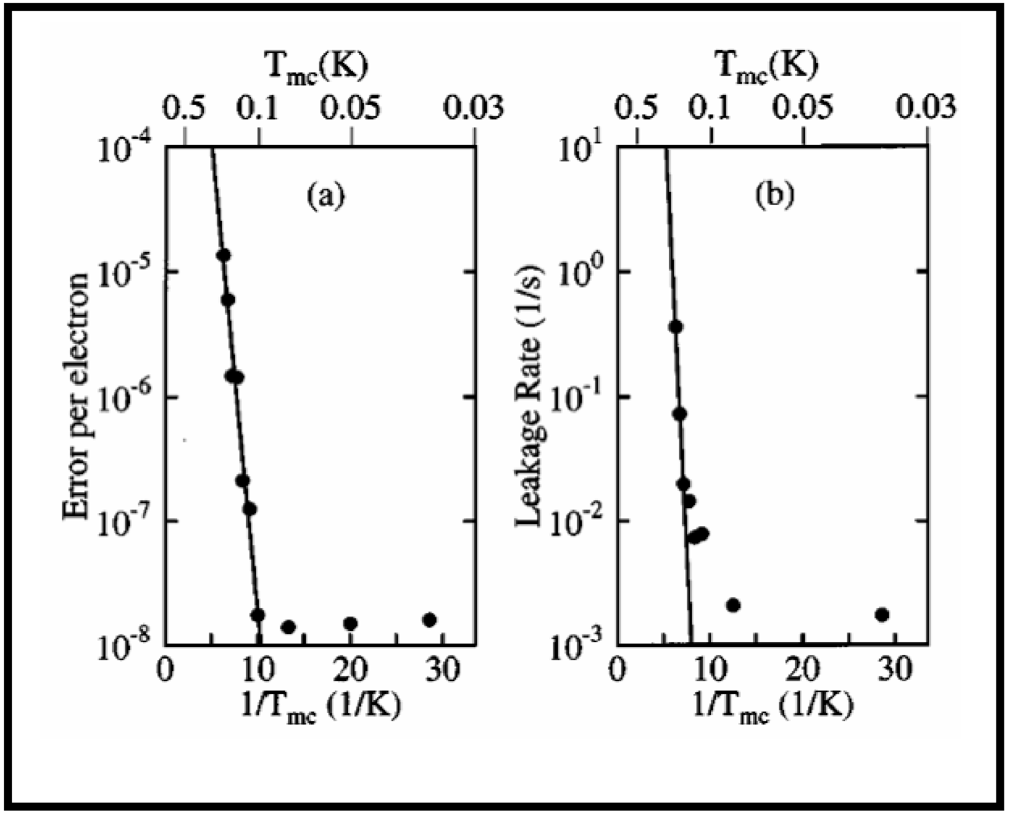


Temperature dependence

- At high T both the error and leakage rate increase exponentially
- At low temp both are independent of T_{MC}
- At low temp a temp-independent error mechanism dominates
- Photon-assisted cotunneling
  - Noise from the slow relaxation of charges trapped in metastable states

$$\varepsilon_{th} = b * \exp(-\Delta E_P / k_B T)$$

$$\Gamma_{th} = \frac{d}{RC} \exp(-\Delta E_h / k_B T)$$



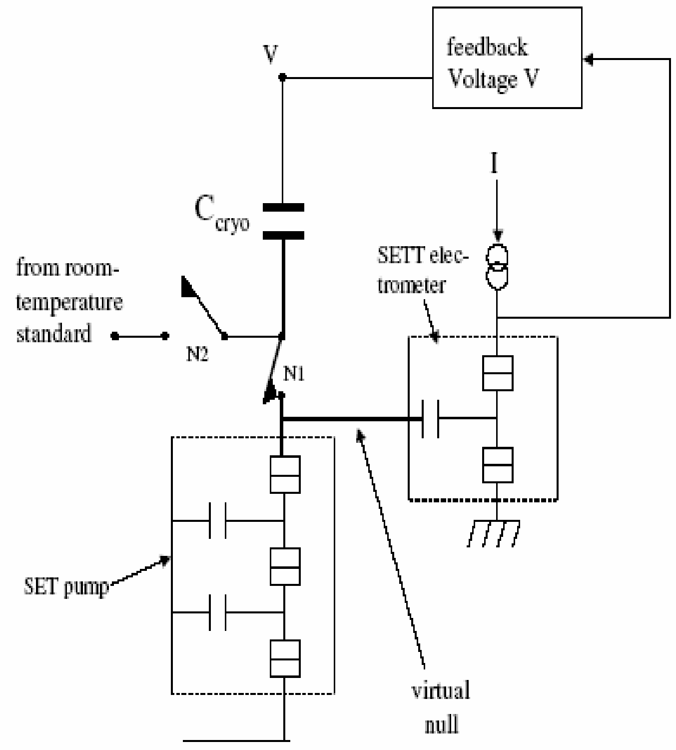
ECCS Operation

Pump Phase: charge capacitor C_{cryo}

electrometer: monitor potential *a*

electron pump: current source

C_{cryo}: standard capacitor

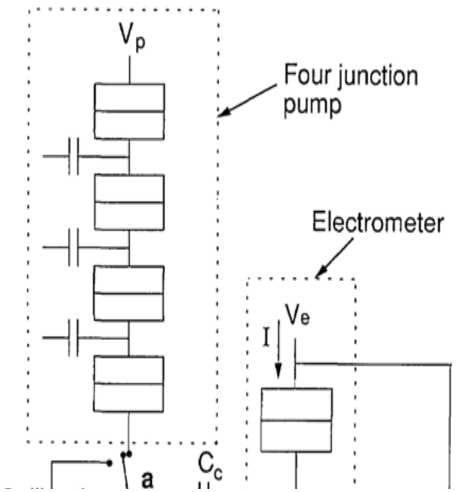


(Zimmerman, 2003)

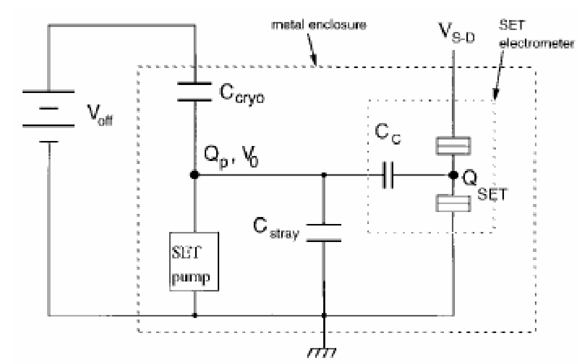
- charge C_{cryo} with N electrons
- measure avg. V across C_{cryo}
- determine C_{cryo}

ECCS Operation

Why null detector/feedback?



(Williams, 1992)



(Zimmerman, 1997)

To avoid errors, voltage across pump must be kept zero

Charge from pump appears across C_{cryo}, not C_{strand}

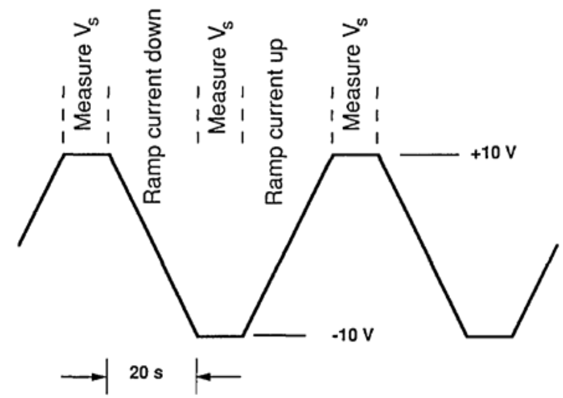
ECCS Operation

Pump at 6.2 MHz for 10s, V_{feed} = 10

$I=e*f \rightarrow I=9.9\times10^{-13}$ (A)

$Q=I*t \rightarrow Q\approx10\times10^{-12}$ (C)

$C=Q/V \rightarrow C\approx1p$ (F)

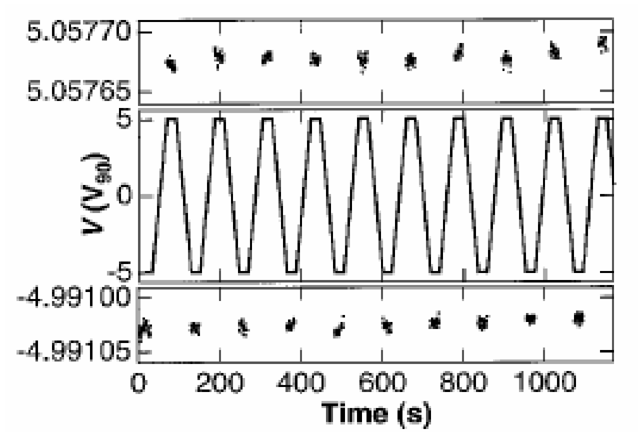


(Williams, 1992)

In practice, repeat 10-100 times

Pump ≈ 10^8 electrons, V_{feed} = 10

$C=Ne/< V_{feed} > \rightarrow C\approx$ 1.8pF

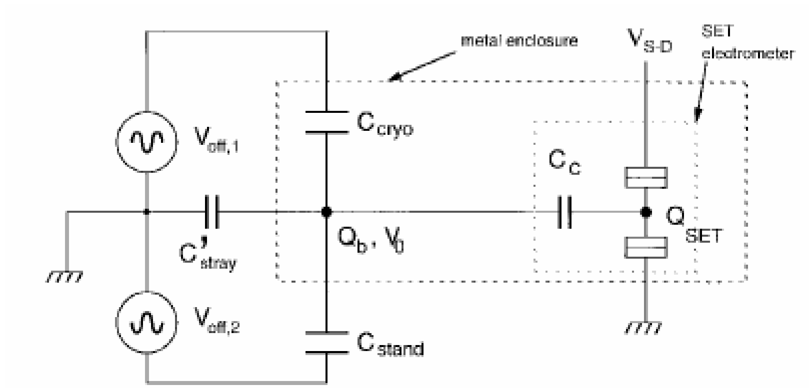


(Zimmerman, 1997)

ECCS Operation

Bridge Phase: Compare with C_{stand} at room temp

- grounded shield V_1, V_2 affect detector through C_{cryo}, C_{stand}
- balanced bridge: no Q_b at balance point
- adjust V_1, V_2 to balance bridge: $V_1*C_{cryo}=V_2*C_{stand}$



(Zimmerman, 1997)

In practice, single source and voltage divider

Capacitance Requirements

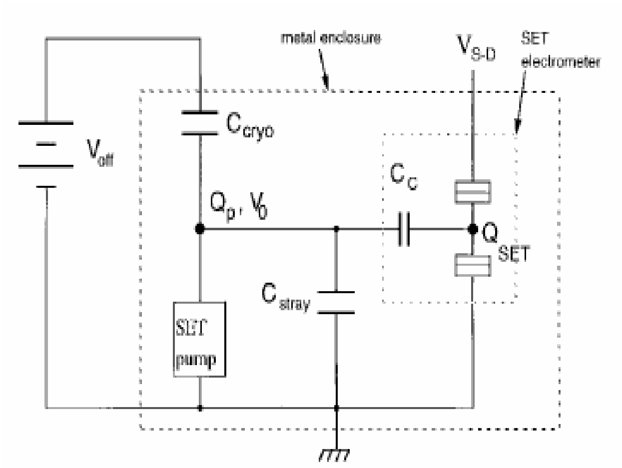
- Pump electrons onto cap
  - stable
  - low loss
- Measure voltage across cap for several seconds
  - hold number of electrons fixed
  - leakage resistance of 10^{22} Ω
- Compare to standard cap at room temp & audio freq
  - stable for minutes or hours
  - low frequency dependence

Cryogenic cap: no freq & volt dependence
parallel resistance 10^{21} Ω

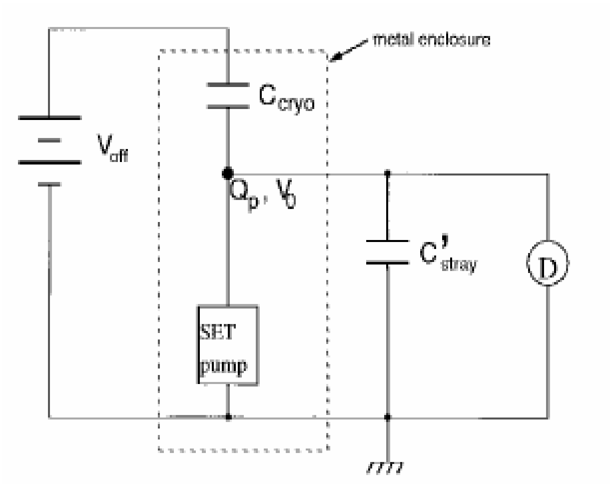
Stray Capacitance

low temp, metal enclosure, spaced closely → lower C_{stray}

perfect null detector: $V_{feed}=Q_p/C_{cryo}$



SET electrometer



conventional electrometer

(Zimmerman, 1997)

$$\left|V_{feed}-\frac{Q_P}{C_{cryo}}\right|=\frac{C_{stray}+C_C+C_{cryo}}{C_{cryo}}\frac{\delta Q_{SET}}{C_C}$$

$$\left|V_{feed}-\frac{Q_P}{C_{cryo}}\right|=\frac{C'_{stray}+C_{cryo}}{C_{cryo}}\delta V_0$$

Line Impedance

R_{filt}: discrete filters used to shield SET from hi-freq noise

R_{line}: disturbed line resistance

Line impedance → uncertainties

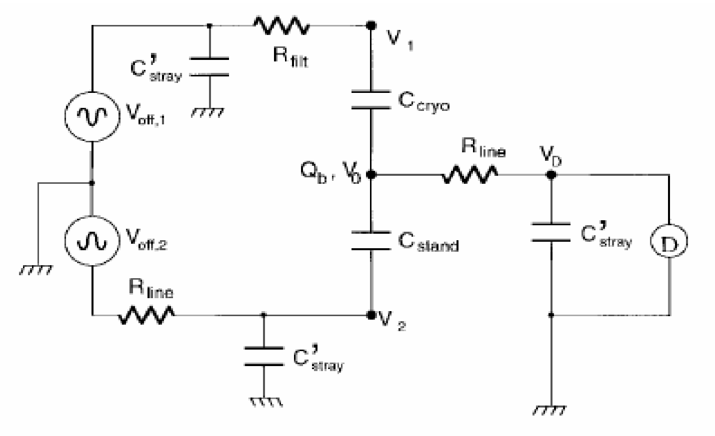
- applied volt to C_{cryo} and C_{stand} different than V_1, V_2

$\omega=10^4$ Hz, uncertainty $<10^{-8}$

→ $R_{filt} < 1\Omega$, $R_{line} < 0.01\Omega$

→ use filters special for SET

(refer to Vion, 1994)



Slide "Stray Capacitance" Notes

$Q_p = Q_{cryo} + Q_{stray} + Q_c$

⇓

$Q_{cryo} = Q_p - (Q_{stray} + Q_c)$

⇓

$$\frac{Q_{cryo}}{C_{cryo}} = \frac{Q_p}{C_{cryo}} - \frac{(Q_{stray} + Q_c)}{C_{cryo}}$$

⇓

$$\frac{Q_{cryo}}{C_{cryo}} = \frac{Q_p}{C_{cryo}} - \frac{(C_{stray} + C_c)V_0}{C_{cryo}}$$

KVL, left-hand loop:

$$V_{off} = \frac{Q_{cryo}}{C_{cryo}} - V_0$$

⇓

$$V_{off} - \frac{Q_{cryo}}{C_{cryo}} = V_0$$

⇓

$$V_{off} - \left(\frac{Q_p}{C_{cryo}} - \frac{(C_{stray} + C_c)V_c}{C_{cryo}}\right) = V_0$$

⇓

$$\left|V_{off} - \frac{Q_p}{C_{cryo}}\right| = \left[(C_{stray} + C_c)/C_{cryo} + 1\right] V_0$$

{min resolvable charge on island $Q_{SET} = \delta Q_{SET}$

$$\left|V_{off} - \frac{Q_p}{C_{cryo}}\right| = \frac{C_{stray} + C_c + C_{cryo}}{C_{cryo}} \; \frac{\delta Q_{SET}}{C_c}$$

Similar Analysis for Conventional Electrometer:

$$\left|V_{off} - \frac{Q_p}{C_{cryo}}\right| = \frac{C'_{stray} + C_{cryo}}{C_{cryo}} \, \delta V_0$$

Slide "Line Impedance" Notes

top loop:

$$I = \frac{V_{off,1}}{Z_{C_{cryo}}} = V_{off,1} \cdot j\omega C_{cryo}$$

$$\frac{V_{off,1} - V_1}{R_{filt}} = C_{cryo} \cdot \omega \cdot V_{off,1}$$

⇓

$$\left|\frac{V_1 - V_{off,1}}{V_{off,1}}\right| = R_{filt}\, C_{cryo}\, \omega$$

$C_{cryo} \to 10^{-12}$, $\omega \to 10^{4}$

$R_{filt} < 1\,\Omega \Rightarrow$ uncertainty $< 10^{-8}$

Similarly, for lower loop:

$$\left|\frac{V_2 - V_{off,2}}{V_{off,2}}\right| = R_{line} \cdot C'_{stray} \cdot \omega$$

$C'_{stray} \to 100 \times 10^{-12}$, $\omega \to 10^{4}$

$R_{line} < 0.01\,\Omega \Rightarrow$ uncertainty $< 10^{-8}$

For resistance in the line to the null detector:

$$I = \frac{V_e}{Z_{C_{stray}}} = V_0 \cdot \omega C_{stray}$$

$$\Rightarrow \left|\frac{V_e - V_0}{V_0}\right| = R_{line}\, C'_{stray} \cdot \omega$$

$C'_{stray} \to 100 \times 10^{-12}$, $\omega \to 10^{4}$

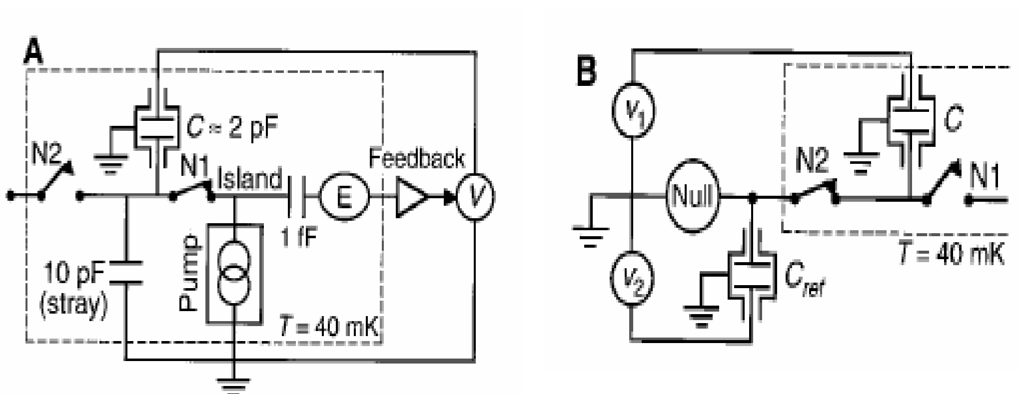
$R_{line} < 1\,m\Omega \Rightarrow$ uncertainty $< 10^{-8}$

Results

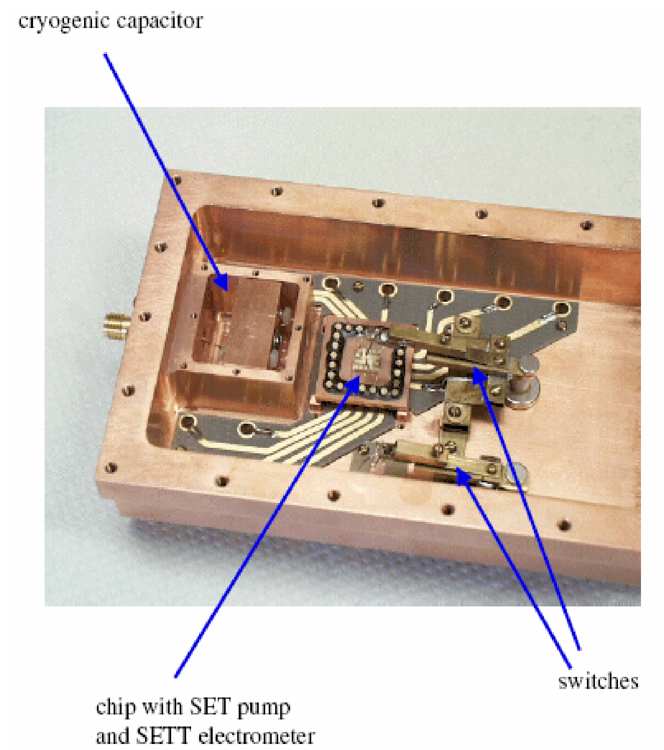
Relative uncertainty of 10^{-6}

prominent uncertainty source:

charge offset noise floor of detector due to its moving charged defects



(Zimmerman, 1999)



(Zimmerman, 2003)

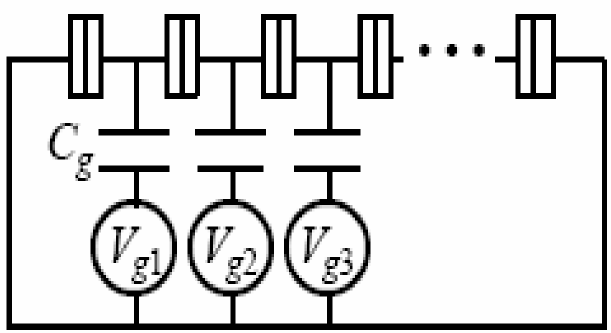
Challenges

- reduce frequency dependence of cryogenic capacitor
- reduce input noise of electrometer
- reduce filter impedance
- reduce the probability of the tunneling induced by out of equilibrium photons from the external circuitry
  - reduce electromagnetic noise
- run thorough analyses of the system including all uncertainties, noise sources, parasitic & co-tunneling effects

Calculation of Capacitance

Apparatus

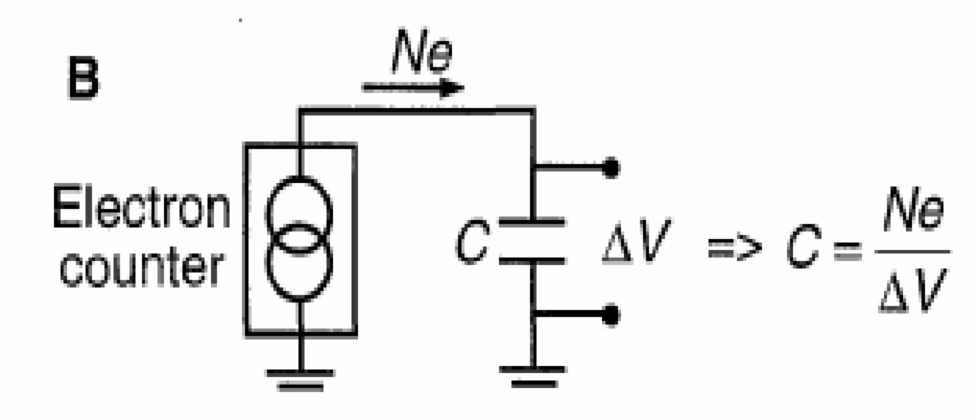
- In order to accurately define capacitance, we need the following:
  - An accurate pumping mechanism (7 Junction electron pump)
  - A close to ideal Cryogenic Capacitor
  - An accurate Electrometer



Keller, 2000

Procedure

- The first step of the process is to pump a known quantity (10^8) of electrons on to the cryogenic capacitor
- The capacitance can be calculated with the formula:



Keller, et al (1999)

Definition of Capacitance

- Capacitance can be defined by the formula:

$$C_{\text{SET}}^{(\text{SI})} = \frac{Ne}{\Delta V^{(\text{SI})}} \qquad C_{\text{SET}}^{(1990)} = \frac{Ne}{\Delta V^{(1990)}}$$

Keller, 2000

- Why are there 2 definitions?
  – The unit for Voltage
- Because of the 2 different standards, there are 2 different methods of defining capacitance
- Each standard has different uncertainties

Standards

- The SI unit system is the authoritative standard for defining units
- The experiments used to define SI units can be cumbersome and are repeated only when needed
- A 1990 agreement created a new set of units (denoted by a 90 in subscript) to improve consistency
- The new units (Voltage, Resistance) have a lot less uncertainty than the SI system units

Why is uncertainty important?

- The difference in values between the SI and the Subscript-90 units can give 2 different theoretical values
- Example:
  - For N = 10^8 and $\{\Delta V\}_{90}$ = 10 the values of capacitance are:

$$C_{\mathrm{SET}}^{(\mathrm{SI})} = 1.602\ 176\ 456(6)\ \mathrm{pF}$$

$$C_{\mathrm{SET}}^{(1990)} = 1.602\ 176\ 491\ 6...\ \mathrm{pF}_{90}$$

Keller (2000)

- Comparison between the two values gives us a relative difference of 2.2 X 10^{-8}

Calculating Capacitance

- The investigators chose to use the subscript-90 units because of the lower uncertainty and consistency
- The formula then becomes: $C = \frac{Ne}{\{\langle \Delta V \rangle\}_{SI} \mathrm{V}}$
- Initial estimates of uncertainty suggest that the combined uncertainty of the result will be no less that 1ppm

Actual Uncertainty

- To calculate the actual uncertainty, we need to define e in terms of more fundamental units

$$e = (4\alpha/\mu_0 c)(1/K_J)$$

- K_J is a fundamental constant that is used in the definition of the voltage (Josephson junction)

$$K_J = 2e/h$$

Keller, et al (1999)

Actual Uncertainty cont.

- Substituting for e in the formula, we get:

$$C = \frac{Ne}{\{\langle \Delta V \rangle\}_{SI} \mathrm{V}} = \frac{N(4\alpha/\mu_0 c)(1/K_J}{\{\langle \Delta V \rangle\}_{90} \mathrm{V}_{90}} = \frac{N(4\alpha/\mu_0 c)}{\{\langle \Delta V \rangle\}_{90} K_{J\text{-}90} \mathrm{V}}$$

- Where:

$$K_{J\text{-}90} \equiv 483\ 597.9\ \mathrm{GHz/V}$$

$$\mu_0 \equiv 4\pi \times 10^{-7}\ \mathrm{N/A^2}$$

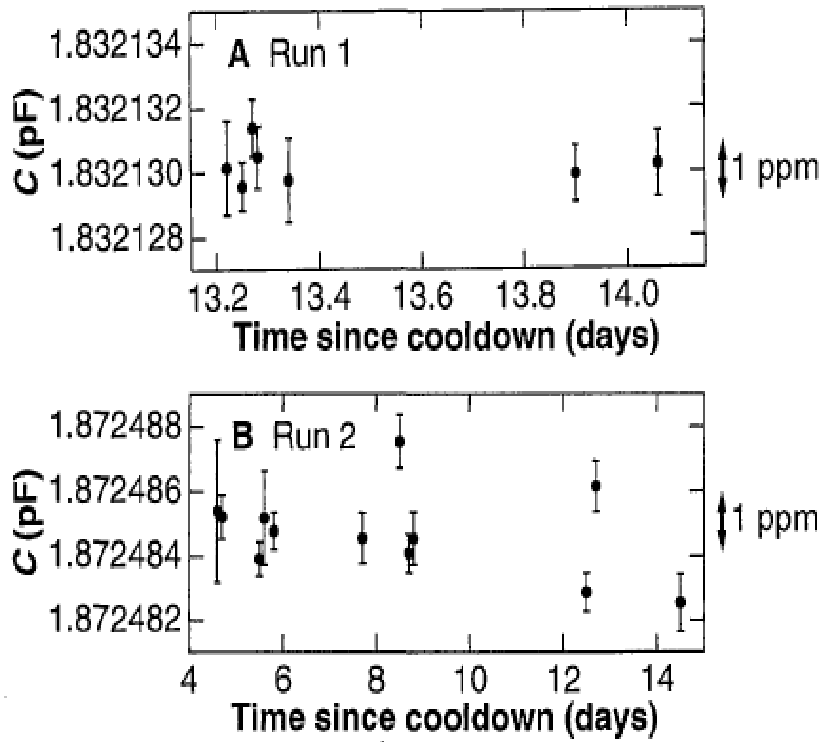
- And $\alpha = 7.29735308 \times 10^{-3}$

Uncertainty cont.

- Based on these fundamental values, the uncertainty of the value of C can be reduced to approx. 0.01 ppm
- This value depends heavily on low uncertainties in the values of N and $\{\langle V\rangle\}_{90}$
- Even though sub-90 values are used, the expression is expressed in SI form

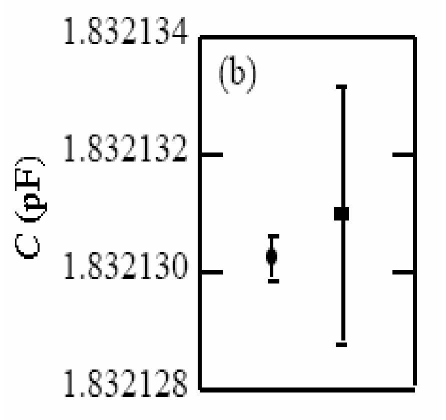
Experimental Results

- Using their 7 junction electron pump, the investigators were able to get the following results:



Comparing to the existing standard

- The following graph shows the comparison between the electron counting (EC) method and NIST primary capacitance standard:



- The results of the experiment show that the EC method gives a more certain value

Applications

- An accurate Capacitance standard can be very useful in a laboratory situation
- The apparatus used by the investigators allows them to tune a capacitor at room temperature (using an AC bridge)

Areas for further progress

- Although the results of this experiment can be used as a new standard for capacitance, there is some room for improvement
- A better cryogenic capacitor and lower uncertainties in the fundamental constants used in the calculations are needed to improve on this standard

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