Capacitance Standard based on Counting Electrons

By:
Zeinab Mousavi
Stephanie Teich-McGoldrick
Jaspreet Wadhwa
SI Electrical Base Units

Electrical Units: m, s, kg, A

– Fundamental Units: same for all times and places
  
  **second**: time taken by 9,192,631,770 cycles of radiation that comes from electrons moving between two energy levels of the caesium-133 atom

  **meter**: wavelength of radiation from a transition in KR atom

– Non-fundamental Units: not constant
  
  **kilogram**: mass of a metal cylinder kept in Paris
**Ampere**: current that when flowing in straight parallel wires of infinite length and negligible cross section, separated by a distance of one meter in free space, produces a force between the wires of 0.2 µ Newton per meter of length (1960)

**volt**: based on electrochemical reactions within chemical cells

**ohm**: based on measuring a wire-wound standard resistor against the impedance of a capacitor at known frequency, drift range of -0.7 to 0.7µΩ/year

need higher-reproducibility fundamental standards based on quantum phenomena
1990 Standards

• Quantum Hall Effect $\rightarrow$ resistance standard

Klaus von Klitzing, 1985 Nobel Prize

2D Hall Effect, Low Temp
resistance varies stepwise with magnetic field
step size does not depend on material properties
step size $= h/(i*e^2)$, $i =$ step number

1 Klitzing: hall resistance at 4$^{th}$ step

(Kosmos, 1986)
1990 Standards

• **Josephson Effect → voltage standard**

  Brian Josephson, 1962
  two superconductors separated by thin insulator
  Cooper pairs tunnel through junction
  applied DC voltage → oscillation of frequency
  1 Volt: voltage required to produce 483,597.9 GHz

\[
 f_j = \frac{2 e \Delta V}{h}
\]
Calculable Capacitor

Parallel Plate Formula: \( C = \frac{\varepsilon_0 A}{d} \) neglects fringing fields effect

Calculable Capacitor:
Special geometry rejects effects of fringing fields capacitance depends on only one length

\[
\frac{\Delta C}{\Delta L} = \varepsilon_0 \ln\left(\frac{2}{\pi}\right)
\]

✓ no uncertainty in relationship
✓ measurement of displacement
✓ can determine impedance

(Zimmerman, 1997)
Electron Counting Capacitance Standard (ECCS)

Capacitance: transfer of charge between two conductors creates potential difference  \[ C = \frac{Q}{\Delta V} \]

- Single Electron Transistor (SET): detect single electron  \[ C = \frac{Ne}{\Delta V} \]

→ Capacitance standard based on quantization of charge  
Use small current to charge a small capacitor to a large voltage in a short time

Three components of SET Capacitance Standard:
1. electrometer
2. 7-junction electron pump
3. cryogenic vacuum-gap capacitor
SET Device

Metal tunnel junction
capacitance $C$, current $I(V)$, bias volt $V$
tunneling $\Rightarrow E_{\text{change}} = eQ/C - e^2/2C$
$R_t$: tunneling resistance

\[ R_t \gg h/e^2 \quad \text{and} \quad kT \ll e^2/2C \quad \Rightarrow \quad \text{Coulomb Blockade} \]
no tunneling for $|V|<e/2C$

$C \approx 0.1\text{fF}$, $e^2/2C \approx 10\text{K}$, Temp $\approx 0.1\text{K}$ (-460F)
SET effects dominate thermal fluctuation

(Fulton, 1987)
SET Electrometer

Two tunnel junctions:
each has capacitance $C$
electrode $a$: island electrically isolated from circuit
Input: potential $U$ coupled to island $a$ thru $C_0$

Lower potential barrier across junction (through $U$)
$\rightarrow$ current passes through
$\rightarrow$ device current linearly proportional to $U$

(Williams, 1992)
Accuracy of electron counting using a 7-junction electron pump
The electron pump

- Based on Coulombic tunneling
- A series of metal islands separated by tunnel junctions
- A gate electrode coupled capacitively to each island
The electron pump

- Changing the gate voltages changes the Coulomb blockade at each junction
- Allows individual electrons to be sent down the chain of islands
- Current -- 1pA
Important as a standard for Capacitance

- A way to count e-
- As a capacitance standard
  - Use electron pump to put e- onto Cyro Cap
  - Stop pumping e- (hold mode)
  - Compare $C_{CRYO}$ with another capacitor at room temp
  - Determine the value of conventional room temp capacitor in terms of e-
  - Room temp capacitor can now be used as a basis for calibrations

Zimmerman et al. Measurement Standards and Technology, 2003
Important as a standard for Capacitance

- Requirements
  - Ability to put $10^8$ e− on 1pF capacitor
  - Uncertainty $\pm 1$ e− (10 ppb)
  - Small leakage current when off
Design

• Minimize error
• Maximize Coulomb Blockade
  - Small junctions to reduce junction capacitance
  - Small islands to reduce self-capacitance
  - Low k constant substrate to reduce stray capacitance
  - Minimize cross-capacitance
Fabrication

- Two-angle evaporation of Al
- PMMA bi-layer mask patterned with EBL
Circuit Design

- Pumps connected to external island
- Electrometer (based on tunnel junction)
- Switch
  - Closed to obtain I/V curve of pump
  - Open to detect pumped e-

All components except the switch were fabricated on a single chip
Circuit Design cont.

- Plot of voltage on external island ($V_P$) vs time
- 1 e- repeatedly pumped on/off island
- Jump of $7.6eV = 1e-$ on/off
Accuracy of electron pump

- Pumping 1 e- on/off external island
- Measure $V_p$ vs. time
- Pumping rate faster than can measure
Accuracy of electron pump

- Pumping rate = 5.05 MHz
- Avg error rate 1 error/13s
- Error per pumped e-, 15 ppb
Leakage Rate

- Hold mode (e- pumped onto islands and gate pulses turned off)
- Each jump in figure is 1e- leaking through the pump
Leakage Rate

- Hold mode (e- pumped onto islands and gate pulses turned off)
- Each jump in figure is 1e- leaking through the pump

Avg. hold time 600s
Pumping speed errors

- $t_P = \text{pump time}$
- $t_W = \text{wait time}$
- $\frac{1}{(t_P + t_W)} = \text{overall pump time}$
Pumping speed errors

- Time each junction is activated must be long compared to RC to avoid errors due to missed tunneling events. 
  Increases at small $t_p$ 
  Decreases at large $t_p$

- $\varepsilon_{th} = \exp(-at_P/RC)$
Pumping speed errors

- Inset graph
- Error rate as pump rate is varied holding $t_P$ constant and varying $t_W$
- Overall pump rate can be adjusted without
Temperature dependence

- At high T both the error and leakage rate increase exponentially.
- At low temp both are independent of $T_{MC}$.
- At low temp a temp-independent error mechanism dominates.
- Photon-assisted cotunneling
  - Noise from the slow relaxation of charges trapped in metastable states.

\[
\varepsilon_{th} = b \exp(-\Delta E_p / k_B T) \\
\Gamma_{th} = \frac{d}{RC} \exp(-\Delta E_h / k_B T)
\]
ECCS Operation

Pump Phase: charge capacitor $C_{\text{cryo}}$

electrometer: monitor potential $a$

electron pump: current source

$C_{\text{cryo}}$: standard capacitor

- charge $C_{\text{cryo}}$ with $N$ electrons
- measure avg. $V$ across $C_{\text{cryo}}$
- determine $C_{\text{cryo}}$

(Zimmerman, 2003)
ECCS Operation

Why null detector/feedback?

To avoid errors, voltage across pump must be kept zero

Charge from pump appears across $C_{\text{cryo}}$, not $C_{\text{strand}}$

(Williams, 1992)

(Zimmerman, 1997)
ECCS Operation

Pump at 6.2 MHz for 10s, $V_{\text{feed}} = 10$

$I = e \cdot f \rightarrow I = 9.9 \times 10^{-13}$ (A)

$Q = I \cdot t \rightarrow Q \approx 10 \times 10^{-12}$ (C)

$C = Q / V \rightarrow C \approx 1 \text{pF}$ (F)

In practice, repeat 10-100 times

Pump $\approx 10^8$ electrons, $V_{\text{feed}} = 10$

$C = N_e / \langle V_{\text{feed}} \rangle \rightarrow C \approx 1.8 \text{pF}$

(Williams, 1992)

(Zimmerman, 1997)
ECCS Operation

Bridge Phase: Compare with $C_{\text{stand}}$ at room temp

• grounded shield $V_1$, $V_2$ affect detector through $C_{\text{cryo}}$, $C_{\text{stand}}$
• balanced bridge: no $Q_b$ at balance point
• adjust $V_1$, $V_2$ to balance bridge: $V_1*C_{\text{cryo}}=V_2*C_{\text{stand}}$

(Zimmerman, 1997)

In practice, single source and voltage divider
Capacitance Requirements

• Pump electrons onto cap
  – stable
  – low loss
• Measure voltage across cap for several seconds
  – hold number of electrons fixed
  – leakage resistance of $10^{22} \, \Omega$
• Compare to standard cap at room temp & audio freq
  – stable for minutes or hours
  – low frequency dependence
    Cryogenic cap: no freq & volt dependence
      parallel resistance $10^{21} \, \Omega$
Stray Capacitance

low temp, metal enclosure, spaced closely \(\rightarrow\) lower \(C_{stray}\)
perfect null detector: \(V_{feed} = \frac{Q_p}{C_{cryo}}\)

\[
V_{feed} - \frac{Q_P}{C_{cryo}} = \frac{C_{stray} + C_C + C_{cryo}}{C_{cryo}} \frac{\partial Q_{SET}}{C_C}
\]

\[
V_{feed} - \frac{Q_P}{C_{cryo}} = \frac{C'_{stray} + C_{cryo}}{C_{cryo}} \delta V_0
\]
Line Impedance

R_$\text{filt}$: discrete filters used to shield SET from hi-freq noise
R_$\text{line}$: disturbed line resistance
Line impedance $\rightarrow$ uncertainties
  - applied volt to $C_{\text{cryo}}$ and $C_{\text{stand}}$ different than $V_1$, $V_2$
  \[ \omega = 10^4 \text{ Hz}, \text{ uncertainty} < 10^{-8} \]
  $\rightarrow$ R$_{\text{filt}} < 1\Omega$, R$_{\text{line}} < 0.01\Omega$
  $\rightarrow$ use filters special for SET
  (refer to Vion, 1994)
\[ Q_p = Q_{core} + Q_{stray} + Q_c \]
\[ Q_{core} = Q_p - (Q_{stray} + Q_c) \]
\[ \frac{Q_{core}}{C_{core}} = \frac{Q_p}{C_{core}} - \frac{(Q_{stray} + Q_c)}{C_{core}} \]
\[ \frac{Q_{core}}{C_{core}} = \frac{Q_p}{C_{core}} - \frac{(Q_{stray} + Q_c) V_0}{C_{core}} \]

**Notes:**
- **Left-hand loop:**
  \[ V_{eff} = \frac{Q_{core}}{C_{core}} - V_0 \]
  \[ V_{eff} = \frac{Q_{core}}{C_{core}} - V_0 \]
  \[ V_{eff} - \frac{Q_{core}}{C_{core}} = V_0 \]
  \[ V_{eff} - \frac{Q_{core}}{C_{core}} = \frac{(Q_{stray} + Q_c) V_0}{C_{core}} \]
  \[ \left| V_{eff} - \frac{Q_p}{C_{core}} \right| = \frac{(Q_{stray} + Q_c) V_0}{C_{core}} + \left| V_0 \right| \]

(minimal resolvable charge on island \( Q_{瑟} = 5.0 \times 10^{-5} \))

\[ V_{eff} - \frac{Q_p}{C_{core}} \geq \frac{C_{stray} + C_{core} + C_{瑟}}{C_{core}} \]

**Similar Analysis for Conventional Electromagnets:**
\[ \left| V_{eff} - \frac{Q_p}{C_{core}} \right| = \frac{C_{stray} + C_{core} + C_{瑟}}{C_{core}} \]

**Notes:**
- **Top loop:**
  \[ I = \frac{V_{eff}}{E_{core}} = \frac{V_{eff}}{V_0 + \sigma C_{core}} \]
  \[ \frac{V_0 - V_1}{R_{in1}} = \frac{C_{core}}{u_0} \cdot V_0 \]
  \[ \frac{V_1 - V_{eff} + V_1}{V_{eff} + V_1} = \frac{R_{in1} C_{core}}{V_0} \]
  \[ R_{in1} < 1 \Omega \Rightarrow \text{uncertainty} < 10^{-8} \]

Similarly, for lower loop:
\[ \frac{V_0 - V_{eff} + V_1}{V_{eff} + V_1} = \frac{R_{in2} C_{core}}{V_0} \]
\[ R_{in2} < 1 \Omega \Rightarrow \text{uncertainty} < 10^{-8} \]

For resistance in the \( U_1n \) to the null detector:
\[ I = \frac{V_0}{E_{stray}} = V_0 \cdot \sigma C_{stray} \]
\[ \Rightarrow \left| \frac{V_0 - V_0}{V_0} \right| = \frac{R_{in1} C_{stray}}{V_0} \]
\[ R_{in1} < 1 \Omega \Rightarrow \text{uncertainty} < 10^{-8} \]
Results

Relative uncertainty of $10^{-6}$

Prominent uncertainty source: charge offset noise floor of detector due to its moving charged defects

(Zimmerman, 1999)
Challenges

• reduce frequency dependence of cryogenic capacitor
• reduce input noise of electrometer
• reduce filter impedance
• reduce the probability of the tunneling induced by out of equilibrium photons from the external circuitry
  – reduce electromagnetic noise
• run thorough analyses of the system including all uncertainties, noise sources, parasitic & co-tunneling effects
Calculation of Capacitance
Apparatus

In order to accurately define capacitance, we need the following:

- An accurate pumping mechanism (7 Junction electron pump)
- A close to ideal Cryogenic Capacitor
- An accurate Electrometer

Keller, 2000
Procedure

• The first step of the process is to pump a known quantity ($10^8$) of electrons on to the cryogenic capacitor

• The capacitance can be calculated with the formula:

\[ C = \frac{N_e}{\Delta V} \]

Definition of Capacitance

• Capacitance can be defined by the formula:

\[ C_{SET}^{(\text{SI})} = \frac{N_e}{\Delta V^{(\text{SI})}} \quad C_{SET}^{(1990)} = \frac{N_e}{\Delta V^{(1990)}} \]

• Why are there 2 definitions?
  – The unit for Voltage
• Because of the 2 different standards, there are 2 different methods of defining capacitance
• Each standard has different uncertainties

Keller, 2000
Standards

• The SI unit system is the authoritative standard for defining units.
• The experiments used to define SI units can be cumbersome and are repeated only when needed.
• A 1990 agreement created a new set of units (denoted by a 90 in subscript) to improve consistency.
• The new units (Voltage, Resistance) have a lot less uncertainty than the SI system units.
Why is uncertainty important?

• The difference in values between the SI and the Subscript-90 units can give 2 different theoretical values

• Example:
  – For $N = 10^8$ and $\{\Delta V\}_90 = 10$ the values of capacitance are:
    
    $C^{\text{SI}}_{\text{SET}} = 1.602\,176\,456(6) \, \text{pF}$
    
    $C^{(1990)}_{\text{SET}} = 1.602\,176\,491\,6... \, \text{pF}_90$

• Comparison between the two values gives us a relative difference of $2.2 \times 10^{-8}$

Keller (2000)
Calculating Capacitance

• The investigators chose to use the subscript-90 units because of the lower uncertainty and consistency.

• The formula then becomes:

$$C = \frac{Ne}{\langle\Delta V\rangle_{st\,V}}$$

• Initial estimates of uncertainty suggest that the combined uncertainty of the result will be no less that 1ppm.
Actual Uncertainty

• To calculate the actual uncertainty, we need to define $e$ in terms of more fundamental units

$$ e = \left(\frac{4\alpha}{\mu_0 c}\right)\left(1/K_J\right) $$

• $K_J$ is a fundamental constant that is used in the definition of the voltage (Josephson junction)

$$ K_J = \frac{2e}{\hbar} $$

Actual Uncertainty cont.

- Substituting for $e$ in the formula, we get:

$$C = \frac{N e}{\langle \Delta V \rangle_{SI} V} = \frac{N(4\alpha/\mu_0 c)(1/K_J)}{\langle \Delta V \rangle_{90} V_{90}} = \frac{N(4\alpha/\mu_0 c)}{\langle \Delta V \rangle_{90} K_{J,90} V}$$

- Where:

$$K_{J,90} \equiv 483\,597.9 \text{ GHz/V}$$

$$\mu_0 \equiv 4\pi \times 10^{-7} \text{ N/A}^2$$

- And $\alpha = 7.29735308 \times 10^{-3}$
Uncertainty cont.

• Based on these fundamental values, the uncertainty of the value of C can be reduced to approx. 0.01 ppm

• This value depends heavily on low uncertainties in the values of N and \{<V>\}_90

• Even though sub-90 values are used, the expression is expressed in SI form
Experimental Results

• Using their 7 junction electron pump, the investigators were able to get the following results:
Comparing to the existing standard

• The following graph shows the comparison between the electron counting (EC) method and NIST primary capacitance standard:

• The results of the experiment show that the EC method gives a more certain value
Applications

• An accurate Capacitance standard can be very useful in a laboratory situation

• The apparatus used by the investigators allows them to tune a capacitor at room temperature (using an AC bridge)
Areas for further progress

• Although the results of this experiment can be used as a new standard for capacitance, there is some room for improvement
• A better cryogenic capacitor and lower uncertainties in the fundamental constants used in the calculations are needed to improve on this standard
References