Quantum Theory of Light
EECS 638/PHYS542/AP609
Midterm Exam

Instructor: Professor S.C. Rand
Date: March 6, 2001
Duration: 1.5 hours

Useful Formulae:

Vector model evolution matrices for $\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$ in the case of

(i) Nutation:

$$\begin{bmatrix} \frac{\chi^2 + \Delta^2 \cos\theta}{\Omega^2} & \frac{-\Delta \sin\theta}{\Omega} & \frac{-\Delta\chi}{\Omega^2}(1-\cos\theta) \\ \frac{\Delta}{\Omega}\sin\theta & \cos\theta & \frac{\chi}{\Omega_R}\sin\theta \\ -\frac{\Delta\chi}{\Omega}(1-\cos\theta) & -\frac{\chi}{\Omega_R}\sin\theta & \frac{\Delta^2 + \chi^2 \cos\theta}{\Omega^2} \end{bmatrix}; \ \theta = \chi\tau; \ \chi = \frac{\mu E}{\hbar}; \ \Omega_R = \sqrt{\Delta^2 + \chi^2}$$

(ii) free induction decay:

$$\begin{bmatrix} e^{-\frac{(t-t_0)}{T_2}} \cos\Delta(t-t_0) & -e^{-\frac{(t-t_0)}{T_2}} \sin\Delta(t-t_0) & 0 \\ e^{-\frac{(t-t_0)}{T_2}} \sin\Delta(t-t_0) & e^{-\frac{(t-t_0)}{T_2}} \cos\Delta(t-t_0) & 0 \\ 0 & 0 & e^{-\frac{(t-t_0)}{T_1}} \end{bmatrix}; \ \Delta = \omega_0 - \omega$$

$$R_1 = \tilde{\rho}_{21} + \tilde{\rho}_{12}$$
$$R_2 = i(\tilde{\rho}_{21} - \tilde{\rho}_{12})$$
$$R_3 = \rho_{22} - \rho_{11}$$

Name____

Signature____

1. **(25 marks total)**

(a) Write down the equation of motion for the density matrix of a closed 2-level system, including a formal term to represent decay. **(1 mark)**

(b) Write out the equations for the time dependence of ρ_{11}, ρ_{22}, ρ_{12} and ρ_{21} assuming that a light field $E(t) = \frac{1}{2} E_0 e^{i\omega t} + c.c.$ couples ground level |1> to excited state |2> via a transition dipole moment μ, and that de-excitation of level |2> occurs by spontaneous decay. **(8 marks)**

(c) Assume in the case of resonant optical excitation that steady-state, oscillatory solutions for the level populations exist, of the form

$$\rho_{11} = \tilde{\rho}_{11}(1 - \cos\xi t)$$
$$\rho_{22} = \tilde{\rho}_{22}(1 + \cos\xi t)$$

Use closure to find consistent values for $\tilde{\rho}_{11}$ and $\tilde{\rho}_{22}$. (Note: Do not try to derive the population expressions above - take them to be given). **(4 marks)**

(d) Find the slowly varying amplitude $\tilde{\rho}_{12}$ of the polarization $\rho_{12} = \tilde{\rho}_{12} e^{i\omega t}$. **(7 marks)**

(e) What would you expect the value of ξ to be, and why does $\tilde{\rho}_{12}$ periodically go to zero (what is the physical reason)? **(4 marks)**

(f) Does the absorption of the system go to zero periodically (show and explain)? **(1 marks)**

2. **(25 marks total)**

(a) Give a general analytic expression for the macroscopic, radiative polarization P inside a 2-level system in terms of the trace of relevant operators and then evaluate it in terms of specific matrix elements assuming there is an allowed transition with a transition dipole moment μ between the two levels. **(3 marks)**

(b) Find the Bloch vector of a 2-level system, initially in the ground state, that is irradiated by a resonant π-pulse of short duration $\tau_p=\tau$ ($\tau<<T_1$, $\tau>>T_2$). Take the time of observation to be twice the pulse duration ($t=2\tau$). **(7 marks)**

(c) Calculate and explain the value of polarization P observed at time $t=2\tau$? **(3 marks)**

(d) Draw the Bloch vector in a suitable coordinate system at times $t=1.5\tau$ and $t=2\tau$.**(2 marks)**

(e) What is the excited state population at $t=2\tau$? **(2 marks)**

(f) What is the sign of the absorption at $t=2\tau$? Would you expect a probe pulse to experience loss (positive absorption) or gain (negative absorption)? **(2 mark)**

(g) If a second pulse, a $\frac{\pi}{2}$ pulse, is applied at time $t=2\tau$ ($\tau<<T_1$, $\tau>>T_2$), what is the polarization at the end of the second pulse? **(3 marks)**

(h) Assuming the system to be inhomogeneously broadened, would you expect an echo at time $t=4\tau$? Why or why not? **(3 marks)**