QUANTUM THEORY OF LIGHT
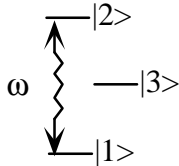## EECS 638/PHYS 542/AP609
MIDTERM EXAM #2

Instructor: Professor S. Rand
Date: April 17, 1999

1. **(20 marks total)**

   (a) Write down the equation of motion for the density matrix ρ of a closed 3-level system, including a formal term to represent decay. **(1 mark)**

   (b) Write out the equations for the time dependence of ρ_{11}, ρ_{22}, ρ_{12} and ρ_{21} assuming that a light field interaction $V = -\mu_{12}Ee^{i\omega t} + c.c.$ couples only levels $|1\rangle$ and $|2\rangle$, and that the excited levels exhibit spontaneous decay. **(8 marks)**

   (c) Under steady-state conditions, find the ratio of populations in states $|2\rangle$ and $|3\rangle$. **(3 marks)**

   (d) Find the slowly varying amplitude $\tilde{\rho}_{12}$ of the polarization $\rho_{12} = \tilde{\rho}_{12}e^{i\omega t}$. **(6 marks)**

   (e) In the limiting case that no decay is possible from intermediate level $|3\rangle$, what is the ground state population $\rho_{11}(\infty)$? **(2 marks)**

2. **(15 marks total)** In a 2-level atom system subjected to a strong field, dressed states are created in the form

   $|D_1(n)\rangle = \sin\theta\,|1,n\rangle + \cos\theta\,|2,n\text{-}1\rangle$
   $|D_2(n)\rangle = \cos\theta\,|1,n\rangle - \sin\theta\,|2,n\text{-}1\rangle$

   where $\sin 2\theta = \dfrac{2|g|\sqrt{n}}{\Omega}$, $\cos 2\theta = \dfrac{\Delta}{\Omega}$, $\Omega = \sqrt{\Delta^2 + 4|g|^2 n}$ and g is related to the electric field amplitude.

   (a) What is a suitable operator representation for the transition dipole moment $\hat{\mu}$ of the atom? **(3 marks)**

   (b) Calculate $< D_1(n)\,|\,\hat{\mu}\,|\,D_2(n+1) >$. **(6 marks)**

   (c) Can the dressed atom emit or absorb light on the transition between $|D_1(n)\rangle$ and $|D_2(n)\rangle$? Justify your answer. **(6 marks)**