1. (20 marks total)

(a) Write down the equation of motion for the density matrix $\rho$ of a closed 3-level system, including a formal term to represent decay. (1 mark)

$$
\omega
\begin{array}{c}
|2> \\
|3> \\
|1>
\end{array}
$$

(b) Write out the equations for the time dependence of $\rho_{11}$, $\rho_{22}$, $\rho_{12}$ and $\rho_{21}$ assuming that a light field interaction $V = -\mu_{12}E e^{i\omega t} + c.c.$ couples only levels $|1>$ and $|2>$, and that the excited levels exhibit spontaneous decay. (8 marks)

(c) Under steady-state conditions, find the ratio of populations in states $|2>$ and $|3>$. (3 marks)

(d) Find the slowly varying amplitude $\tilde{\rho}_{12}$ of the polarization $\rho_{12} = \tilde{\rho}_{12} e^{i\omega t}$. (6 marks)

(e) In the limiting case that no decay is possible from intermediate level $|3>$, what is the ground state population $\rho_{11}(\infty)$? (2 marks)

2. (15 marks total) In a 2-level atom system subjected to a strong field, dressed states are created in the form

$$
|D_1(n)> = \sin \theta |1,n> + \cos \theta |2,n-1>
|D_2(n)> = \cos \theta |1,n> - \sin \theta |2,n-1>
$$

where $\sin^2 \theta = 2|g|\sqrt{n}/\Omega$, $\cos^2 \theta = \Delta/\Omega$, $\Omega = \sqrt{\Delta^2 + 4|g|^2 n}$ and $g$ is related to the electric field amplitude.

(a) What is a suitable operator representation for the transition dipole moment $\hat{\mu}$ of the atom? (3 marks)

(b) Calculate $< D_1(n) | \hat{\mu} | D_2(n+1)>$. (6 marks)

(c) Can the dressed atom emit or absorb light on the transition between $|D_1(n)>$ and $|D_2(n)>$? Justify your answer. (6 marks)