Homework Set 6 **EECS 651** **Due: Wednesday, March 21, 2007**

Due Wednesday in class. There is a grace period until Thursday 9 AM.

1. A DPCM system with 1st-order variable-length coding is to be designed for a stationary autogressive Gaussian source with mean zero, variance 1, correlation coefficient .9. The system should have rate approximately 4 and MSE as small as possible.

   (a) Specify the predictor and the quantizer. You may use the high resolution approximations discussed in class and the notes.

   (b) Find the MSE. You may use the approximate high resolution analysis discussed in class and in the notes.

2. (a) Design and describe a DPCM encoder with fixed-length encoding with rate 3 and with a linear predictor of order two, for a zero-mean, stationary Gaussian source with autocorrelation function:

$$R_X(k) = \frac{-128}{105}\,\frac{1}{4^{|k|}} + \frac{64}{21}\,\frac{1}{2^{|k|}} .$$

   The distortion should be as small as you can make it. Find the resulting distortion. Hints: In designing the code, you may use the assumptions we used in our high resolution analysis, and you may use the scalar quantization tables.

   (b) Show that a DPCM code with a higher order predictor would do no better. For this part, you may assume that the rate is large.

   (c) Find, approximately, the OPTA function assuming no restriction on the order of the linear predictor.

3. A discrete-time, stationary Gaussian random process has zero mean and spectral density

$$S(\omega) = 17 - 15 \cos 2\omega .$$

   (a) Find the autocorrelation function $R(k)$ of this process. (Hint: One can do this problem without using the inverse Fourier transform formula.

   (b) Find the minimum mean-squared error for the best linear predictor for X_i from all past symbols.

4. Let $\underline{X} = (X_1,...,X_k)$ be a real-valued zero-mean, random vector with covariance matrix $K = [K_{i,j}]$. Show that if K is singular, then there exists some i such that X_i is a linear combination of the other random variables in the sense that $E[X_i - \Sigma_{j\neq i} b_j X_j]^2 = 0$ from some choice of $b_1,...,b_k$. Hint: As one of the steps, you may wish to show that if $\underline{K}_i = \Sigma_{j\neq i} b_j \underline{K}_j$ for some i and some choice of coefficients b_j, $j = 1,...,k, j\neq i$, then $E[X_i - \Sigma_{j\neq i} b_j X_j]^2 = 0$.

5. A continuation of Problem 4, Homework 5. CD recordings are made with a uniform scalar quantizer with fixed-length coding and rate 16 bits/sample. Assuming that music samples are Gaussian with variance σ^2 and assuming that the level spacing Δ is chosen optimally for this source model, in HW 4, Problem 2 we estimated the SNR to be 84.6 dB, and we found that SNR would be 94.8 dB with optimal scalar quantization with first-order variable-length coding.

   Now suppose further that the source is modelled as stationary, Gaussian and Markov with correlation coefficient 0.9. Estimate the SNR that would be attained by

   (a) DPCM with fixed-length coding

   (b) DPCM with variable-length coding.

   (c) The answer to those of Problem 4, Homework 5.

6. (a) Design a 2-dimensional (fixed-rate) transform code with rate $R = 1.5$ bit/sample for a first-order Gaussian autoregressive source with mean 0**,** variance 1 and correlation coefficient $\rho = .9$. The code should have as small MSE as possible. You need to specify the transform and the scalar quantizers, as well as give an overall block diagram or other complete description of the encoder and decoder. Do not use high resolution analysis in the design of the scalar quantizers. Assume the scalar quantizers have integer rates. You will need to use the scalar quantization tables included with Homework 2.

   (b) Compute the MSE of this code.

   (c) Compare the MSE to that of an optimal fixed-rate scalar quantizer for this source with as nearly the same rate as possible. (Assume the rate is $\log_2 M$, where M is the size of the quantizer.)

7. In this problem you will experiment with Matlab.

   From the homework auxiliary files webpage, download the Matlab script dpcm_template.m and the sound file energy.wav .

   The Matlab script is the template of a program that implements DPCM (with fixed-length binary encoding) of the sound file energy.wav, computes MSE, plots a segment of the original and encoded signals, and replays the original and the encoded signals. It is missing only the specification of the scalar quantizer and the predictor. Look for the question marks, which you must replace with something suitable.

   (a) Find a first-order linear predictor and a quantizer with four levels that results in as small MSE as possible when encoding the first two seconds of the sound file. You need not get the absolutely smallest MSE, but you need to have a reasonable methodology. For example it suffices to follow the design methodology discussed in class and the notes. Extra credit will be given to whoever gets the smallest MSE.

   In your homework paper: Explain your design methodology. Give the specification of your filter and your quantizer. State the resulting SNR. Comment on the quality of the DPCM encoded sound and the appearance of the plotted reconstructed signal. Attach your Matlab script to the end of your homework paper.

   (b) Find a first-order linear predictor and a quantizer that attains SNR 20 dB or more with as small a rate as possible. You need not find the absolutely smallest possible rate, but you need to have a reasonable design methodology. Stick to quantizers with number of levels equal to a power of 2. You may use any of Matlab's design tools, but you need not use them. Extra credit will be given to whoever gets the smallest rate.

   In your homework paper: Explain your design methodology. Give the specification of your filter and your quantizer. State the resulting rate and SNR. Comment on the quality of the DPCM encoded sound and the appearance of the plotted reconstructed signal. Attach your Matlab script to the end of your homework paper.