Homework Set 7 **EECS 651** **Due: Monday, March 26, 2007**

Due Monday in class. There is a grace period until Tuesday 9 AM.

1. Suppose a high-rate 2-dimensional transform code is to be designed for a Gaussian random vector $\underline{X} = (X_1,X_2)$ with zero means and covariance matrix $K = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$

   using an orthogonal transform of the form $T = \begin{bmatrix} a & \sqrt{1-a^2} \\ \sqrt{1-a^2} & -a \end{bmatrix}$.

   (a) Find the covariance matrix of $\underline{U} = T\underline{X}$.

   (b) Find the value of a that optimizes this transform for use with high-rate transform coding and either fixed-length for variable-length coding.

   (c) With the optimized value of a found in (b), with scalar quantizers optimized for the resulting transform, and for some large rate R, find the gain of this system in dB over direct scalar quantization of $\underline{X}$ with the same rate. (Assume that either the transform code and scalar quantizer both use fixed-length coding, or both use variable-length coding.)

2. This problem is concerned with designing fixed-rate transform coders with a different criteria, instead of designing for minimum MSE.

   Assume the source is stationary and Gaussian with zero mean. Consider k-dimensional transform coding based on an orthogonal transform T. Let $\sigma_1^2 > ... > \sigma_k^2$ be the variance of the transform coefficients. Assume that a large target rate R is specified and that fixed-length coding is used.

   (a) Choose $R_1,...,R_k$ such that $\frac{1}{k}\sum_{i=1}^{k} R_i = R$ and σ_i^2/D_i is the same for all i and the overall MSE D is as small as possible subject to these constraints. Find the value of D.

   (Note: This criteria demands that each coefficient be quantized with the same SNR. It might be appropriate when permissible quantization noise is proportional to coefficient variance. For example, if T is a discrete Fourier transform then our ability to tolerate noise in a Fourier coefficient is roughly proportional to the variance of the coefficient. So this criteria leads to a perceptually designed transform code, rather than a transform code designed to have minimum MSE.)

   (b) Compare D to the MSE of optimum scalar quantization with rate D.

3. Assume R is large. A k-dimensional transform code is optimized for rate R, variable-length coding, and a zero-mean, stationary Gaussian source with covariance matrix K. (As usual, the variable-length coding of each coefficient is first order.) In this problem we wish to show that its point density is the same as that for optimal k-dimensional vector quantization designed for first-order variable-length coding.

   (a) Show that the point density of optimal k-dimensional vector quantization designed for first-order variable-length coding is

   $$\Lambda^*(\underline{x}) = \frac{2^{kR}}{(2\pi e)^{k/2}|K|^{1/2}}$$

   (b) Show that when the transform and the bit allocation are optimized, the point density for the ith scalar quantizer of the transform code is

   $$\Lambda_i(u) = 2^R \frac{\sigma_i}{(2\pi e)^{1/2}|K|^{1/k}} \quad \text{where } \sigma_i^2 \text{ is the variance of the ith transform coefficient.}$$

   (c) Find an expression for the point density $\Lambda_{pr}(\underline{u})$ of the product quantizer made up of the k scalar quantizers.

   (d) Find an expression for the point density $\Lambda_{tr}(\underline{x})$ of the transform code, and show that it is the same as point density in part (a). Hint: Problem 8 of Homework 6 might be useful.

   (e) Find the inertial profile of the transform code.