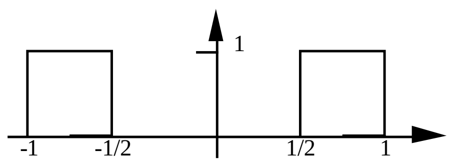
Homework 2 **EECS 651** **Due: Fri., Jan. 19, 2001**
in class, before lecture begins

1. Consider three points in two-dimensional space that are not colinear. Show that the perpendicular bisectors between each pair of points meet at one point.

2. Suppose $C = \{\underline{w}_1, \underline{w}_2\}$ is a minimum MSE k-dimensional VQ with two codevectors for random vector $\underline{X} = (X_1,\ldots,X_k)$. Show that there exists numbers a and b such that $E\underline{X} = a\,\underline{w}_1 + b\,\underline{w}_2$; $E\underline{X}$ is located on a line between $\underline{w}_1$ and $\underline{w}_2$. Give the values of a and b.

3. A vector quantizer must be designed to quantize at rate 2 bits/sample a source that emits 1000 samples/sec. The device available for encoding can perform 10^6 arithmetic operations per second (floating point or otherwise) and has storage for 100,000 floating point numbers. Find the largest dimension that the VQ can use. (Don't worry about decoding.)

4. (a) Find as many 3-level scalar quantizers (M=3) as possible that satisfy the two optimality properties for the density $f_X(x)$ shown below.



(b) Which of the quantizers is optimal? What is its MSE?

(c) Find the best 3-level "symmetric" quantizer whose levels and thresholds are symmetric about the origin. (The answer to this should convince you that the best quantizers for symmetric densities are not always symmetric. But what if the number of levels is even. Must the best quantizer with an even number of levels be symmetric for a symmetric density? I'll leave this as an open question for you to think about.)

5. This problem should be done analytically without using a computer.

(a) Find a scalar quantizer with three levels that satisfies the optimality critera for the Laplacian pdf

$$p(x) = \frac{1}{2}\, e^{-|x|} .$$

If you can find more than one, choose the one with smallest MSE.

(b) Find the resulting MSE. and SNR (in dB).

6. The scalar quantizer shown below is optimum for a random variable X with pfd $f_X(x)$. The probabilities of the three levels are .5, .3 and .2 . The MSE is $D = .2$.

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---------×--------------|-------------×---------|--------×--------
        -2            -.5             1         2        3
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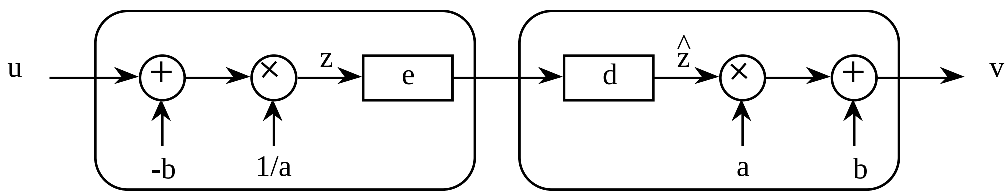
Find EX, $E\,XQ(X)$ and $\mathrm{var}(X)$.

7. In this problem we show how a quantizer designed for a random variable X can be modified to obtain a quantizer for a related random variable. Suppose a scalar quantizer, called Quantizer A, is designed for random variable X, with pdf $f_X(x)$. It has size M, thresholds $t_1,\ldots,t_{M-1}$, levels $w_1,\ldots,w_M$, binary codewords $\underline{c}_1,\ldots,\underline{c}_M$, quantization rule Q, encoding rule e, and decoding rule d. It could be nonuniform, and it does not have to be optimal any sense. And suppose we need a quantizer for random variable U, whose density $f_U(u)$ is related to that of X via

$$f_U(u) = \frac{1}{|a|} f_X\left(\frac{u-b}{a}\right), \text{ where } a \neq 0$$

For example, this would be the case if $U = aX + b$.

Consider the Quantizer B shown below which precedes the encoder e with an addition and a multiplication and follows the decoder d with a multiplication and an addition.



encoder e' of Quantizer B　　　decoder d' of Quantizer B

(a) For Quantizer B, find the size M', thresholds $t'_1,\ldots,t'_{M'-1}$, levels $w'_1,\ldots,w'_{M'}$, binary codewords $\underline{c}'_1,\ldots,\underline{c}'_{M'}$, quantization rule Q', encoding rule e', and decoding rule d' in terms of a, b and the corresponding parameters or functions of Quantizer 1. (It might help to draw yourself an example of Quantizer A and Quantizer B.)

(b) Show that the input to e is a random variable Z with the same pdf as X. (You may need to remind yourself of how to find the pdf of one random variable that is a function of another.)

(c) Show the MSE, denoted $D_{B,U}$, of Quantizer B operating on U is related to $D_{A,X}$, the MSE of Quantizer A operating on X, via

$$D_{B,U} = a^2 D_{A,X} .$$

8. Let C_k and C_m be vq codebooks with rate R and dimensions k and m, respectively. Let $\{X_i\}$ be a stationary source, and let D_k and D_m denote their MSE distortions when used with their respective Voronoi partitions. Let

$$C = C_k \times C_m = \left\{(x_1,\ldots,x_{k+m}) : (x_1,\ldots,x_k) \in C_k \text{ and } (x_{k+1},\ldots,x_{k+m})\right\}$$

C is said to be the *product* of C_k and C_m. Alternatively, C is said to be a *product quantizer*.

(a) Find the dimension, size and rate of the VQ with codebook C.

(b) Show that the MSE distortion $D(C)$ of C, when used with its Voronoi partition on the given source, satisfies

$$D = \frac{k}{k+m} D_k + \frac{m}{k+m} D_m$$