Homework 4 **EECS 651** **Due: Friday., Feb. 9, 2001**
in class, before lecture begins

1. In this problem, you will estimate the SNR of an audio compact disc (CD) and compare the result to that attainable by if other quantization methods were used. Here's what you need to know. Audio CD's are encoded with a fixed-rate, uniform scalar quantizer (USQ) with rate 16. Let [-L,L] denote the support region of the uniform scalar quantizer. Assume the audio source is (first-order) autogressive, stationary and Gaussian with zero mean, variance σ^2, and correlation coefficient $\rho=0.9$

   (a) Assuming $L = 10\sigma$ (I'm guessing this is reasonable), estimate the SNR in dB.

   (b) Estimate the SNR in dB assuming (for this part only) that the audio source is uniformly distributed between -L and L.

   (c) Estimate the largest SNR in dB that could be attained by any fixed-rate scalar quantizer (uniform or nonuniform) with rate 16.

   (d) Estimate the largest SNR in dB that could be attained by any fixed-rate k-dimensional vector quantizer with rate 16, for $k = 2,3,4,8$.

   (e) Repeat (d) assuming the vq can have any dimension whatsover.

   (f) Make a table showing how much more (in percent) audio could be stored on a CD with the methods of parts of (c), (d) and (e) than the with uniform scalar quantization with $L = 10\sigma$, assuming these other methods attain the same SNR as in Part (a).

2. Derive the formula for the Zador factor β_k assuming $\underline{X}$ is a k-dimensional IID random vector with Laplacian marginal densities.

3. A VQ is needed for a (first-order) autoregressive, stationary Gaussian source with correlation coefficient $\rho = .95$. It must have rate 4 or less and signal-to-noise ratio 32.5 dB or more. Determine whether or not there exists a suitable VQ. If yes, estimate the smallest possible dimension.

4. Show that $\beta_1 \geq \frac{1}{\sigma^2} 2^{2h}$ where $h = -\int_{-\infty}^{\infty} f(x) \log_2 f(x)\, dx$.

   (Hints: You might try Jensen's inequality or $\ln x \leq x-1$. Also, $h = E[\frac{3}{2} f^{-2/3}(X)]$.)

5. Do there exist prefix codes with the following sets of codeword lengths?

   (a) {2,2,3,3,3,5,6,6,6,6,7} .

   (b) {2,3,3,3,4,4,4,4,4}

   (c) {2,2,2,3,3}

   (d) For any set for which there does exist a code, draw the binary tree of a code with these lengths.

6. Consider an IID source with the following set of probabilities:

$$\{.25,.2,.1,.1,.1,.1,.05,.05,.05\}.$$

(a) Find the entropy of the source.

(b) Find two different prefix codes (first-order) with minimum rate. The codes should have different sets of lengths.

(c) Compare the entropy and the rate of the codes found in (b). Do they differ by a "reasonable" amount?

7. Find an example of a source for which $R^* \geq H + .9$. Hint: a binary source will suffice. This shows that R^* can be very close to $H + 1$.

8. Show by example that a prefix code with lengths $l_i = \lceil -\log_2 P_i \rceil$ does not necessarily have minimum average length.

9. Show that if each probability in the set $\{P_1,...,P_M\}$ is a negative power of 2, then the Shannon code is an optimal prefix code.