Homework 5 **EECS 651** **Due: Friday., Feb. 23, 2001**
in class, before lecture begins

1. A binary IID source has p(0) = .995 and p(1)=.005.

   (a) Find a fixed-to-variable length block lossless source code with rate no larger than 0.4 . (It should be as simple as possible.)

   (b) Is it possible to find a fixed-to-variable length block lossless source code with rate less than .1? If so, what can you say about how large its input block length must be?

2. Suppose we are told that a certain stationary source $\{X_k\}$ with alphabet $A = \{1,2,...,M\}$ has

   $$R_3^* = .95 \quad R_4^* = .92$$

   where R_k^* denotes the least rate of any block to variable length prefix code with input blocklength k. What can you deduce about the values of H_∞ and H_k for k = 1,2,3, ... ? In other words, using the given information, find the tightest possible bounds to H_∞ and to each H_k .

3. Consider the "runlength" code shown below. This is a variable-length to fixed-length code, unlike the fixed-length to variable-length we have considered in class. Let the source be IID source with p(0) = .995 and p(1)=.005

source sequence	run length	binary codeword
1	0	000
01	1	001
001	2	010
0001	3	011
00001	4	100
000001	5	101
0000001	6	110
0000000	7	111

   (a) Explain why this code is uniquely encodable and decodable.

   (b) Find the average length of the encoded source sequences.

   (c) Find the rate of this code.

   (d) Compare the rate and complexity of this code to that found in Problem 2a.

4. Let $\{Z_k\}$ be a binary IID source with alphabet $A_Z = \{0,1\}$ and $p_Z(1) = q$. Suppose $\{X_k\}$ is a stationary binary source with alphabet $A_X = \{0,1\}$ and suppose that

$$X_k = X_{k-1} \oplus X_{k-2} \oplus Z_k ,$$

where $\oplus$ denotes modulo 2 addition, and where Z_k is independent of $X_{k-1}, X_{k-2}, X_{k-3}, \ldots$.

(a) Show that the source is Nth-order Markov for some appropriately chosen N.

(b) Find an expression for H_k and H_∞ .

For the remaining parts assume $q = 0.1$.

(c) Make a sketch of how H_k approaches H_∞ as k increases.

(d) Find minimum rate FL-VL block prefix codes for block lengths 1, 2 and 3. Compute their rates and plot them vs. source length on the sketch of part (c).

(e) Can you design a conditional code that is better than those in (d), taking into account rate and complexity?

5. Assume high rate in this problem.

(a) Show that when variable-rate coding is applied to scalar quantizers with levels and thresholds that are optimal for fixed-rate coding, then the SNR gain will be two-thirds the SNR gain of optimal SQ-VR over optimal SQ-FR.

(b) For Gaussian and Laplacian densities, make a table of the SNR gains of VR coding applied to optimal FR quantizers over the performance of optimal FR quantizers, and also for the gains of optimal SQ-VR over optimal SQ-FR.

6. (a) Find a high-resolution expression for the probability of the cell containing $\underline{x}$ assuming the quantizer is optimized for variable-rate coding and compare the expression to that for fixed-rate coding.

(b) Repeat the above with "probability" replaced by "distortion" contribution.

7. Show that for scalar quantizers optimized for variable-rate coding, the levels should be centroids but the thresholds need not need be halfway between the levels.