Homework 6 **EECS 651** **Due: Friday., March 16, 2001**
in class, before lecture begins

1. Let $\underline{X} = (X_1,...,X_k)$ be a real-valued zero-mean, random vector with covariance matrix $K = [K_{i,j}]$. Show that if K is singular, then there exists some i such that X_i is a linear combination of the other random variables. Hint: Let $\underline{K}_j$ denote the jth row of K. Show that if $\underline{K}_i = \Sigma_{j\neq i} b_j \underline{K}_j$ for some i and some choice of coefficients b_j, $j = 1,...,k, j\neq i$, then $E[X_i - \Sigma_{j\neq i} b_j X_j]^2 = 0$.

2. (a) Design a 2-dimensional (fixed-rate) transform code with rate $R = 1$ bit/sample for a first-order Gaussian autoregressive source with variance 1 and correlation coefficient $\rho = .9$. The code should have as small MSE as possible. (You'll have to specify the transform and the scalar quantizers, as well as give an overall block diagram.) Do not use high resolution analysis. But you will need to use the scalar quantization tables distributed in class and posted on the website.

   (b) Compute the MSE of this code.

   (c) Compare the MSE to that of an optimal scalar quantizer with rate 1.

3. (a) Estimate how much more music (in percent) an audio CD could store if it used fixed-rate transform coding. There is no limit on the dimension of the transform.

   The conventional scheme with which to compare is described in Problem 1a of HW 4. The transform code must attain the same SNR.

   Assume the source is the one described in Problem 1 of HW 4.

   (b) What is the smallest dimension k such that transform coding with this dimension and the rate found in (a) comes with .5 dB of the desired SNR?