- 1. A variable-rate k-dimensional transform code is optimized for a zero-mean, stationary Gaussian source.
 - (a) Show that its point density is the optimal point density for variable-rate k-dimensional vector quantizers.
 - (b) Find its inertial profile.
- 2. (a) Derive the OPTA function for k-dimensional transform coding in which the scalar quantizers are replaced by two-dimensional vector quantizers. That is, one vq is applied to (U_1,U_2) , another is applied to (U_3,U_4) , and so on. Assume k is even, and assume the source is zero mean, stationary and Gaussian. Your answer should include both the fixed-rate and variable-rate cases. You may parallel the derivation given in class, and you may use without rederivation any fact that was proven in class which is useful here. (Just remember to state the fact that you are using.)

(b) Would the OPTA function change if instead of applying 2-dimensional vq's to (U_1,U_2) , (U_3,U_4) , oen applied two-dimensional vq's to (U_1,U_k) , (U_2,U_{k-1}) , ...?

3. (a) Design and describe a fixed-rate DPCM code with rate 3 and with a linear predictor of order two, for a zero-mean, stationary Gaussian source with autocorrelation function:

$$R_X(k) \; = \; \frac{-128}{105} \; \frac{1}{4^{|k|}} \; + \; \frac{64}{21} \; \frac{1}{2^{|k|}} \; . \label{eq:RXk}$$

The distortion should be as small as you can make it. Hints: In designing the code, you may use the assumptions we used in our high resolution analysis, and you may use the scalar quantization table that was used in the previous homework.

(b) This part is not required and will not be graded. Show that a DPCM code with a higher order predictor would do no better.

(c) This part is not required and will not be graded. Find, approximately, the OPTA function assuming no restriction on the order of the linear predictor.