What Happens as k and n Change?

As usual, consider a stationary source.

Recall:

$$\delta(k,n,R) \cong \sigma^2 m_k^* \alpha_{k,n} 2^{-2R}$$

m_k^* decreases subadditively to $m_\infty^* = \frac{1}{2\pi e}$

$$\alpha_{k,n} = \begin{cases} \beta_k , & n = 0 \\ \frac{1}{\sigma^2} 2^{2h_{kn}} , & n \geq 1 \end{cases}$$

β_k decreases submultiplicatively to β_∞

2^{2h_k} decreases monotonically to 2^{2h_∞}

Therefore,

$\alpha_{k,0}$ decreases submultiplicatively with k to β_∞

$\alpha_{k,n}$ decreases monotonically with k to $2^{2h_\infty}/\sigma^2$ for $n \geq 1$

$\alpha_{k,n}$ decreases monotonically with n to $2^{2h_\infty}/\sigma^2$

Key Fact: $2^{2h_\infty}/\sigma^2 = \beta_\infty$ (proved later)

Therefore,

> $\alpha_{k,n}$ decreases monotonically with n or k to $\beta_\infty = 2^{2h_\infty}$

Conclusions

(1) The least distortion of vector quantization with rate R or less, with any dimension, and with fixed-rate coding or with variable-rate encoding of any order is

$$\delta(R) \cong \sigma^2 m_\infty^* \beta_\infty 2^{-2R}$$

Among other things, this says that the best possible performance with variable-rate coding is no better than the best possible performance with fixed-rate coding.

(2) Increasing n with k fixed:

$\delta(k,n,R)$ decreases monotonically to the limit

$$\delta(k,\infty,R) \cong \sigma^2 m_k^* \beta_\infty 2^{-2R} \cong \frac{m_k^*}{m_\infty^*} \delta(R) \qquad \text{(space filling loss)}$$

> Therefore, for large n and arbitrary k,
>
> $$\delta(k,n,R) \cong \frac{m_k^*}{m_\infty^*} \delta(R)$$

Among other things, this shows that one needs large k in order to approach the best possible performance.