1. A VQ is needed for a (first-order) autoregressive, stationary Gaussian source with correlation coefficient $\rho = .95$. It must have rate 4 or less and signal-to-noise ratio 32.5 dB or more. Determine whether or not there exists a suitable VQ. If yes, estimate the smallest possible dimension.

2. Show that $\beta_1 \geq \frac{1}{\sigma^2} 2^{2h}$ where $h = - \int_{-\infty}^{\infty} f(x) \log_2 f(x) \, dx$. This will turn out to be the key step in comparing optimal quantizers with fixed and variable-length binary codebooks. Hints: You can derive this using the inequality $\ln x \leq x - 1$, or with Jensen's inequality, which says that if a function $g$ is convex $\cap$, then $E g(X) \leq g(EX)$. It may be useful to start by trying to find an upper bound to $h$, starting with the expression $h = E \left[ \frac{3}{2} \log f^{2/3}(X) \right]$.

3. Find an expression for the "oblongitis" suffered by scalar quantization for a Laplacian random variable. Check your answer by evaluating your expression and comparing to the answer given in the lecture notes.

4. Do there exist prefix codes with the following sets of codeword lengths?
   (a) $\{2,2,3,3,5,6,6,6,7\}$
   (b) $\{2,3,3,4,4,4,4\}$
   (c) $\{2,2,2,3\}$
   (d) For any set for which there does exist a code, draw the binary tree of a code with these lengths.

5. Find an example of a set of probabilities for which $R^* \geq H + .9$. Hint: a binary source will suffice. This shows that $R^*$ can be very close to $H + 1$.

6. Show by example that a prefix code with lengths $l_i = \left\lceil - \log_2 P_i \right\rceil$ does not necessarily have minimum average length.

7. Show that if each probability in the set $\{P_1, \ldots, P_M\}$ is a negative power of 2, then the Shannon code is an optimal prefix code.

8. Consider an IID source with the following set of probabilities:
   \{.25,.2,.1,.1,.1,.1,.05,.05,.05\}.
   (a) Find the entropy of the source.
   (b) Find two different prefix codes (first-order) with minimum rate. The codes should have different sets of lengths.
   (c) Compare the entropy and the rate of the codes found in (b). Do they differ by a "reasonable" amount?