EECS 730 Due Jan. 27, 2009

Homework Set No. 1

Problem 1: Prove the following vector-dyadic identities, where $\overline{a}$ and $\overline{b}$ are vectors and $\overline{\overline{A}}$ and $\overline{\overline{B}}$ are dyads.

I. $(\overline{a} \times \overline{b})\cdot \overline{\overline{A}} = \overline{a} \cdot (\overline{b}\times \overline{\overline{A}}) = -\overline{b} \cdot (\overline{a}\times \overline{\overline{A}})$

II. $(\overline{\overline{A}} \times \overline{a}) \cdot \overline{b} = \overline{\overline{A}} \cdot (\overline{a} \times \overline{b}) = -(\overline{\overline{A}} \times \overline{b}) \cdot \overline{a}$

III. $(\overline{a}\times \overline{\overline{A}}) \times \overline{b} = \overline{a} \times (\overline{\overline{A}} \times \overline{b})$

IV. $(\overline{a}\times \overline{\overline{A}})\cdot \overline{\overline{B}} = a \times (\overline{\overline{A}} \cdot \overline{\overline{B}})$

V. $(\overline{\overline{I}} \times \overline{a})\cdot \overline{\overline{A}} = \overline{a}\times \overline{\overline{A}}$

Problem 2: Prove the following differential dyadic identities:

I. $\nabla(\overline{a} \times \overline{b}) = (\nabla\overline{a}) \times \overline{b} - \nabla\overline{b} \times \overline{a}$

II. $\nabla \cdot (\overline{a}\times \overline{\overline{A}}) = (\nabla \times \overline{a})\cdot \overline{\overline{A}} - \overline{a} \cdot \nabla\times \overline{\overline{A}}$

III. $\nabla \times (\overline{a}\overline{b}) = (\nabla \times \overline{a})b - \overline{a} \times \nabla\overline{b}$

Problem 3: Prove $\overline{\overline{B}} = \overline{C}\times \overline{\overline{I}}$ is anti-symmetric.

Problem 4: Momentum Conservation Theorem: Starting from the Lorentz force law show that

$$\frac{\partial\overline{G}}{\partial t} + \nabla\cdot \overline{\overline{T}} = -\overline{f}$$

where $\overline{f}$ is the force field per unit volume, and

$$\overline{G} = \overline{D} \times \overline{B}$$

is the momentum density vector, and

$$\overline{\overline{T}} = \frac{1}{2}(\overline{D} \cdot \overline{E} + \overline{B} \cdot \overline{H})\, \overline{\overline{I}} - \overline{DE} - \overline{BH}$$

is referred to as Maxwell stress tensor.

Problem 5: For each of the following constitutive relations, state whether the given medium is

(1) Isotropic/anisotropic/bianisotropic,

(2) Linear/nonlinear,

(3) Homogeneous/inhomogeneous.

(a) Cholesteric liquid crystals can be modeled by a spiral structure with constitutive relations given by

$$\overline{D} = \begin{pmatrix} \varepsilon(1+\delta\cos Kz) & \varepsilon\delta\sin Kz & 0 \\ \varepsilon\delta\sin Kz & \varepsilon(1-\delta\cos Kz) & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} \cdot \overline{E}$$

where the spiral direction is along the z axis.

(b) In view of the optical activities in quartz crystals, the constitutive relations for a quartz crystal are phenomenologically described as

$$E_j = \sum_{i=j}^{3} k_{ij} D_i + \frac{1}{\mu_0 \varepsilon} G_{ij} \frac{\partial}{\partial t} B_i$$

$$H_j = \frac{1}{\mu_0} B_j - \frac{1}{\mu_0 \varepsilon_0} \sum_{i=j}^{3} G_{ij} \frac{\partial}{\partial t} D_i$$

(c) When a magnetic field $\overline{B_0}$ is applied to a conductor carrying a current, an electric field $\overline{E}$ is developed. This is called *Hall effect*. Assuming the conduction carrier drifts with a mean velocity $\overline{v}$ proportional to $R\sigma\overline{E}$, the constitutive relation that takes care of the Hall effect is given by

$$\overline{J} = \sigma(\overline{E} + R\sigma\overline{E} \times \overline{B_0})$$

where σ is the conductivity and R is the Hall coefficient. For copper, $\sigma \approx 6.7 \times 10^7 mho/m$ and $R \approx= 5.5 \times 10^{-11} m^3/coul$.

(d) The phenomenon of pyroelectricity in a crystal is observed when it is heated. The constitutive relation for a *pyroelectric material* can be written as

$$\overline{D} = \overline{D}_0 + \overline{\varepsilon} \cdot \overline{E}$$

where a spontaneous term $\overline{D_0}$ exists even in the absence of an external field.

(e) The phenomenon in which dipole moments are induced in a crystal by mechanical stress is called *piezoelectricity*. A piezoelectric material is characterized by a piezoelectric tensor $\gamma_{i,kl} = \gamma_{lk}$ such that

$$D_i = D_{0i} + \varepsilon\delta_{ij}E - k + \gamma_{i,kl}s_{kl}$$

where s_{kl} is the stress tensor to the second order in electric fields. All pyroelectric media are also piezoelectric.

(f) An isotropic dielectric can exhibit the *Kerr effect* when placed in an electric field. In this case the permittivity can be written as

$$\varepsilon_{ij} = \varepsilon\delta_{ij} + \sigma E_i E_j$$

where ε is the unperturbed permittivity. The principle axis of ε_{ij} coincides with the electric field.