

Dyadic Green's Function for Stratified Media[†]

Layered dielectric media is encountered in many electromagnetic problems such as analysis of multi-layered substrate/superstrate microwave circuits and printed antennas as well as modeling the terrain with a dielectric stratified medium. In this section a procedure for evaluating the dyadic Green's function for a multi-layered (with planar interfaces) medium with arbitrary positions of the source and observation points is outlined. Also formal expressions for the dyadic Green's function in terms of continuous spectrum of plane waves are derived.

In order to evaluate the Green's function, the behavior of electromagnetic plane waves in layered media should be characterized first. In the following section reflection and transmission of plane waves by a multi-layered medium is discussed.

1 Plane Waves in Layered Media

Consider a stratified medium with planar interfaces parallel to the x-y plane of a coordinate system. Suppose the parameters of the n th layer are denoted by μ_n and ϵ_n and its boundary with the $(n + 1)$ th layer is at $z = -d_n$. It is also assumed that the top (region 0) and bottom (region $N + 1$) layers are semi-infinite (see Figure 1).

Without loss of generality let us assume that a plane wave is illuminating the interface from above whose direction of propagation is given by

$$\hat{k}_i = \frac{1}{k_0} (k_{0x}\hat{x} - k_{0z}\hat{z}) \quad (1)$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0}$. Similar to waveguide problems the field quantities can be decomposed into transverse electric (TE) and transverse magnetic (TM) waves.² For TM waves all field quantities can be expressed in terms of axial component of the electric field (E_z) and for TE waves they can be expressed in terms of the axial component of the magnetic

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²R.F. Harrington, "Time-Harmonic Electromagnetic Fields," pp. 130, New York: Mc Graw-Hill, 1961.

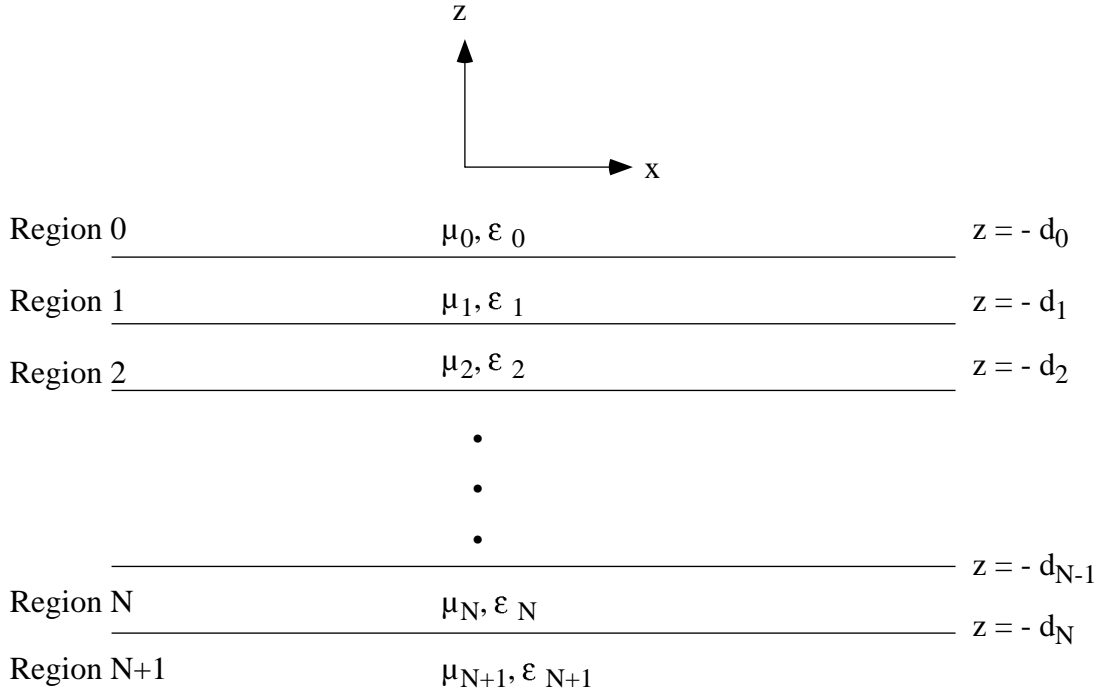


Figure 1: A stratified medium.

field (H_z). Because of the symmetry of the problem $\frac{\partial}{\partial y}$ of all field components are zero. It is also assumed that the dependency on z in the n th layer is of $e^{\pm ik_{nz}z}$.

For TE waves, H_{nz} can be obtained from the solution to Helmholtz's equation

$$\left(\frac{\partial^2}{\partial x^2} + (k_n^2 - k_{nz}^2) \right) H_{nz} = 0 \quad (2)$$

where $k_n = \omega \sqrt{\mu_n \epsilon_n}$. Other field quantities in terms of H_{nz} are given by

$$H_{nx} = \frac{1}{k_n^2 - k_{nz}^2} \frac{\partial^2}{\partial x \partial z} H_{nz} \quad (3)$$

$$E_{ny} = \frac{-i\omega\mu_n}{k_n^2 - k_{nz}^2} \frac{\partial}{\partial x} H_{nz} \quad (4)$$

TE and TM waves are dual of each other under the standard duality relationship $E \rightarrow H, H \rightarrow -E$, and $\mu \rightleftharpoons \epsilon$. Hence, once the field quantities for a TE incidence is obtained, the results for a TM excitation can be obtained using the duality relationships. For the incident TE wave

$$H_z^i = H_0 e^{-ik_0 z} e^{ik_0 x} \quad (5)$$

The phase matching condition mandates that the x -dependency of all field quantities be the same as that of the incident wave.

The total field quantity in region n , excluding its dependency on x , is given by

$$H_{nz} = A_n e^{ik_{nz}z} + B_n e^{-ik_{nz}z} \quad (6)$$

where A_n and B_n represent the coefficients of the wave components propagating in positive and negative z directions respectively. In the zeroth region it is recognized that

$$B_0 = H_0 \quad (7)$$

$$A_0 = R^{TE} H_0 \quad (8)$$

where R^{TE} is the total magnetic field reflection coefficient for TE (horizontally polarized) waves. Since the most bottom region ($(N+1)$ th region) is semi-infinite there would be no upward traveling wave, i.e.,

$$A_{N+1} = 0 \quad (9)$$

$$B_{N+1} = T^{TE} H_0 \quad (10)$$

where T^{TE} is the total transmission coefficient.

Evaluating H_x and E_y from (3) and (4) and then enforcing the continuity of tangential electric and magnetic fields at the interface between the n th and $(n+1)$ th region (at $z = -d_n$) we get

$$k_{nz} [A_n e^{-ik_{nz}d_n} - B_n e^{ik_{nz}d_n}] = k_{(n+1)z} [A_{n+1} e^{-ik_{(n+1)z}d_n} - B_{n+1} e^{ik_{(n+1)z}d_n}] \quad (11)$$

$$\mu_n [A_n e^{-ik_{nz}d_n} + B_n e^{ik_{nz}d_n}] = \mu_{(n+1)} [A_{n+1} e^{-ik_{(n+1)z}d_n} + B_{n+1} e^{ik_{(n+1)z}d_n}] \quad (12)$$

There are $N+2$ regions and therefore the number of unknowns are $2N+2$ (B_0 and A_{N+1} are known). On the other hand there are $N+1$ boundaries and $2N+2$ equations for the $2N+2$ unknowns. A simple procedure to solve for the unknowns is as follows:

1. Find R^{TE} by recursively relating $\frac{A_{N+1}}{B_{N+1}} = 0$ to $\frac{A_N}{B_N}$ etc.
2. Once A_0 is found relate A_{n+1}, B_{n+1} to A_n, B_n to find all coefficients.

Solving (11), (12) for A_n and B_n results in

$$A_n e^{-ik_n z d_n} = \frac{1}{2} \left[\frac{\mu_{n+1}}{\mu_n} + \frac{k_{(n+1)z}}{k_{nz}} \right] \left\{ A_{n+1} e^{-ik_{(n+1)z} d_n} + r_{n(n+1)}^{TE} B_{n+1} e^{ik_{(n+1)z} d_n} \right\} \quad (13)$$

$$B_n e^{+ik_n z d_n} = \frac{1}{2} \left[\frac{\mu_{n+1}}{\mu_n} + \frac{k_{(n+1)z}}{k_{nz}} \right] \left\{ r_{n(n+1)}^{TE} A_{n+1} e^{-ik_{(n+1)z} d_n} + B_{n+1} e^{ik_{(n+1)z} d_n} \right\} \quad (14)$$

where

$$r_{n(n+1)}^{TE} = \frac{\mu_{n+1} k_{nz} - \mu_n k_{(n+1)z}}{\mu_{n+1} k_{nz} + \mu_n k_{(n+1)z}}$$

is the Fresnel reflection coefficient for a horizontally polarized wave. Taking the ratio of (13) and (14), we obtain

$$\frac{A_n}{B_n} e^{-2ik_n z d_n} = \frac{\left[\frac{A_{n+1}}{B_{n+1}} e^{-2ik_{(n+1)z} d_{(n+1)}} \right] e^{2ik_{(n+1)z} (d_{n+1} - d_n)} + r_{n(n+1)}^{TE}}{\left[\frac{A_{n+1}}{B_{n+1}} e^{-2ik_{(n+1)z} d_{(n+1)}} \right] e^{2ik_{(n+1)z} (d_{n+1} - d_n)} r_{n(n+1)}^{TE} + 1} \quad (15)$$

This is a recurrence relation which can be used to find A_0/B_0 starting from $A_{N+1}/B_{N+1} = 0$. In order to solve for T^{TE} and the rest of the coefficients, we rearrange (11) and (12) to get A_{n+1} and B_{n+1} in terms of A_n and B_n . In matrix notation we have

$$\begin{bmatrix} A_{n+1} e^{-ik_{(n+1)z} d_{n+1}} \\ B_{n+1} e^{ik_{(n+1)z} d_{n+1}} \end{bmatrix} = \overset{=TE}{V}_{(n+1)n} \begin{bmatrix} A_n e^{-ik_n z d_n} \\ B_n e^{ik_n z d_n} \end{bmatrix} \quad (16)$$

where $\overset{=TE}{V}_{(n+1)n}$ is referred to as the forward propagation matrix given by

$$\overset{=TE}{V}_{(n+1)n} = \frac{1}{2} \left(\frac{k_{nz}}{k_{(n+1)z}} + \frac{\mu_n}{\mu_{n+1}} \right) \begin{bmatrix} e^{-ik_{(n+1)z} (d_{n+1} - d_n)} & -r_{n(n+1)}^{TE} e^{ik_{(n+1)z} (d_{n+1} - d_n)} \\ -r_{n(n+1)}^{TE} e^{ik_{(n+1)z} (d_{n+1} - d_n)} & e^{ik_{(n+1)z} (d_{n+1} - d_n)} \end{bmatrix} \quad (17)$$

Knowing A_0 and B_0 , (16) can be used to find A_n and B_n ($n \in \{1, \dots, N+1\}$).

For TM polarized waves, the duality relations can be used to find the desired quantities.

2 Dyadic Green's Function For a Multi-layer Media

Let us consider a multi-layer medium similar to the one shown in Figure 1. As discussed before if the electric dyadic Green's function for such media can be defined, then the electric field anywhere in the medium can be obtained from

$$\bar{E}(r) = ik_n Z_n \iiint \bar{G}(\bar{r}, \bar{r}') \cdot \bar{J}_e(\bar{r}') dv' \quad (18)$$

This dyadic Green's function satisfies the following continuity conditions:

$$\begin{aligned} \lim_{r_i, r_j \rightarrow r} \mu_i \hat{n}(\bar{r}) \times \bar{G}(\bar{r}_i, \bar{r}') - \mu_j \hat{n} \times \bar{G}(\bar{r}_j, \bar{r}') &= 0 \\ \lim_{r_i, r_j \rightarrow r} \hat{n}(\bar{r}) \times \left[\nabla \times \bar{G}(\bar{r}_i, \bar{r}') \right] - \hat{n}(r) \times \left[\nabla \times \bar{G}(\bar{r}_j, \bar{r}') \right] &= 0 \end{aligned} \quad (19)$$

where r is a point at the interface between the i th and j th boundary.

The starting point to derive an expression for $\bar{G}(\bar{r}, \bar{r}')$ is the Fourier transform of the free-space dyadic Green's function $\left(\bar{G}^{fs}(\bar{r}, \bar{r}') \right)$. It was found that

$$\bar{G}^{fs}(\bar{r}, \bar{r}') = -\frac{\hat{z}\hat{z}}{k_0^2} \delta(\bar{r} - \bar{r}') + \begin{cases} \frac{i}{8\pi^2} \iint_{-\infty}^{+\infty} \frac{1}{k_{0z}} \left[\hat{e}(k_{0z})\hat{e}(k_{0z}) + \hat{h}(k_{0z})\hat{h}(k_{0z}) \right] e^{i\bar{k}_0 \cdot (\bar{r} - \bar{r}')} dk_{\perp} & z > z' \\ \frac{i}{8\pi^2} \iint_{-\infty}^{+\infty} \frac{1}{k_{0z}} \left[\hat{e}(-k_{0z})\hat{e}(-k_{0z}) + \hat{h}(-k_{0z})\hat{h}(-k_{0z}) \right] e^{i\bar{K}_0 \cdot (\bar{r} - \bar{r}')} dk_{\perp} & z < z' \end{cases}$$

To designate the location of the source and observation points, we use two subscripts for the dyadic Green's function. The first one denotes the layer where the observation point is and the second one denotes the layer where the source point is. For example $\bar{G}_{nq}(\bar{r}, \bar{r}')$ is the expression for the dyadic Green's function when the source and observation points are, respectively in the q th and n th layers.

Suppose the source is in the zeroth region and we are interested in characterizing the expressions for the dyadic Green's function in the n th layer.

First consider an observation point in region 0 with $z < z'$. Plane waves of the form $\left(\frac{1}{k_{0z}} \hat{e}(-k_{0z}) \cdot \bar{J} e^{-i\bar{K}_0 \cdot \bar{r}'} \right) \hat{e}(-k_{0z}) e^{i\bar{K}_0 \cdot \bar{r}}$ are illuminating the multi-layer medium from above. The field in $-d_0 < z < z'$ region is the sum of the incident and reflected waves of the form $\left(\frac{1}{k_{0z}} \hat{e}(-k_{0z}) \cdot \bar{J} e^{-i\bar{K}_0 \cdot \bar{r}'} \right) R^{TE} \hat{e}(k_{0z}) e^{i\bar{k}_0 \cdot \bar{r}}$. Therefore the expression for the dyadic Green's function in region $-d_0 < z < z'$ is given by

$$\begin{aligned} \bar{G}_{00}(\bar{r}, \bar{r}') &= \frac{i}{8\pi^2} \iint_{-\infty}^{+\infty} \frac{1}{k_{0z}} \left\{ \left[R^{TE} \hat{e}(k_{0z}) e^{i\bar{k}_0 \cdot \bar{r}} + \hat{e}(-k_{0z}) e^{i\bar{K}_0 \cdot \bar{r}} \right] \hat{e}(-k_{0z}) e^{-i\bar{K}_0 \cdot \bar{r}'} \right. \\ &\quad \left. + \left[R^{TM} \hat{h}(k_{0z}) e^{i\bar{k}_0 \cdot \bar{r}} + \hat{h}(-k_{0z}) e^{i\bar{K}_0 \cdot \bar{r}} \right] \hat{h}(-k_{0z}) e^{-i\bar{K}_0 \cdot \bar{r}'} \right\} dk_{\perp} \end{aligned} \quad (20)$$

The plane waves in the n th region can be decomposed into upward- and downward-traveling waves with unknown coefficients that can be characterized using the continuity

conditions of the dyadic Green's function. The expression for the dyadic Green's function with the source point in the zeroth region and the observation point in the n th region is given by

$$\begin{aligned} \bar{\bar{G}}_{n0}(\bar{r}, \bar{r}') = \frac{i}{8\pi^2} \iint_{-\infty}^{+\infty} \frac{1}{k_{0z}} \left\{ [A_n \hat{e}_n(k_{nz}) e^{i\bar{k}_n \cdot \bar{r}} + B_n \hat{e}_n(-k_{nz}) e^{i\bar{K}_n \cdot \bar{r}}] \hat{e}(-k_{0z}) e^{-i\bar{K}_0 \cdot \bar{r}'} \right. \\ \left. + [C_n \hat{h}_n(k_{nz}) e^{i\bar{k}_n \cdot \bar{r}} + D_n \hat{h}_n(-k_{nz}) e^{i\bar{K}_n \cdot \bar{r}}] \hat{h}(-k_{0z}) e^{-i\bar{K}_0 \cdot \bar{r}'} \right\} dk_{\perp} \end{aligned} \quad (21)$$

In the lowest layer ($(N+1)$ th) there is no upward-traveling wave and therefore

$$\begin{aligned} \bar{\bar{G}}_{(N+1)0}(\bar{r}, \bar{r}') = \frac{i}{8\pi^2} \iint_{-\infty}^{+\infty} \frac{1}{k_{0z}} \left\{ T^{TE} \hat{e}_{N+1}(-k_{(N+1)z}) e^{i\bar{K}_{(N+1)} \cdot \bar{r}} \hat{e}(-k_{0z}) e^{-i\bar{K}_0 \cdot \bar{r}'} \right. \\ \left. + T^{TM} \hat{h}_{N+1}(-k_{(N+1)z}) e^{i\bar{K}_{(N+1)} \cdot \bar{r}} \hat{e}(-k_{0z}) e^{-i\bar{K}_0 \cdot \bar{r}'} \right\} dk_{\perp} \end{aligned} \quad (22)$$

At the interface between the n th and $(n+1)$ th regions ($z = -d_n$), the boundary condition for the dyadic Green's function mandates that $\mu_n \hat{z} \times \bar{\bar{G}} = \mu_{n+1} \hat{z} \times \bar{\bar{G}}$. But $\hat{z} \times \hat{e}_n(\pm k_{nz}) = \hat{z} \times \hat{e}_{n+1}(\pm k_{nz})$ and $\hat{z} \times \hat{h}_n(\pm k_{nz}) = \pm \frac{k_{nz}}{k_n} \hat{e}(\pm k_{nz})$ which can be used in conjunction with the boundary condition given in (19) to generate the following two equations:

$$\begin{aligned} \mu_n (A_n e^{-ik_{nz}d_n} + B_n e^{ik_{nz}d_n}) &= \mu_{n+1} (A_{n+1} e^{-ik_{(n+1)z}d_n} + B_{n+1} e^{ik_{(n+1)z}d_n}) \quad (23) \\ \mu_n \frac{k_{nz}}{k_n} [C_n e^{-ik_{nz}d_n} - D_n e^{ik_{nz}d_n}] &= \mu_{n+1} \frac{k_{(n+1)z}}{k_{(n+1)}} [C_{n+1} e^{-ik_{(n+1)z}d_n} - D_{n+1} e^{ik_{(n+1)z}d_n}] \quad (24) \end{aligned}$$

The boundary condition also mandates the continuity $\hat{z} \times (\nabla \times \bar{\bar{G}})$. Noting that $\nabla \times (e^{i\bar{k}_n \cdot \bar{r}} \hat{e}) = i e^{i\bar{k}_n \cdot \bar{r}} \bar{k}_n \times \hat{e}$ and $\hat{z} \times (\bar{k}_n \times \hat{e}) = k_{nz} \hat{e}$ this boundary condition renders the following equation:

$$k_{nz} [A_n e^{-ik_{nz}d_n} - B_n e^{ik_{nz}d_n}] = k_{(n+1)z} [A_{n+1} e^{-ik_{(n+1)z}d_n} - B_{n+1} e^{ik_{(n+1)z}d_n}] \quad (25)$$

In a similar manner one can show that $\hat{z} \times [\nabla \times (e^{i\bar{k}_n \cdot \bar{r}} \hat{h}(k_{nz}))] = ik_n \hat{z} \times \hat{e}(k_{nz}) e^{i\bar{k}_n \cdot \bar{r}}$ which together with the application of continuity of $\hat{z} \times \nabla \times \bar{\bar{G}}$ provides the following equation:

$$k_n [C_n e^{-ik_{nz}d_n} + D_n e^{ik_{nz}d_n}] = k_{(n+1)} [C_{n+1} e^{-ik_{(n+1)z}d_n} + D_{n+1} e^{ik_{(n+1)z}d_n}] \quad (26)$$

Noting that $A_0 = R^{TE}$, $B_0 = 1$, $B_{N+1} = T^{TE}$, $C_0 = R^{TM}$, $D_0 = 1$, and $C_{N+1} = 0$, a procedure similar to what was outlined before can be followed to find R^{TE} , R^{TM} first and then A_n , B_n , C_n , and D_n coefficients. The ratio of $\frac{A_n}{B_n} e^{-2ik_n z d_n}$ is given by (15) and

$$\frac{C_n}{D_n} e^{-2ik_n z d_n} = \frac{\left[\frac{C_{n+1}}{D_{n+1}} e^{-2ik_{(n+1)z} d_{(n+1)}} \right] e^{2ik_{(n+1)z} (d_{n+1} - d_n)} + r_{n(n+1)}^{TM}}{\left[\frac{C_{n+1}}{D_{n+1}} e^{-2ik_{(n+1)z} d_{(n+1)}} \right] r_{n(n+1)}^{TM} e^{2ik_{(n+1)z} (d_{n+1} - d_n)} + 1} \quad (27)$$

where $r_{n(n+1)}^{TM}$ is the Fresnel reflection coefficient for TM waves and is given by

$$r_{n(n+1)}^{TM} = \frac{\epsilon_{n+1} k_{nz} - \epsilon_n k_{(n+1)z}}{\epsilon_{n+1} k_{nz} + \epsilon_n k_{(n+1)z}} \quad (28)$$

Using (27) and (15), R^{TE} and R^{TM} (i.e., A_0 and C_0) are obtained. Then using forward propagation matrices the coefficients in all layers can be obtained. The forward propagation matrix for TE waves is given by (16) and for TM waves is defined by

$$\begin{bmatrix} C_{n+1} e^{-ik_{(n+1)z} d_{n+1}} \\ D_{n+1} e^{ik_{(n+1)z} d_{n+1}} \end{bmatrix} = \bar{V}_{(n+1)n}^{TM} \begin{bmatrix} C_n e^{-ik_n z d_n} \\ D_n e^{ik_n z d_n} \end{bmatrix} \quad (29)$$

where

$$\bar{V}_{(n+1)n}^{TM} = \frac{1}{2} \frac{k_n}{k_{n+1}} \left(1 + \frac{\epsilon_{n+1}}{\epsilon_n} \frac{k_{nz}}{k_{(n+1)z}} \right) \begin{bmatrix} e^{ik_{(n+1)z} (d_{n+1} - d_n)} - r_{n(n+1)}^{TM} e^{-ik_{(n+1)z} (d_{n+1} - d_n)} \\ -r_{n(n+1)}^{TM} e^{ik_{(n+1)z} (d_{n+1} - d_n)} e^{ik_{(n+1)z} (d_{n+1} - d_n)} \end{bmatrix} \quad (30)$$

Once \bar{G}_{n0} (\bar{r}, \bar{r}') is characterized, \bar{G}_{0n} (\bar{r}, \bar{r}') can easily be obtained using the symmetry property of dyadic Green's functions. Hence, the dyadic Green's function when the source is in the n th layer and the observation point is the 0th region is given by:

$$\bar{G}_{0n}(\bar{r}, \bar{r}') = \left[\bar{G}_{n0}(\bar{r}', \bar{r}) \right]^T \quad (31)$$

3 Far Field Expression For Dyadic Green's Function

In calculation of the electric field generated by a current distribution, being either impressed current in antenna problems or induced current in scattering problems, equation (18) can be used. In many practical situations the observation point is far from the source ($k_0 r \gg 1$) which allows for the development of a simplified expression for the calculation of the electric field. Let us assume the observation point is in the zeroth region of Figure 1. In this case $z > z'$ and therefore

$$\begin{aligned} \bar{G}_{00}(\bar{r}, \bar{r}') = \frac{i}{8\pi^2} \iint_{-\infty}^{+\infty} \frac{e^{i\bar{k}_0 \cdot \bar{r}}}{k_{0z}} \left\{ \hat{e}(k_{0z}) \left[R^{TE} \hat{e}(-k_{0z}) e^{-i\bar{K}_0 \cdot \bar{r}'} + \hat{e}(k_{0z}) e^{-i\bar{k}_0 \cdot \bar{r}'} \right] \right. \\ \left. + \hat{h}(k_{0z}) \left[R^{TM} \hat{h}(-k_{0z}) e^{-i\bar{K}_0 \cdot \bar{r}'} + \hat{h}(k_{0z}) e^{-i\bar{k}_0 \cdot \bar{r}'} \right] \right\} dk_{\perp} \end{aligned} \quad (32)$$

since $k_0 r \gg 1$, (32) can be evaluated using the stationary phase approximation. Denoting the observation point by

$$\bar{r} = r [\cos \phi_s \sin \theta_s \hat{x} + \sin \phi_s \sin \theta_s \hat{y} + \cos \theta_s \hat{z}]$$

The exponent function is given by

$$\bar{k}_0 \cdot \bar{r} = r \left[k_x \cos \phi_s \sin \theta_s + k_y \sin \phi_s \sin \theta_s + \sqrt{k_0^2 - k_x^2 - k_y^2} \cos \theta_s \right]$$

Since $k_0 r$ is a large quantity, the integrand is highly oscillatory and its contribution to the overall integral comes from a region where the variation of the phase is slow. This stationary phase point can be obtained from the solution

$$\begin{aligned} \frac{\partial}{\partial k_x} (\bar{k}_0 \cdot \bar{r}) &= 0 \\ \frac{\partial}{\partial k_y} (\bar{k}_0 \cdot \bar{r}) &= 0 \end{aligned} \quad (33)$$

and is given by

$$\begin{aligned} k_x^s &= k_0 \sin \theta_s \cos \phi_s \\ k_y^s &= k_0 \sin \theta_s \sin \phi_s \\ k_z^s &= k_0 \cos \theta_s \end{aligned} \quad (34)$$

According to the stationary phase approximation

$$\iint_{-\infty}^{+\infty} A(x_1, x_2) e^{if(x_1, x_2)} dx_1 dx_2 \simeq A(x_1^s, x_2^s) e^{if(x_1^s, x_2^s)} \frac{2\pi i}{\sqrt{f_{11}f_{22} - f_{12}^2}} \quad (35)$$

where $A(x_1, x_2)$ is any smooth function and

$$f_{ij} = \frac{\partial^2}{\partial x_i \partial x_j} f$$

In (35), x_1^s and x_2^s are the stationary phase points derived from the solution of

$$\frac{\partial f}{\partial x_1} = 0; \quad \frac{\partial f}{\partial x_2} = 0$$

Using (35), (32) can be approximated by

$$\begin{aligned} \bar{G}_{00}(\bar{r}, \bar{r}') \simeq \frac{e^{ik_0 r}}{4\pi r} \left\{ \left[\hat{e}(k_z^s) \hat{e}(-k_z^s) R_s^{TE} + \hat{h}(k_z^s) \hat{h}(-k_z^s) R_s^{TM} \right] e^{-i\bar{K}_s \cdot \bar{r}'} \right. \\ \left. + \left[\hat{e}(k_z^s) \hat{e}(k_z^s) + \hat{h}(k_z^s) \hat{h}(k_z^s) \right] e^{-i\bar{k}_s \cdot \bar{r}'} \right\} \end{aligned} \quad (36)$$

Where R_s^{TE} and R_s^{TM} are the Fresnel reflection coefficient evaluated at the stationary phase point given by (34). The first term of (36) is the contribution from the layered medium, and in far field, this contribution appears to be originated from the mirror image point. However, depending on the polarization, each component of the image field is modified by the appropriate Fresnel reflection coefficient.