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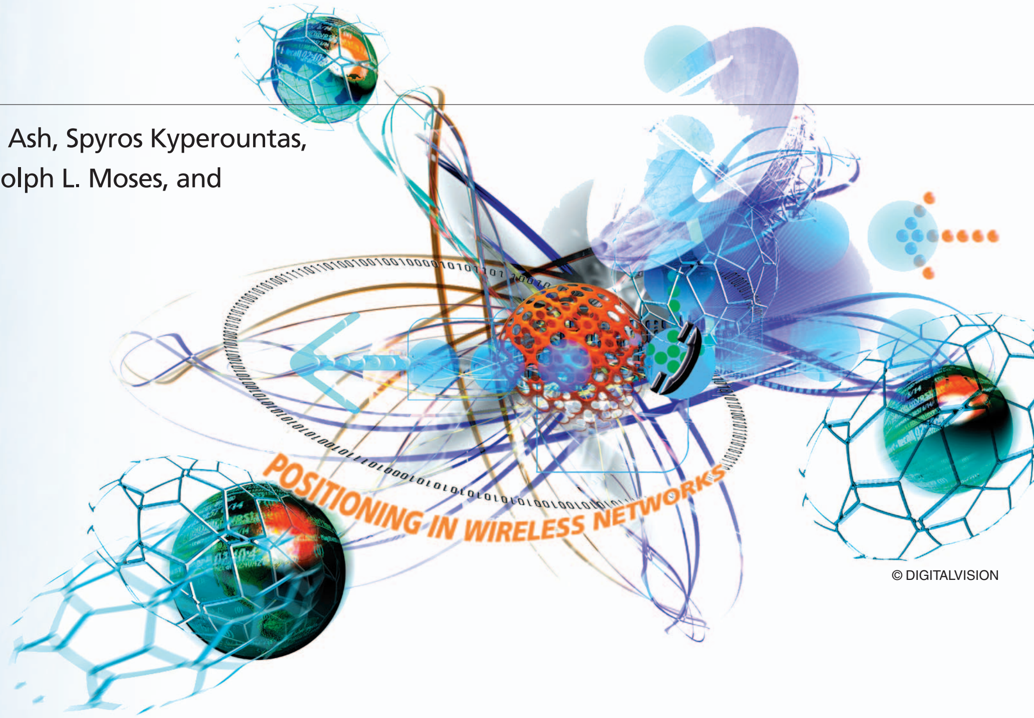

Locating the Nodes

[Cooperative localization in wireless sensor networks]

Accurate and low-cost sensor localization is a critical requirement for the deployment of wireless sensor networks in a wide variety of applications. Low-power wireless sensors may be many hops away from any other sensors with a priori location information. In cooperative localization, sensors work together in a peer-to-peer manner to make measurements and then form a map of the network. Various application requirements (such as scalability, energy efficiency, and accuracy) will influence the design of sensor localization systems. In this article, we describe measurement-based statistical models useful to describe time-of-arrival (TOA), angle-of-arrival (AOA), and received-signal-strength (RSS) measurements in wireless sensor networks. Wideband and ultra-wideband (UWB) measurements, and RF and acoustic media are also discussed. Using the models, we show how to calculate a Cramér-Rao bound (CRB) on the location estimation precision possible for a given set of measurements. This is a useful tool to help system designers and researchers select measurement technologies and evaluate localization algorithms. We also briefly survey a large and growing body of sensor localization algorithms. This article is intended to emphasize the basic statistical signal processing background necessary to understand the state-of-the-art and to make progress in the new and largely open areas of sensor network localization research.

INTRODUCTION

Dramatic advances in RF and MEMS IC design have made possible the use of large networks of wireless sensors for a variety of new monitoring and control applications [1]–[5]. For example, smart structures will actively respond to earthquakes and make buildings safer; precision agriculture will reduce costs and environmental impact by watering and fertilizing only where necessary and will improve quality by monitoring storage conditions after harvesting; condition-based

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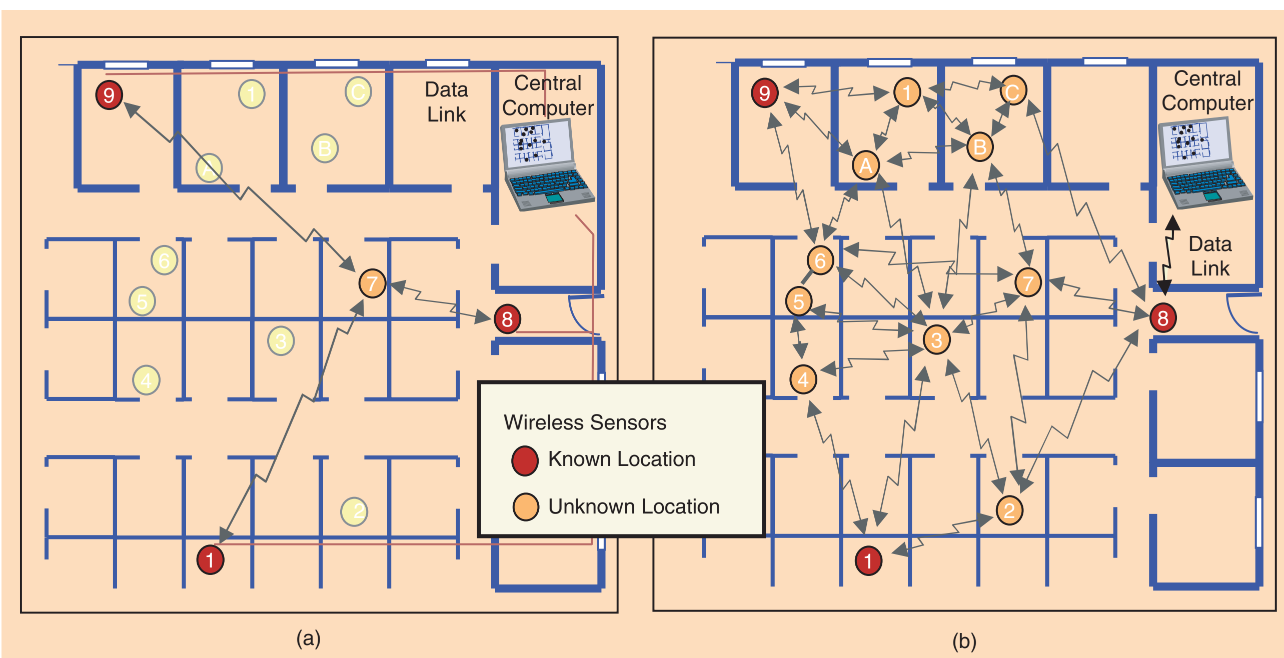
maintenance will direct equipment servicing exactly when and where it is needed based on data from wireless sensors; traffic monitoring systems will better control stoplights and inform motorists of alternate routes in the case of traffic jams; and environmental monitoring networks will sense air, water, and soil quality and identify the source of pollutants in real time.

Automatic localization of the sensors in these wireless networks is a key enabling technology. The overwhelming reason is that a sensor's location must be known for its data to be meaningful. As an additional motivation, sensor location information (if it is accurate enough) can be extremely useful for scalable, "geographic" routing algorithms. Note also that location itself is often the data that needs to be sensed; localization can be the driving force for wireless sensor networks in applications such as warehousing and manufacturing logistics.

To make these applications viable with possibly vast numbers of sensors, device costs will need to be low (from a few dollars to a few cents depending on the application), sensors will need to last for years or even decades without battery replacement, and the network will need to organize without significant human moderation. Traditional localization techniques are not well suited for these requirements. Including a global positioning system (GPS) receiver on each device is cost and energy prohibitive for many applications, not sufficiently robust to jamming for military applications, and limited to outdoor applications. Local positioning systems (LPS) [6] rely on high-capability base stations being deployed in each coverage area, an expensive burden for most low-configuration wireless sensor networks.

Instead, we consider the problem in which some small number m of sensors, called reference nodes, obtain their coordinates (either via GPS or from a system administrator during startup) and the rest, n unknown-location nodes, must determine their own coordinates. If sensors were capable of high-power transmission, they would be able to make measurements to multiple reference nodes. Positioning techniques presented in other articles in this special issue, for cellular mobile station (MS) location or location in wireless local area networks (WLANs), could be applied. However, low-capability, energy-conserving devices will not include a power amplifier, will lack the energy necessary for long-range communication, and may be limited by regulatory constraints on transmit power. Instead, wireless sensor networks, and thus localization techniques, will be multihop (a.k.a. "cooperative" localization), as shown in Figure 1. Rather than solving for each sensor's position one at a time, a location solver (analogous to the system of masses connected by springs shown in Figure 2) will estimate all sensor positions simultaneously.

Such localization systems are an extension of techniques used in or proposed for WLAN and cellular MS location, as described elsewhere in this issue. We still allow unknown-location devices to make measurements with known-location references, but in cooperative localization, we additionally allow unknown-location devices to make measurements with other unknown-location devices. The additional information gained from these measurements between pairs of unknown-location devices enhances the accuracy and robustness of the localization system. In the considerable literature, such systems have alternatively been described as "cooperative," "relative," "distributed," "GPS-free," "multihop," or "network" localization; "self-localization;" "ad-hoc" or "sensor" positioning; or "network calibration." In this



[FIG1] (a) Traditional multilateration or multiangulation is a special case in which measurements are made only between an unknown-location sensor and known-location sensors. In (b) cooperative localization, measurements made between any pairs of sensors can be used to aid in the location estimate.

article, we use "cooperative" localization [7] to emphasize the communication and measurements between many pairs of sensors required to achieve localization for all sensors.

MOTIVATING APPLICATION EXAMPLE: ANIMAL TRACKING

If cooperative localization can be implemented as described above, many compelling new applications can be enabled. For the purposes of biological research, it is very useful to track animals over time and over very wide ranges [8]. Such tracking can answer questions about animal behavior and interactions within their own species as well as with other species. Using current practices, tracking is a very difficult, expensive process that requires bulky tags that rapidly run out of energy. A typical practice is to attach VHF transmitter collars to the animals to be tracked and then triangulate their location by driving (or flying) to various locations with a directional antenna. Alternatively, GPS-based collars can be used, but these are limited by cost concerns and offer only a short lifetime due to high energy consumption. Using wireless sensor networks can dramatically improve the abilities of biological researchers (as demonstrated by "ZebraNet" [8]). Using multihop routing of location data through the sensor network enables low transmit powers from the animal tags. Furthermore, interanimal distances, which are of particular interest to animal behaviorists, can be estimated using pair-wise measurements and cooperative localization methods without resorting to GPS. The end result of the longer battery lifetimes is less frequent recollaring of the animals being studied.

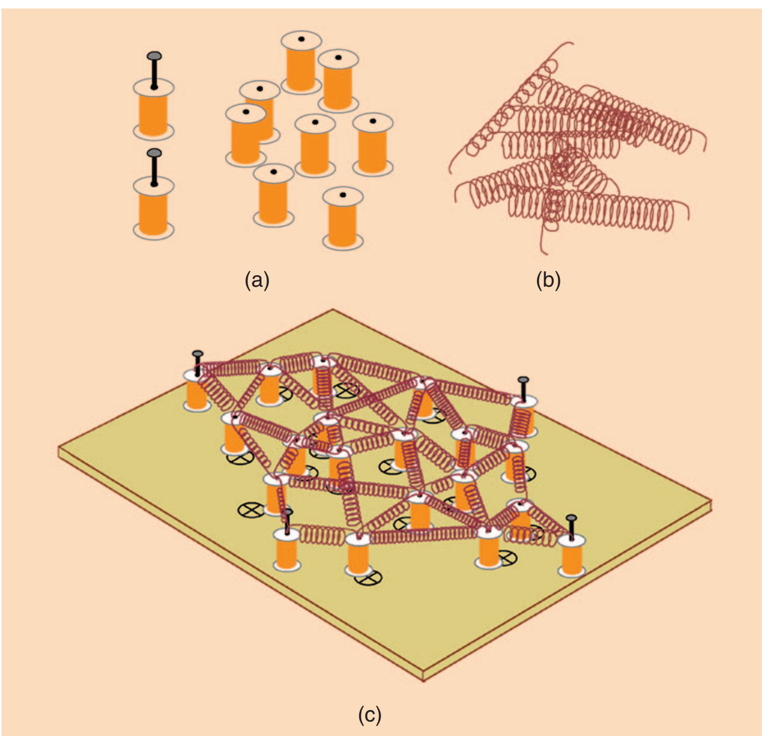
MOTIVATING APPLICATION EXAMPLE: LOGISTICS

As another example, consider deploying a sensor network in an office building, manufacturing floor, and warehouse. Sensors already play a very important role in manufacturing. The monitoring and control of machinery has traditionally been wired, but making these sensors wireless reduces the high cost of cabling and makes the manufacturing floor more dynamic. Automatic localization of these sensors further increases automation.

Also, boxes and parts to be warehoused as well as factory and office equipment are all tagged with sensors when first brought into the facility. These sensors monitor storage conditions (temperature and humidity) and help control the heating, ventilation, and air conditioning (HVAC) system. Sensors on mobile equipment report their location when the equipment is lost or needs to be found (e.g., during inventory), and even contact security if the equipment is about to "walk out" of the building. Knowing where parts and equipment are when they are critically required reduces the need to have duplicates as backup, savings which could pay for the wireless sensor network itself.

Radio-frequency identification (RFID) tags, such as those now required by Walmart on pallets and cartons entering its warehouses [9], represent a first step in warehouse logistics. RFID tags are only located when they pass within a few feet of a reader, thus remaining out of access most of their time in the warehouse. Networked wireless sensors, however, can be queried and located as long as they are within range (on the order of 10 m) of the closest other wireless sensor.

The accuracy of cooperative localization increases with the density of sensors, as we show later in a numerical example. Thus, having heterogeneous sensors of varied purposes, all participating in the same network helps drive localization errors down.



[FIG2] **Cooperative localization is analogous to finding the resting point of (a) masses (spools of thread) connected by a network of (b) springs. First, reference nodes are nailed to their known coordinates on a board. Springs have a natural length equal to measured ranges and can be compressed or stretched. They are connected to the pair of masses whose measured range they represent. After letting go, the (c) equilibrium point of the masses represent a minimum-energy localization estimate; the actual node locations are indicated by ⊗.**

COOPERATION REQUIREMENT: STANDARDIZATION

One way to ensure that heterogeneous sensors can "cooperate" to improve localization performance is to pursue standardization of wireless sensor networks. Two major sensor network standards are the IEEE 802.15.4 physical (PHY) layer and medium access control (MAC) layer standard for low-rate wireless personal area networks (LR-WPANs) and the ZigBee networking and application layer standard [10]. These standards enable localization information to be measured between pairs of sensors. In particular, RSS can be measured in the 802.15.4 PHY standard via the link quality indication (LQI), which reports the signal strength associated with a received packet to higher layers. Finally, we note that these standards are specifically tailored to low-power, long-life sensors. The 802.15.4 standard allows duty cycles of less than 0.1%; in active mode, devices will consume on the order of 40 mA, and in sleep mode, on the order of 40 μA.

PROBLEM STATEMENT

Before going into detail, it is useful to formally state the cooperative sensor location estimation problem. The two-dimensional (2-D) localization problem demonstrated in Figures 1 and 2 is the estimation of $2n$ unknown-location node coordinates $\boldsymbol{\theta} = [\boldsymbol{\theta}_x, \boldsymbol{\theta}_y]$

$$\boldsymbol{\theta}_x = [x_1, \ldots, x_n], \quad \boldsymbol{\theta}_y = [y_1, \ldots, y_n] \tag{1}$$

given the known reference coordinates $[x_{n+1}, \ldots, x_{n+m}, y_{n+1}, \ldots, y_{n+m}]$, and pair-wise measurements $\{X_{i,j}\}$, where $X_{i,j}$ is a measurement between devices i and j. While we treat the 2-D case here, extension to a three-dimensional (3-D) case appends a vector $\boldsymbol{\theta}_z$ to parameter vector $\boldsymbol{\theta}$ [11]. Measurements $X_{i,j}$ could be any physical reading that indicates distance or relative position, such as TOA, AOA, RSS, or connectivity (whether or not two devices can communicate). We do not assume full measurements, so we define the set $H(i)$ to be the set of sensors with which sensor i makes measurements. Clearly, $i \notin H(i)$, and $H(i) \subset \{1, \ldots, n+m\}$. Note that these measurements could be attained via different modalities, e.g., RF, infrared (IR), acoustics [12], [13], or a combination of these [14]. Finally, TOA can be measured using different signaling techniques, such as direct-sequence spread-spectrum (DS-SS) [15], [16] or UWB [17]–[19]. We discuss these measurement methods in the section on measurement characterization.

Some research has further assumed that various nodes may have imperfect prior information about their coordinates; for example, reference coordinates obtained from GPS may be random, but from a known distribution. Also, other localization research has focused on truly "relative" location, i.e., when no references exist ($m = 0$) and an arbitrary coordinate system can be chosen. This is also called "beacon-free" sensor localization in the article by Sun et al. [88]. These are important directions of research, but to simplify the discussion in this article, we leave these extensions to [12], [20], and [21].

MOTIVATION AND OUTLINE

The main goal of this article is to provide an introduction to the sensor location estimation problem from a signal processing perspective. We review both theoretical estimation bounds and methods and the algorithms being applied to the cooperative localization problem. We believe that signal processing methods will be very useful for aiding system design decisions as well as in localization algorithms themselves.

We discuss reasons for unavoidable limits to localization accuracy and present measurement-based statistical models for RSS, TOA, and AOA measurements. We then use these models to present lower bounds on sensor location estimation variance. The scope of this article does not include a detailed description of localization algorithms for sensor networks. We do, however, describe the main categories of approaches and provide references to the growing literature on localization algorithms.

WHY ARE MEASUREMENTS RANDOM?

Range and angle measurements used for localization are measured in a physical medium that introduces errors. Generally, these measurements are impacted by both time-varying errors and static, environment-dependent errors. Time-varying errors (e.g., due to additive noise and interference) can be reduced by averaging multiple measurements over time. Environment-dependent errors are the result of the physical arrangement of objects (e.g., buildings, trees, and furniture) in the particular environment in which the sensor network is operating. Since the environment is unpredictable, these errors are unpredictable and we model them as random. However, in a particular environment, objects are predominantly stationary. Thus, for a network of mostly stationary sensors, environment-dependent errors will be largely constant over time.

The majority of applications of wireless sensor networks involve mostly stationary sensors. Because time delay is acceptable in these applications, each pair of sensors will make multiple measurements over time and average the results together to reduce the impact of time-varying errors. We must characterize the statistics of these measurements after time-averaging to determine the performance of localization in wireless sensor networks.

Measurement experiments have provided much data about the statistics of RSS, TOA, and AOA in sensor networks, which we will discuss in this article. We begin by discussing the methodology of these measurement experiments.

MEASUREMENT CHARACTERIZATION

Ideally, statistical characterization of sensor network measurements would proceed as follows: 1) Deploy K wireless sensor networks, each with N sensors positioned with the identical geometry in the same type of environment, but with each network in a different place. For example, we might test a sensor network deployed in a grid in K different office buildings. 2) In each deployment, make many measurements between all possible pairs of devices. 3) Repeat each measurement over a short time period and compute the time average. 4) Then, the joint distribution (conditional on the particular sensor geometry) of the time-averaged measurements could be characterized. To our knowledge, no such wide-scale measurements have been presented due to the huge scale of the task. First, a large K would be required to characterize the joint distribution. Second, the result would only be valid for that particular N and those particular sensor coordinates. Each different geometry would require a different measurement experiment!

Measurements attained to date have made simplifying assumptions about the measurement model. Basically, it is assumed that measurements in a network are independent and from the same family of distributions. The independence assumption, which says that observing an error in one link does not provide any information about whether or not errors occur in different links, is a simplifying assumption [22]. Large obstructions may affect a number of similarly positioned links in a network. Considering correlations between links would make the analysis more difficult, and future research is needed to characterize the effects of link dependencies.

The second simplifying assumption is the choice of a family of distributions. We tend to subtract from each measurement its ensemble mean and then assume that the error (the difference) is characterized by a particular parameterized distribution (such as a Gaussian, log-normal, or mixture distribution). We then use the measurements to estimate the parameters of the distribution, such as the variance. With this method, one set of parameters can be used to characterize the whole set of measurements.

Other articles in this issue also discuss statistical models for location measurements. Compared to Gustafsson and Gunnarsson [87], who present models for RSS, TOA, and AOA measurements useful for cellular mobile system (MS) location and tracking, our focus is on the shorter-range, low-antenna, sensor network environment. Gezici et al. investigate UWB measurement models for both RSS and TOA in much greater detail than in this article, in which UWB is just one of multiple measurement modalities [89].

As an online supplement to this article [23], we provide a set of TOA and RSS measurements from a 44-node indoor sensor network originally reported in [24], to allow researchers to test localization algorithms on measured data. Next, we discuss what those measurements and many other measurements of RSS, TOA, and AOA have indicated about the distributions of the error in pair-wise sensor measurements.

RSS

RSS is defined as the voltage measured by a receiver's received signal strength indicator (RSSI) circuit. Often, RSS is equivalently reported as measured power, i.e., the squared magnitude of the signal strength. We can consider the RSS of acoustic, RF, or other signals. Wireless sensors communicate with neighboring sensors, so the RSS of RF signals can be measured by each receiver during normal data communication without presenting additional bandwidth or energy requirements. RSS measurements are relatively inexpensive and simple to implement in hardware. They are an important and popular topic of localization research. Yet, RSS measurements are notoriously unpredictable. If they are to be a useful part of a robust localization system, their sources of error must be well understood.

MAJOR SOURCES OF ERROR

In free space, signal power decays proportional to d^{-2}, where d is the distance between the transmitter and receiver. In real-world channels, multipath signals and shadowing are two major sources of environment dependence in the measured RSS. Multiple signals with different amplitudes and phases arrive at the receiver, and these signals add constructively or destructively as a function of the frequency, causing frequency-selective fading. The effect of this type of fading can be diminished by using a spread-spectrum method (e.g., direct-sequence or frequency hopping) that averages the received power over a wide range of frequencies. Spread-spectrum receivers are an acceptable solution since spread-spectrum methods also reduce interference in the unlicensed bands in which wireless sensors typically operate. The measured received power using a wideband method (as the bandwidth $\to \infty$) is equivalent to measuring the sum of the powers of each multipath signal [25].

Assuming that frequency-selective effects are diminished, environment-dependent errors in RSS measurements are caused by shadowing, such as the attenuation of a signal due to obstructions (furniture, walls, trees, buildings, and more) that a signal must pass through or diffract around on the path between the transmitter and receiver. As discussed at the start of this section, these shadowing effects are modeled as random (as a function of the environment in which the network is deployed). An RSS model considers the randomness across an ensemble of many deployment environments.

STATISTICAL MODEL

Typically, the ensemble mean received power in a real-world, obstructed channel decays proportional to d^{-n_p}, where n_p is the path-loss exponent, typically between two and four. The ensemble mean power at distance d is typically modeled as

$$\bar{P}(d) = P_0 - 10 n_p \log \frac{d}{d_0}, \qquad (2)$$

where P_0 is the received power (dBm) at a short reference distance d_0.

The difference between a measured received power and its ensemble average, due to the randomness of shadowing, is modeled as log-normal (i.e., Gaussian if expressed in decibels). The log-normal model is based on a wide variety of measurement results [24], [26]–[28] and analytical evidence [29]. The standard deviation of received power (when received power is expressed in dBm), σ_{dB}, is expressed in units of dB and is relatively constant with distance. Typically, σ_{dB} is as low as four and as high as 12 [27]. Thus, the received power (dBm) at sensor i transmitted by j, $P_{i,j}$ is distributed as

$$f\left(P_{i,j} = p|\boldsymbol{\theta}\right) = \mathcal{N}\left(p; \bar{P}(d_{i,j}), \sigma_{\text{dB}}^2\right), \qquad (3)$$

where $\mathcal{N}(x; y, z)$ is our notation for the value at x of a Gaussian probability density function (pdf) with mean y and variance z, $\boldsymbol{\theta}$ is the coordinate parameter vector from (1), and the actual transmitter-receiver separation distance $d_{i,j}$ is given by

$$d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}. \qquad (4)$$

The most important result of the log-normal model is that RSS-based range estimates have variance proportional to their actual range. This is not a contradiction of the earlier statement that σ_{dB} is constant with range. In fact, constant standard deviation in decibels means that the multiplicative factors are constant with range; this explains the proportionality. For example, consider a multiplicative factor of 1.5. At an actual range of 100 m, we would measure a range of 150 m and an error of 50 m; at 10 m, the measured range would be 15 m with an error of 5 m, a factor of ten smaller. This is why RSS errors are referred to as multiplicative, in comparison to the additive TOA errors presented in the section on TOA. Clearly, RSS is most valuable in high-density sensor networks.

CALIBRATION AND SYNCHRONIZATION

In addition to the path loss, measured RSS is also a function of the calibration of both the transmitter and receiver. Depending on the expense of the manufacturing process, RSSI circuits and transmit powers will vary from device to device. Also, transmit powers can change as batteries deplete. Sensors might be designed to measure and report their own calibration data to their neighbors.

Alternatively, each sensor's transmit power can be considered an unknown parameter to be estimated. This means that the unknown vector $\boldsymbol{\theta}$ described earlier is augmented to include the actual transmit power of each sensor along with its coordinates. Or, analogous to time difference of arrival (TDOA) measurements, we can consider only the differences between RSS measured at pairs of receivers [30]. The RSS difference between two sensors indicates information about their relative distance from the transmitter and removes the dependency on the actual transmit power. We leave discussion of localization algorithms to a later section.

TOA

TOA is the measured time at which a signal (RF, acoustic, or other) first arrives at a receiver. The measured TOA is the time of transmission plus a propagation-induced time delay. This time delay, $T_{i,j}$, between transmission at sensor i and reception at sensor j, is equal to the transmitter-receiver separation distance, $d_{i,j}$, divided by the propagation velocity, v_p. This speed for RF is approximately 10^6 times as fast as the speed of sound; as a rule of thumb, for acoustic propagation, 1 ms translates to 1 ft (0.3 m), while for RF, 1 ns translates to 1 ft.

The cornerstone of time-based techniques is the receiver's ability to accurately estimate the arrival time of the line-of-sight (LOS) signal. This estimation is hampered both by additive noise and multipath signals.

MAJOR SOURCES OF ERROR: ADDITIVE NOISE

Even in the absence of multipath signals, the accuracy of the arrival time is limited by additive noise. Estimation of time delay in additive noise is a relatively mature field [31]. Typically, the TOA estimate is the time that maximizes the cross-correlation between the received signals and the known transmitted signal. This estimator is known as a simple cross-correlator (SCC). The generalized cross-correlator (GCC) derived by Knapp and Carter [32] (the maximum likelihood estimator (MLE) for the TOA) extends the SCC by applying prefilters to amplify spectral components of the signal that have little noise and attenuate components with large noise. As such, the GCC requires knowledge (or estimates) of the signal and noise power spectra.

For a given bandwidth and signal-to-noise ratio (SNR), our time-delay estimate can only achieve a certain accuracy. The CRB provides a lower bound on the variance of the TOA estimate in a multipath-free channel. For a signal with bandwidth B in (hertz), when B is much lower than the center frequency, F_c (Hz), and signal and noise powers are constant over the signal bandwidth [33]

$$\text{var}(\text{TOA}) \geq \frac{1}{8\pi^2\, B\, T_s F_c^2\, \text{SNR}}, \tag{5}$$

where T_s is the signal duration in seconds. By designing the system to achieve sufficiently high SNR, the bound predicted by the CRB in (5) can be achieved in multipath-free channels. Thus (5) provides intuition about how signal parameters like duration, bandwidth, and power affect our ability to accurately estimate the TOA. For example, doubling either the transmission power or the bandwidth will cut ranging variance in half. This CRB on TOA variance is complementary to the bound that will be presented for location variance because the location variance bound requires, as an input, the variance of the TOA estimates.

MAJOR SOURCES OF ERROR: MULTIPATH

TOA-based range errors in multipath channels can be many times greater than those caused by additive noise alone. Essentially, all late-arriving multipath components are self-interference that effectively decrease the SNR of the desired LOS signal. Rather than finding the highest peak of the cross-correlation, in the multipath channel, the receiver must find the first-arriving peak because there is no guarantee that the LOS signal will be the strongest of the arriving signals. This can be done by measuring the time that the cross-correlation first crosses a threshold. Alternatively, in template-matching, the leading edge of the cross-correlation is matched in a least-squares (LS) sense to the leading edge of the auto-correlation (the correlation of the transmitted signal with itself) to achieve subsampling time resolutions [16]. Generally, errors in TOA estimation are caused by two problems:

- *Early-arriving multipath*. Many multipath signals arrive very soon after the LOS signal, and their contributions to the cross-correlation obscure the location of the peak from the LOS signal.
- *Attenuated LOS*. The LOS signal can be severely attenuated compared to the late-arriving multipath components, causing it to be "lost in the noise" and missed completely; this leads to large positive errors in the TOA estimate.

In dense sensor networks, in which any pair of sensors can measure TOA, we have the distinct advantage of being able to measure TOA between nearby neighbors. As the path length

ULTRA-WIDEBAND AND LOCALIZATION

UWB communication employs narrow pulses of very short (subnanosecond) duration that result in radio signals that are broadly spread in the frequency domain. The article by Gezici et al. [89] provides a detailed overview of UWB-based localization. A signal is considered to be UWB if either its fractional bandwidth (the ratio of its bandwidth to its center frequency) is larger than 0.2 or if it is a multiband signal with total bandwidth greater than 500 MHz. In 2003, the U.S. Federal Communications Commission (FCC) approved the commercialization and operation of UWB devices for public safety and consumer applications. Among the envisaged applications are wireless networking and localization. Standardization of UWB is underway, including the development of a high-bit-rate UWB physical layer that supports peer-to-peer ranging, in IEEE task group 802.15.3a, and potentially in IEEE task group 802.15.4a [35].

The very high bandwidth of UWB leads to very high temporal resolution, making it ideal for high-precision radiolocation applications. Implementations of UWB-based range measurements, reported in [17]–[19] and [41], have demonstrated RMS ranging errors of 0.4 to 5 feet (0.12 to 1.5 m).

decreases, the LOS signal power (relative to the power in the multipath components) generally increases [34]. Thus, the severely attenuated LOS problem is only severe in networks with large intersensor distances.

While early-arriving multipath components cause smaller errors, they are very difficult to combat. Generally, wider signal bandwidths are necessary to obtain greater temporal resolution. The peak width of the autocorrelation function is inversely proportional to the signal bandwidth. A narrow autocorrelation peak enhances the ability to pinpoint the arrival time of a signal and helps in separating the LOS signal cross-correlation contribution from the contributions of the early-arriving multipath signals. Wideband direct-sequence spread-spectrum (DS-SS) or UWB signals (see "Ultra-Wideband and Localization") are popular techniques for high-bandwidth TOA measurements. However, wider bandwidths require higher speed signal processing, higher device costs, and possibly higher energy costs. Standards proposed to the IEEE 802.15 Alternative PHY Task Group 3a quote receiver power consumptions on the order of 200 mW [35]. And, although high-speed circuitry typically means higher energy consumption, the extra bandwidth can be used to lower the time-average power consumption. Transferring data packets in less time means spending more time in standby mode.

Finally, note that time delays in the transmitter and receiver hardware and software add to the measured TOA. While the nominal delays are typically known, variance in component specifications and response times can be an additional source of TOA variance.

STATISTICAL MODEL

Measurements have shown that for short-range measurements, measured time delay can be roughly modeled as Gaussian

$$f\left(T_{i,j} = t|\theta\right) = \mathcal{N}\left(t; d_{i,j}/v_p + \mu_T, \sigma_T^2\right), \quad (6)$$

where μ_T and σ_T^2 are the mean and variance of the time delay error, $\boldsymbol{\theta}$ is defined in (1), $d_{i,j}$ is given in (4), and v_p is the propagation velocity. Wideband DS-SS measurements reported in [24] supported the Gaussian error model and showed $\mu_T = 10.9$ ns and $\sigma_T = 6.1$ ns. UWB measurements conducted on a mostly empty Motorola factory floor showed $\mu_\mathrm{T} = 0.3$ ns and $\sigma_T = 1.9$ ns. This mean error μ_T can be estimated (as a nuisance parameter) by the localization algorithm so that it can be subtracted out [17].

However, the presence of large errors can complicate the Gaussian model. These errors make the tails of the distribution of measured TOA heavier than Gaussian and have been modeled using a mixture distribution. With a small probability, the TOA measurement results from a different, higher-variance distribution, as described in [36] as well as by Gustafsson and Gunnarsson [87]. Localization systems should be designed to be robust to these large errors, also called NLOS errors. For TOA measurements made over time in a changing channel, the TOAs that include excess delays can be identified and ignored [36]. Even in static channels, if the number of range measurements to a device is greater than the minimum required, the redundancy can be used to identify likely NLOS errors [37], [38]. Localization algorithm robustness is further addressed in a later section.

CALIBRATION AND SYNCHRONIZATION

If wireless sensors have clocks that are accurately synchronized, then the time delay is determined by subtracting the known transmit time from the measured TOA. Sensor network clock synchronization algorithms have reported precisions on the order of 10 μs [39]. Because of the difference in propagation speed, such clock accuracies are adequate for acoustic signals [14] but not for RF signals.

For TOA in asynchronous sensor networks, a common practice is to use two-way (or round-trip) TOA measurements. In this method, one sensor transmits a signal to a second sensor, which immediately replies with its own signal. At the first sensor, the measured delay between its transmission and its reception of the reply is twice the propagation delay plus a reply delay internal to the second sensor. This internal delay is either known, or measured and sent to the first sensor to be subtracted. Multiple practical two-way TOA methods have been reported in the literature [15], [18], 40], [41]. Generally each pair of sensors measures round-trip TOA separately in time. But, if the first sensor has the signal processing capability, multiple sensors can reply at the same time, and two-way TOAs can be estimated simultaneously using multiuser interference cancellation [40].

The state of each sensor's clock (its bias compared with absolute time) can also be considered to be an unknown parameter and included in the parameter vector $\boldsymbol{\theta}$. In this case, one-way TOA is measured and input to a localization algorithm that estimates both the sensor coordinates and the biases of each sensor's clock [42]. The difference between the arrival times of the same signal at two sensors is called the TDOA. A TDOA measurement does not depend on the clock bias of the transmitting sensor. For decades, TDOA methods have been used in source localization for locating asynchronous transmitters; they find application in GPS and cellular localization. Under certain weak conditions, it has been shown that TOA with clock bias (treated as an unknown parameter) is equivalent to TDOA [43].

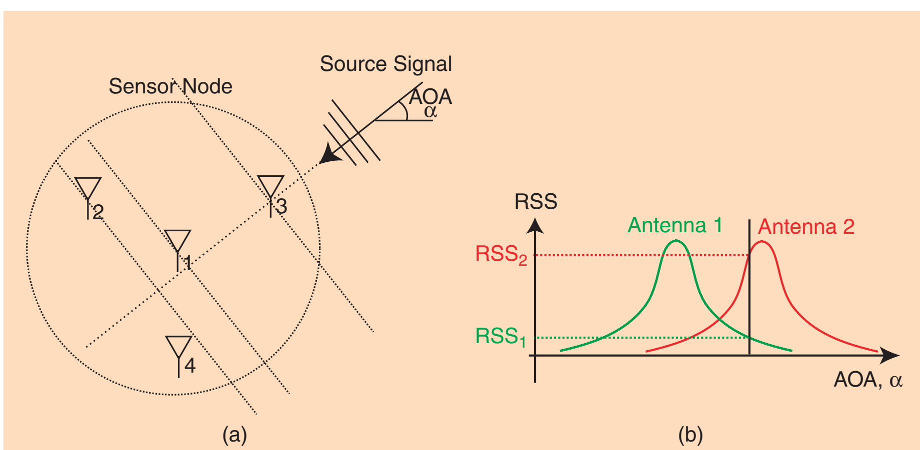
AOA

By providing information about the direction to neighboring sensors rather than the distance to neighboring sensors, AOA measurements provide localization information complementary to the TOA and RSS measurements discussed above.

There are two common ways that sensors measure AOA (as shown in Figure 3). The most common method is to use a sensor array and employ so-called array signal processing techniques at the sensor nodes. In this case, each sensor node is comprised of two or more individual sensors (microphones for acoustic signals or antennas for RF signals) whose locations with respect to the node center are known. A four-element Y-shaped microphone array is shown in Figure 3(a). The AOA is estimated from the differences in arrival times for a transmitted signal at each of the sensor array elements. The estimation is similar to time-delay estimation discussed in the section on TOA measurements but generalized to the case of more than two array elements. When the impinging signal is narrowband (that is, its bandwidth is much less than its center frequency), then a time delay τ relates to a phase delay ϕ by $\phi = 2\pi f_c \tau$ where f_c is the center frequency. Narrowband AOA estimators are often formulated based on phase delay. See [44]–[46] for more detailed discussions on AOA estimation algorithms and their properties.

A second approach to AOA estimation uses the RSS ratio between two (or more) directional antennas located on the sensor [see Figure 3(b)]. Two directional antennas pointed in different directions, such that their main beams overlap, can be used to estimate the AOA from the ratio of their individual RSS values.

Both AOA approaches require multiple antenna elements, which can contribute to sensor device cost and size. However, acoustic sensor arrays may already be required in devices for many environmental monitoring and security applications, in which the purpose of the sensor network is to identify and locate acoustic sources [47]. Locating the sensors themselves using acoustics in these applications is a natural extension. RF antenna arrays imply large device size unless center frequencies are very high. However, available bandwidth and decreasing manufacturing costs at millimeter-wave frequencies may make them desirable for sensor network applications. For example, at 60 GHz, higher attenuation due to oxygen absorption helps to mitigate multipath and accurate indoor AOA measurements have been demonstrated [48].



[FIG3] AOA estimation methods. (a) AOA is estimated from the TOA differences among sensor elements embedded in the node; a four-element Y-shaped array is shown. (b) AOA can also be estimated from the RSS ratio RSS_1/RSS_2 between directional antennas.

MAJOR SOURCES OF ERROR AND STATISTICAL MODEL

AOA measurements are impaired by the same sources discussed in the TOA section: additive noise and multipath. The resulting AOA measurements are typically modeled as Gaussian, with ensemble mean equal to the true angle to the source and standard deviation σ_α. Theoretical results for acoustic-based AOA estimation show standard deviation bounds on the order of $\sigma_\alpha = 2^\circ$ to $\sigma_\alpha = 6^\circ$, depending on range [49]. Estimation errors for RF AOA on the order of $\sigma_\alpha = 3^\circ$ have been reported using the RSS ratio method [50].

CALIBRATION AND SYNCHRONIZATION

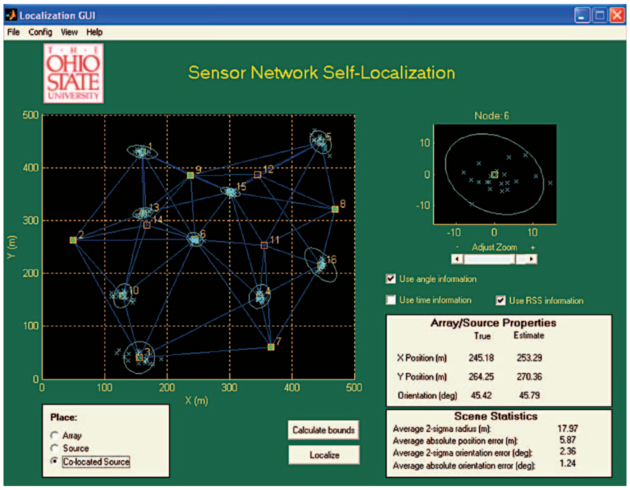
It is not likely that sensors will be placed with known orientation. When sensor nodes have directionality, the network localization problem must be extended to consider each sensor's orientation as an unknown parameter to be estimated along with position. In this case, the unknown vector $\boldsymbol{\theta}$ is augmented to include the orientation of each sensor.

The models presented earlier are sufficient to find bounds on localization performance in cooperative localization. These lower bounds are not a function of the particular localization algorithm employed. Thus we present some of these performance limits in the following section before discussing current algorithm research.

LIMITS ON LOCALIZATION COVARIANCE

The CRB provides a means for calculating a lower bound on the covariance of any unbiased location estimator that uses RSS, TOA, or AOA measurements. Such a lower bound provides a useful tool for researchers and system designers. Without testing particular estimation algorithms, a designer can quickly find the "best-case" using particular measurement technologies.

Researchers who are testing localization algorithms, like those presented in the later section on algorithms for location estimation, can use the CRB as a benchmark for a particular algorithm. If the bound is nearly achieved, then there is little reason to continue working to improve that algorithm's accuracy. Furthermore, the bound's functional dependence on particular parameters helps to provide insight into the behavior of cooperative localization.



[FIG4] Lower bounds and Monte Carlo ML estimates can be calculated interactively using this MATLAB-based GUI developed by Joshua Ash at Ohio State University and freely available online [53]. Sensors can be placed arbitrarily and their capabilities and a priori location information given. The user may select any combination of AOA, TOA, and RSS measurements.

The bound on estimator covariance is a function of the following:

1) number of unknown-location and known-location sensors
2) sensor geometry
3) whether localization is in two or three dimensions
4) measurement type(s) implemented (i.e., RSS, TOA, or AOA)
5) channel parameters (such as σ_{dB} and n_p in RSS, σ_T in TOA, or σ_α in AOA measurements)
6) which pairs of sensors make measurements (network connectivity)
7) nuisance (unknown) parameters that must also be estimated (such as clock bias for TOA or orientation for AOA measurements).

As an online supplement to this article, we provide public access to a multifeatured MATLAB-based code and GUI for the calculation of the localization CRB [53], as shown in Figure 4. This code can determine bounds when any combination of RSS, TOA, and AOA measurements is used. It allows the inclusion of device orientation and clock biases as unknown nuisance parameters. Sensors can be arranged visually using the GUI and the bound can be calculated. For each sensor, the GUI displays the CRB by plotting the lower bound on the 2-σ uncertainty ellipse. The tool also includes the ability to run Monte Carlo simulations that estimate sensor parameters and coordinates

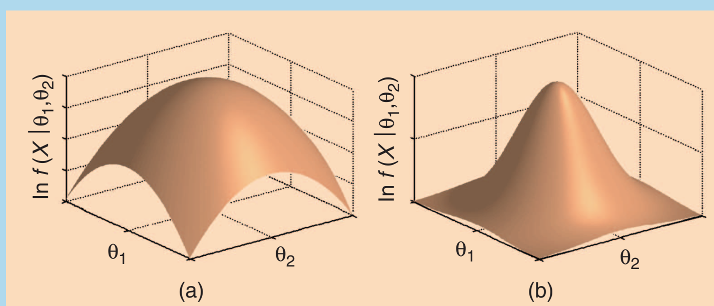
WHAT IS THE CRAMÉR-RAO BOUND?

The Cramér-Rao bound (CRB) provides a lower bound on the variance achievable by any unbiased location estimator [54]. The bound is useful as a guideline: knowing the best an estimator can possibly do helps us judge the estimators that we implement. Essentially, the CRB is a general uncertainty principle that we apply in this article to location estimation. It is surprising, to those without a priori knowledge of the CRB, that we can calculate the lower bound on estimation variance without ever considering a single estimation method. All that is needed to calculate a CRB is the statistical model of the random measurements, i.e., $f(\mathbf{X}|\theta)$, where $\mathbf{X}$ is the random measurement and θ are the parameters that are to be estimated from the measurements. Any unbiased estimator $\hat{\theta}$ must satisfy

$$\text{Cov}(\hat{\theta}) \geq \left\{E\left[-\nabla_\theta(\nabla_\theta \ln f(\mathbf{X}|\theta))^T\right]\right\}^{-1}, \tag{11}$$

where $\text{Cov}(\hat{\theta})$ is the covariance of the estimator, E[·] indicates expected value, ∇_θ is the gradient operator w.r.t. the vector θ, and superscript ^{T} indicates transpose.

The bound is very similar to sensitivity analysis, applied to random measurements. The CRB is based on the curvature of the log-likelihood function, $\ln f(\mathbf{X}|\theta)$. Intuitively, if the curvature of the log-likelihood function is very sharp like the example plot in Figure 5(b), then the optimal parameter estimate can be accurately identified. Conversely, if the log-likelihood is broad with small curvature like the graph in Figure 5(a), then estimating the optimal will be more difficult.



[FIG5] Example log-likelihood functions for two-parameter estimation with (a) small and (b) large curvature. The variance bound will be higher in example (a) than in (b).

The CRB is limited to unbiased estimators. Such estimators provide coordinate estimates that, if averaged over enough realizations, are equal to the true coordinates. Although unbiased estimation is a very desirable property, some bias might be tolerated to reduce variance; in such cases, the bound can be adapted [55].

using the MLE that will be discussed. The Monte Carlo coordinate estimates are plotted on screen for comparison with the covariance bound.

In this section, we present some analytical results for the CRB. Our purpose is both to show how simple it is to calculate and to demonstrate it as a means to compare the three measurement methods presented earlier. To keep the formulation short, we make two simplifying assumptions: first, we address 2-D (rather than 3-D) localization and, second, we assume that channel and device parameters (orientation for AOA, transmit powers and n_p for RSS, and clock biases for TOA) are known. Analysis of bounds without these assumptions are left to [11], [12], [42], [51], and [52], which have presented details of these analytical CRBs for a variety of different measurement types.

CALCULATING THE CRB IN THREE STEPS

Under these two simplifying assumptions, the variance bounds based on measurements of RSS, TOA, and AOA are remarkably similar. Particular differences demonstrate how localization performance varies by measurement type. We show how to calculate the CRB for the estimate of $\boldsymbol{\theta}$ as given in (1) in three steps:

STEP 1: CALCULATE FISHER INFORMATION SUBMATRICES

First, form three $n \times n$ matrices: $\mathbf{F}_{xx}$, $\mathbf{F}_{xy}$, and $\mathbf{F}_{yy}$. As introduced earlier, n is the number of unknown-location sensors. The k, l element of each matrix is calculated as follows:

$$\left[\mathbf{F}_{xx}\right]_{k,l} = \begin{cases} \gamma \sum_{i \in H(k)} (x_k - x_i)^2 / d_{k,i}^s & k = l \\ -\gamma \, I_{H(k)}(l)(x_k - x_l)^2 / d_{k,l}^s & k \neq l \end{cases}$$
$$\left[\mathbf{F}_{xy}\right]_{k,l} = \begin{cases} \gamma \sum_{i \in H(k)} (x_k - x_i)(y_k - y_i) / d_{k,i}^s & k = l \\ -\gamma \, I_{H(k)}(l)(x_k - x_l)(y_k - y_l) / d_{k,l}^s & k \neq l \end{cases}$$
$$\left[\mathbf{F}_{yy}\right]_{k,l} = \begin{cases} \gamma \sum_{i \in H(k)} (y_k - y_i)^2 / d_{k,i}^s & k = l \\ -\gamma \, I_{H(k)}(l)(y_k - y_l)^2 / d_{k,l}^s & k \neq l. \end{cases} \quad (7)$$

Here, γ is a channel constant and s is an exponent, both of which are functions of the measurement type and are given in Table 1; d_{ij} is the true distance between i and j given in (4); and $I_{H(k)}(l)$ is the indicator function, (which allows us to include the information only if sensor k made a measurement with sensor l), $I_{H(k)}(l) = 1$ if $l \in H(k)$, or 0 if not.

STEP 2: MERGE SUBMATRICES TO FORM THE FIM

Next, we form the $2n \times 2n$ Fisher information matrix (FIM) $\mathbf{F}$ corresponding to the $2n$ coordinates in $\boldsymbol{\theta}$ that need to be estimated. For TOA or RSS, we select $\mathbf{F} = \mathbf{F}_{\text{TR}}$, while for AOA, we select $\mathbf{F} = \mathbf{F}_{\text{A}}$, where

$$\mathbf{F}_{\text{TR}} = \begin{bmatrix} \mathbf{F}_{xx} & \mathbf{F}_{xy} \\ \mathbf{F}_{xy}^T & \mathbf{F}_{yy} \end{bmatrix}, \quad \mathbf{F}_{\text{A}} = \begin{bmatrix} \mathbf{F}_{yy} & -\mathbf{F}_{xy} \\ -\mathbf{F}_{xy}^T & \mathbf{F}_{xx} \end{bmatrix}, \quad (8)$$

where $\mathbf{F}_{xx}$, $\mathbf{F}_{xy}$, and $\mathbf{F}_{yy}$ are given in (7), and we use the superscript ^{T} to indicate matrix transpose.

> **CRB AND GEOMETRIC DILUTION OF PRECISION**
>
> The geometric dilution of precision (GDOP), commonly used to describe localization performance of an estimator, is closely related to the CRB. If an estimator uses range measurements with variance σ_d^2 and achieves a location variance [as defined in (9)] of σ_i^2 at sensor i, then its GDOP is defined as GDOP $= \sigma_i / \sigma_d$. If the estimator was efficient (an efficient estimator is one that achieves the CRB), then the CRB on standard deviation is $\sigma_i = \sigma_d$GDOP.

[TABLE 1] DIFFERENCES IN CRB BY MEASUREMENT TYPE.

	CHANNEL CONSTANT γ	EXPONENT s	FIM **F**
TOA	$\gamma = 1/(v_p \sigma_T)^2$	$s = 2$	$\mathbf{F} = \mathbf{F}_{\text{TR}}$
RSS	$\gamma = \left(\frac{10 n_p}{\sigma_{dB} \log 10}\right)^2$	$s = 4$	$\mathbf{F} = \mathbf{F}_{\text{TR}}$
AOA	$\gamma = 1/\sigma_\alpha^2$	$s = 4$	$\mathbf{F} = \mathbf{F}_{\text{A}}$

STEP 3: INVERT THE FIM TO GET THE CRB

The CRB matrix is equal to $\mathbf{F}^{-1}$, the matrix inverse of the FIM. The diagonal of $\mathbf{F}^{-1}$ contains $2n$ values that are the variance bounds for the $2n$ parameters of $\boldsymbol{\theta}$. To say this more precisely, let an estimator of sensor i's coordinates be $\hat{\mathbf{z}}_i = [\hat{x}_i, \hat{y}_i]^T$. If we define the location variance of the estimator to be σ_i^2,

$$\sigma_i^2 \triangleq \text{tr}\left\{\text{cov}_\theta(\hat{\mathbf{z}}_i)\right\} = \text{Var}_\theta(\hat{x}_i) + \text{Var}_\theta(\hat{y}_i), \quad (9)$$

then the CRB asserts that,

$$\sigma_i^2 \geq \left(\mathbf{F}^{-1}\right)_{i,i} + \left(\mathbf{F}^{-1}\right)_{i+n,i+n}. \quad (10)$$

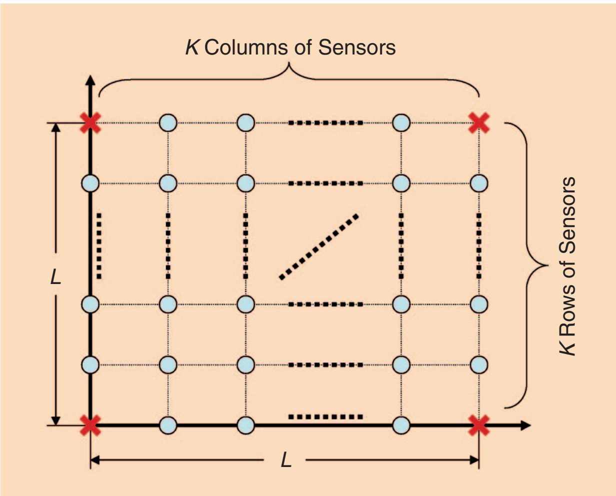
RESULTS SEEN FROM THE CRB

Even without calculating the CRB for a particular sensor network geometry, we can explore the scaling characteristics of the variance bound. What happens when the geometry and connectivity of the network and kept constant, but the dimensions of the network are scaled up proportionally?

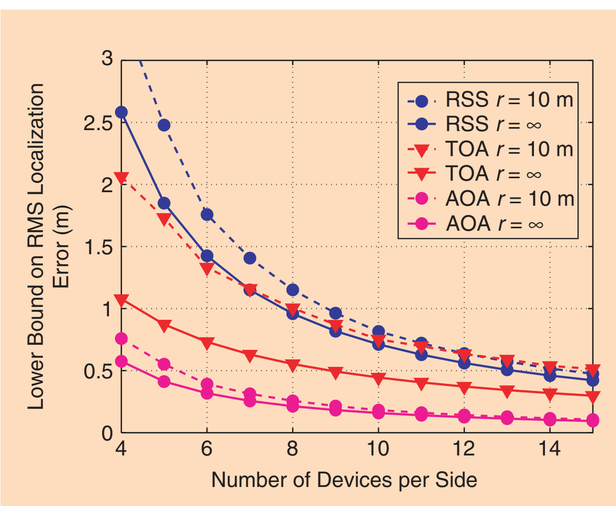
- *TOA:* TOA bounds will remain constant with a scaling of the dimensions. Note that since $s = 2$ for TOA, the fractions in (7) are unitless; if units of the coordinates were feet or even centimeters instead of meters, the ratios would be identical. Instead, the units come from the variance of ranging error, $v_p \sigma_T$.
- *RSS and AOA:* These bounds on standard deviation are proportional to the size of the system. Since $s = 4$ for RSS and AOA, the geometry ratios in (7) have units of $1/\text{distance}^2$, so the variance bound (the inverse) takes its units of distance^2 directly from this ratio. Note that the channel constant γ is unitless for both RSS and AOA.

Of course, channel parameters will change slowly as the path lengths change (TOA measurements over kilometer links would have higher variance than over 10-m links), but these scaling characteristics are good first-order approximations.

Finally, note that the bound on standard deviation of localization error is proportional to $(1/\gamma)^{1/2}$. It makes sense that the localization error is proportional to σ_T for TOA and σ_α for AOA. While not as obvious, we also find from the CRB that for RSS, the proportionality is to σ_{dB}/n_p. An RSS-based localization system operating in a high-path-loss exponent environment (often found when using ground-level antennas), while requiring higher transmit powers from sensors, also allows more accurate sensor localization.



[FIG6] Diagram showing layout of the K^2 sensors from the numerical example given in the test, with four reference sensors ($\times$) and $K^2 - 4$ unknown-location sensors ($\bullet$) in a $L \times L$ square area.



[FIG7] Lower bounds for localization standard deviation for the example described in the text when measurements are RSS (with $\sigma_{dB}/n_p = 1.7$ [24]), TOA (with $\sigma_T = 6.3$ ns [24]), and AOA (with $\sigma_\alpha = 5°$). Parameter r is the radius of connectivity; only pairs of sensors closer than r make measurements, and for $r = \infty$, all pairs make measurements.

NUMERICAL EXAMPLE

Consider a sensor network in a 20-m by 20-m area, with K^2sensors arranged into K rows and K columns, as shown in Figure 6. The four sensors in the corners are reference nodes, while the remaining $K^2 - 4$ are unknown-location nodes. Let's consider what happens to the localization variance bound as K increases, for the cases when measurements are:

1) RSS with $\sigma_{dB}/n_p = 1.7$ [24]
2) TOA with $\sigma_T = 6.1$ ns and $v_p = 3 \cdot 10^8$(m/s) [24]
3) AOA with $\sigma_\alpha = 5°$.

As presented above, the lower bounds for RSS, TOA, and AOA are proportional to these three channel parameters. We start by assuming that each sensor makes measurements with every other sensor in the network. We calculate the RMS value of the localization bound, i.e., $((1/n)\text{tr}\mathbf{F}^{-1})^{1/2}$, which gives an average of the bound over the entire $K^2 - 4$ unknown-location sensors. The result is shown in the solid lines in Figure 7 labeled as $r = \infty$. Next, we consider the realistic case in which each sensor only makes measurements to those sensors located within $r = 10$ m of itself. Of course, sensors will not really know exactly which sensors are within 10 m, but the connectivity implied by the 10-m radius provides a realistic test. In this case, the bound is shown as dotted lines in Figure 7 and labeled as $r = 10$ m.

Comparing performance of the measurement methods for the chosen parameters, AOA outperforms TOA and RSS, while RSS can perform as well as TOA at high sensor densities. Of course, these comparisons are based on the chosen values of the measurement parameters and the chosen geometry shown in Figure 6. As described earlier, these bounds are proportional to $1/\sqrt{\gamma}$ where γ is the channel constant given in Table 1. For example, if it was assumed that $\sigma_\alpha = 10°$ instead of 5°, the standard deviation bound for AOA would be twice that shown in Figure 7. Note that RSS and AOA bounds decrease more rapidly than TOA as the density increases. Also, for RSS and AOA, the difference between the $r = 10$ m and $r = \infty$ lines decreases dramatically as density increases. At high densities, the results show that little additional information comes from the distant sensors (> 10 m). For TOA, however, even distant sensors' measurements can provide significant localization information.

LOCATION ESTIMATION ALGORITHMS

This article has presented performance bounds for location estimators without mentioning particular algorithms. Detailed reviews of such algorithms could easily consume an entire article; in fact, such localization algorithm reviews have been presented [21], [56]. The article by Sun et al. [88] also presents details of many sensor localization algorithms. This article, however, does attempt to describe the general signal processing tools that have been deemed useful in reported cooperative localization algorithms.

While positioning and navigation have a long history (as evidenced in this issue), cooperative localization algorithms must extend existing methods by finding ways to use measurements (of range or angle) between pairs of unknown-location nodes.

The challenge is to allow sensors that are not in range of any known-location devices to be located and, further, to improve the location estimates of all sensors.

If all sensors were in range of multiple reference nodes, they could directly calculate their own locations. For example, in [57], nodes measure RSS from a dense network of reference nodes and calculate their location to be the mean of the locations of the in-range reference nodes. Yet, in most wireless sensor networks, to minimize installation expenses, reference nodes will be sparse. Also, low-energy sensors generally will not be in range of enough references (three or four for 2-D or 3-D localization, respectively).

SIGNAL PROCESSING METHODS WILL BE VERY USEFUL FOR AIDING SYSTEM DESIGN DECISIONS AS WELL AS IN COOPERATIVE LOCALIZATION ALGORITHMS THEMSELVES.

We divide cooperative localization into centralized algorithms, which collect measurements at a central processor prior to calculation, and distributed algorithms, which require sensors to share information only with their neighbors, but possibly iteratively.

CENTRALIZED ALGORITHMS

If the data is known to be described well by a particular statistical model (e.g., Gaussian or log-normal), then the MLE can be derived and implemented [24], [12]. One reason that these estimators are used is that their variance asymptotically (as the SNR ratio goes high) approaches the lower bound given by the CRB. As indicated by the name, the maximum of a likelihood function must be found. There are two difficulties with this approach.

1) *Local maxima:* Unless we initialize the MLE to a value close to the correct solution, it is possible that our maximization search may not find the global maxima.

2) *Model dependency:* If measurements deviate from the assumed model (or model parameters), the results are no longer guaranteed to be optimal.

One way to prevent local maxima is to formulate the localization as a convex optimization problem. In [58], convex constraints are presented that can be used to require a sensor's location estimate to be within a radius r and/or angle range $[\alpha_1, \alpha_2]$ from a second sensor. In [42], linear programming using a "taxi metric" is suggested to provide a quick means to obtain rough localization estimates. More general constraints can be considered if semidefinite programming (SDP) techniques are used [59]. One difficulty that must be overcome in both techniques is their high computational complexity. Toward this end, a distributed SDP-based localization algorithm was presented in [60].

Multidimensional scaling (MDS) algorithms (and Isomap) [61] formulate sensor localization from range measurements as an LS problem [62], [63]. In classical MDS, the LS solution is found by eigen-decomposition, which does not suffer from local maxima. To linearize the localization problem, the classical MDS formulation works with squared distance rather than distance itself, and the end result is very sensitive to range measurement errors. Other MDS-based techniques, not based on eigen-decomposition, can be made more robust by allowing measurements to be weighted according to their perceived accuracy [20].

While MDS and Isomap have complexity $\mathcal{O}(N^3)$, where $N = n + m$ is the total number of sensors, other manifold learning methods (such as local linear embedding (LLE) [64]) are also based on eigen-decomposition, but of sparse matrices, and are $\mathcal{O}(N^2)$. Manifold learning performance has been presented for the case when sensor data records are used as location information [65], and will likely play an important role when using other types of measurements. Also adapted from the statistical learning area, supervised learning approaches localization as a series of detection problems [66]. The covered area is split into smaller, overlapping regions; based on the measurements, each region detects whether or not the sensor is within its boundaries.

DISTRIBUTED ALGORITHMS

There are two big motivations for developing distributed localization algorithms. First, for some applications, no central processor (or none with enough computational power) is available to handle the calculations. Second, when a large network of sensors must forward all measurement data to a single central processor, there is a communication bottleneck and higher energy drain at and near the central processor.

Distributed algorithms for cooperative localization generally fall into one of two categories.

1) *Network multilateration*: Each sensor estimates its multihop range to the nearest reference nodes. These ranges can be estimated via the shortest path between the sensor and reference nodes, i.e., proportional to the number of hops or the sum of measured ranges along the shortest path [67]–[69]. Note that finding the shortest path is readily distributed across the network. When each sensor has multiple range estimates to known positions, its coordinates are calculated locally via multilateration [70], [71].

2) *Successive refinement*: These algorithms try to find the optimum of a global cost function, e.g., LS, weighted LS (WLS) [20], or maximum likelihood (ML). Each sensor estimates its location and then transmits that assertion to its neighbors [7], [72], [73]. Neighbors must then recalculate their location and transmit again, until convergence. A device starting without any coordinates can begin with its own local coordinate system and later merge it with neighboring coordinate systems [74]. Typically, better statistical performance is achieved by successive refinement compared to network multilateration, but convergence issues must be addressed.

Bayesian networks (or factor graphs) provide another distributed successive refinement method to estimate the probability density of sensor network parameters. These methods are particularly promising for sensor localization—each sensor stores a conditional density on its own coordinates, based on its measurements and the conditional density of its neighbors [75]. Alternatively, particle filtering methods (or Monte Carlo estimation methods) have each sensor store a set of "particles," i.e., candidate representations of its coordinates, weighted according to their likelihood [76], [77]. These methods have been used to accurately locate and track mobile robots [78]; they will likely find application in future sensor localization and tracking research.

> WE SHOW HOW TO CALCULATE A CRAMÉR-RAO BOUND ON THE LOCATION ESTIMATION ACCURACY POSSIBLE FOR A GIVEN SET OF MEASUREMENTS.

COMPARISON

Both centralized and distributed algorithms must face the high relative costs of communication. The energy required per transmitted bit could be used, depending on the hardware and the range, to execute 1,000 to 30,000 instructions [47]. Centralized algorithms in large networks require each sensor's measurements to be passed over many hops to a central processor, while distributed algorithms have sensors send messages only one hop (but possibly make multiple iterations). The energy efficiency of centralized and distributed estimation approaches can be compared [79]; in general, when the average number of hops to the central processor exceeds the necessary number of iterations, distributed algorithms will likely save communication energy costs.

There may be hybrid algorithms that combine centralized and distributed features to reduce the energy consumption beyond what either one could do alone. For example, if the sensor network is divided into small clusters, an algorithm could select a processor from within each cluster to estimate a map of the cluster's sensors. Then, cluster processors could operate a distributed algorithm to merge and optimize the local estimates, such as described in [80]. Such algorithms are a promising topic for future research.

FUTURE RESEARCH NEEDS AND CONCLUSION

Ultimately, actual localization performance will depend on many things, including the localization algorithm used, the size and density of the network, the quantity of prior coordinate information, the measurement method chosen, and the accuracies possible from those measurements in the environment of interest (the γ of Table 1). However, based on the characteristics of the variance bounds presented in the section on limits on localization covariance, we can make some broad generalizations. It appears that TOA measurements will be most useful in low-density sensor networks, since they are not as sensitive to increases in interdevice distances as RSS and AOA. Both AOA and TOA are typically able to achieve higher accuracy than RSS; however, that accuracy can come with higher device costs. Because of their scaling characteristics, localization based on RSS and AOA measurements can, without sacrificing much accuracy, avoid taking measurements on longer-distance links; instead, they focus on those links between nearest neighbors. RSS measurements will allow accurate localization in dense networks, and will be very attractive due to their low cost to system designers.

Considerable literature has studied cooperative localization with an emphasis on algorithms; less research has placed the emphasis on localization as estimation. Accordingly, bias and variance performance are often secondary concerns. While a notable algorithm comparison is seen in [81], in general, it is difficult to compare the performance of localization algorithms in the literature. Reporting both bias and variance performance along with the Cramér-Rao lower bound (CRLB) will help provide a reference for comparison.

While simulation will be very valuable, the next step in cooperative localization research is to test algorithms using measured data. However, measurements of RSS, TOA, and AOA in wireless sensor networks have only begun to be reported, largely because of the complexity of such measurement campaigns. To conduct measurements in a N-sensor network requires $\mathcal{O}(N^2)$ measurements, and multiple sensor networks must be measured to develop and test statistical models. Despite the complexity, data from such measurement campaigns will be of key importance to sensor network researchers.

Such measurements should consider joint statistics of RSS, TOA, and AOA. While this article has considered them separately, multiple modalities measured together may provide more information than just the sum of their parts. For example, together, angle and time (spatio-temporal) measurements can cross-check for NLOS errors; if at the leading edge in the receiver's cross-correlation, power from multiple angles are measured, then it is apparent that the leading edge is not a direct LOS signal. This example is part of a bigger issue of determining sufficient statistics of joint spatio-temporal-signal strength measurements, which is still an open research topic.

Three other future directions for cooperative localization research are suggested: mobile sensor tracking, the use of connectivity measurements, and routing using virtual coordinates.

MOBILE SENSOR TRACKING

This article has not discussed sensor mobility. Mobility creates the problem of locating and tracking moving sensors in real time, and also the opportunity to improve sensor localization. Detecting movement of a sensor in a network of communication or energy-constrained nodes is a distributed detection problem that has yet to be fully explored. For the problem of passive tracking of sources in the environment, a review is presented in [47]; but the problem of tracking active sensors has not been sufficiently addressed as a collaborative signal processing problem. The sensor tracking problem is an important aspect of many applications, including the animal tracking and logistics applications discussed earlier.

If a sensor makes multiple measurements to its neighbors as it moves across space, it has the opportunity to reduce environment-dependent errors by averaging over space. The multiple measurements are useful to help improve coordinate estimates for other sensors in the network, not just the mobile node. Researchers have tested schemes that use mobile sources and sensors to achieve cooperative localization [82], [83]; however, further opportunities to exploit mobility remain to be explored.

CONNECTIVITY MEASUREMENTS

Connectivity (a.k.a. proximity) is a binary measurement of whether or not two devices are in communication range of each other. Typical digital receivers have a minimum received power below which it is unlikely that a packet will be correctly decoded. Connectivity can be considered to be a binary quantization of RSS. As a good approximation, when the RSS is below a power threshold, two devices will not be connected; when the RSS is above the threshold, two devices will be connected. As a binary quantization of RSS, it is clear that connectivity is less informative than RSS and will result in higher localization variance bounds [84]. Research in connectivity-based localization is often called “range-free” localization. The assumption that connectivity does not suffer from the same fading phenomena as RSS, and instead that radio coverage is a perfect circle around the transmitter, can be a valuable simplification during algorithm development. However, this assumption cannot be used to accurately test the performance of such algorithms. Range-free localization algorithms can be simulated by generating measurements of RSS between each pair of sensors using the log-normal model of (3) and then considering each pair with an RSS measurement above a threshold power to be connected.

ROUTING USING VIRTUAL COORDINATES

Geographic routing is an application of sensor localization. The use of the coordinates of sensors can reduce routing tables and simplify routing algorithms. Localization errors, however, can adversely impact routing algorithms, leading to longer paths and delivery failures [85]. For the purposes of routing efficiency, actual geographical coordinates may be less useful than “virtual” coordinates [86] (i.e., a representation of a sensor’s ‘location’ in the graph of network connectivity). These virtual coordinates could be in an arbitrary dimension, possibly higher than two or three. There are often paths in multihop wireless networks that consume less power than the shortest, straightest-line path between two nodes, and virtual coordinates may enable a better representation of the network connectivity. The virtual coordinate estimation problem is a dimension reduction problem that inputs each sensor’s connectivity or RSS measurement vector and outputs a virtual coordinate in an arbitrary low dimension, optimized to minimize a communication cost metric. Such research could enable more energy-efficient scalable routing protocols for very large sensor networks.

CONCLUSION

Cooperative localization research will continue to grow as sensor networks are deployed in larger numbers and as applications become more varied. Localization algorithms must be designed to achieve low bias and as low of variance as possible; at the same time, they need to be scalable to very large network sizes without dramatically increasing energy or computational requirements.

This article has provided a window into cooperative localization, which has found considerable application in ad-hoc and wireless sensor networks. We have presented measurement-based statistical models of TOA, AOA, and RSS, and used them to generate localization performance bounds. Such bounds are useful, among other design considerations, as design tools to help choose among measurement methods, select neighborhood size, set minimum reference node densities, and compare localization algorithms.

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