Mobile Sensing of Spatial Fields

Challenges and Opportunities

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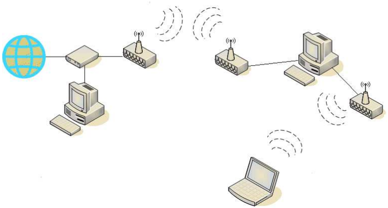
University of Michigan
Oct. 9, 2013

Life in a networked world

- Communication and computing devices are everywhere



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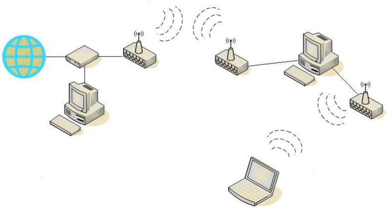

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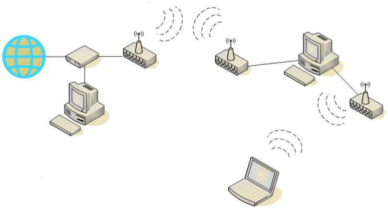
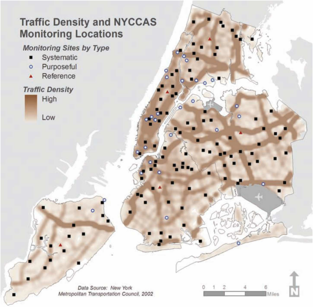


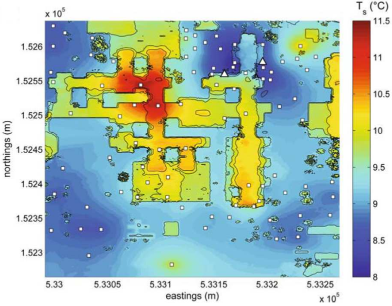
- Sensors are everywhere
- And getting smarter

Berkeley motes: "Smart Dust"

Environment monitoring like never before



Traffic density, New York



Surface temperature, EPFL,

[Nadeau et al., '09]

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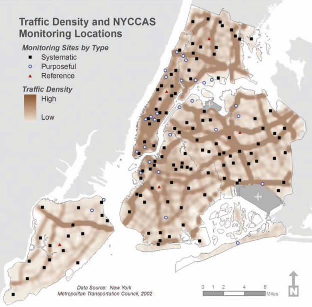

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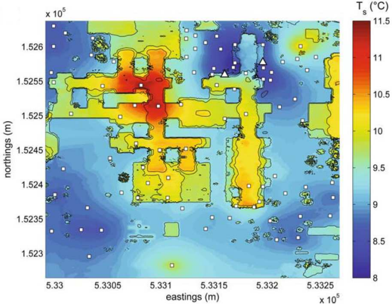

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- Emerging paradigm:
  - Sensing spatial fields with mobile sensors

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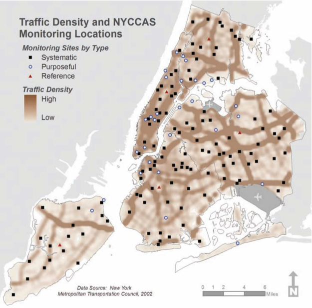

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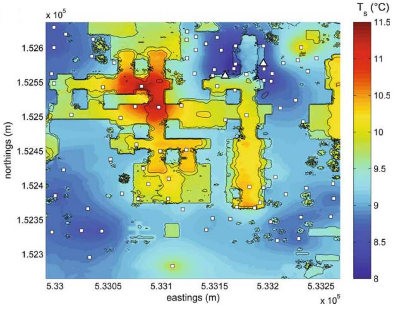

Surface temperature, EPFL,

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- Emerging paradigm:
  - Sensing spatial fields with mobile sensors
  - Offers unique advantages over static sensing

Advantages of mobile sensing

- Mobile sensor can sample at arbitrarily high resolutions along path

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- Mobile sensors can implement spatial anti-aliasing via time-domain filtering (*See later*)

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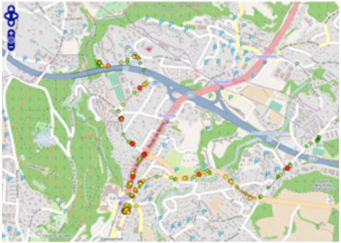
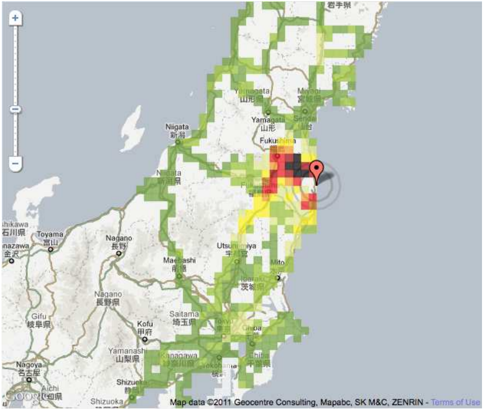
- Mobile sensor can sample at arbitrarily high resolutions along path
- Mobile sensors can implement spatial anti-aliasing via time-domain filtering (*See later*)
- Single mobile sensor can cover a wide area of interest
  - Potentially cost-effective and more practical - e.g. pollution monitoring in a city

Pollution monitoring in Lausanne



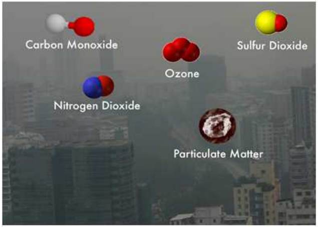
Mobile radiation sampling in Japan

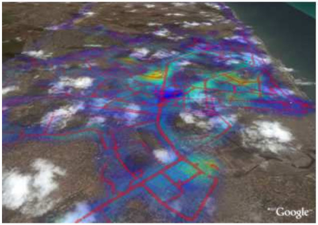


Mobile radiation sampling in Japan

Mobile sensing on roads

Citizen sensing



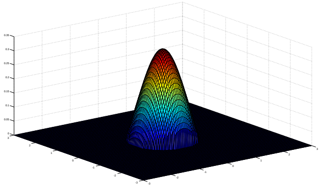


Images from http://www.urban-atmospheres.net/CitizenScience/

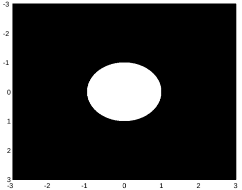
Outline of the talk

- Given: spatially bandlimited field $f : \mathbb{R}^d \mapsto \mathbb{R}$

$$\mathcal{F}(\omega) := \int f(r) e^{-j\langle \omega, r \rangle} dr = 0 \text{ for } \omega \notin \Omega$$



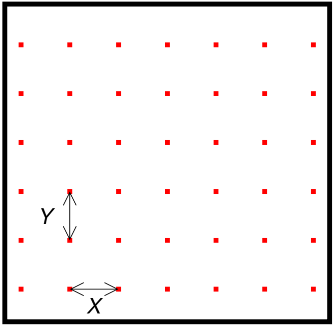
Spectrum $|\mathcal{F}(\omega_x, \omega_y)|$



Support of spectrum Ω

Classical sampling in higher dimensions [PM 1962]

- Sampling on a lattice



Sampling lattice

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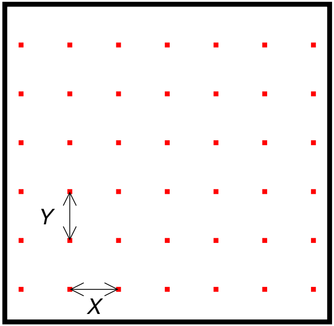

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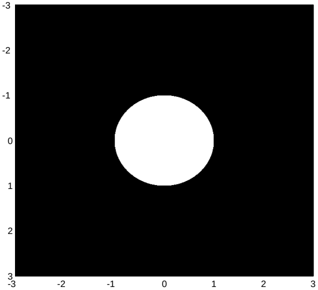

Original spectrum

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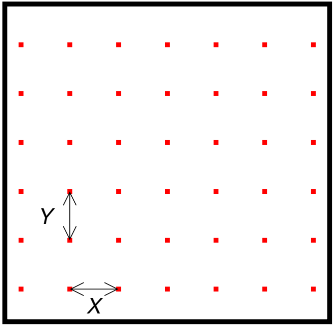

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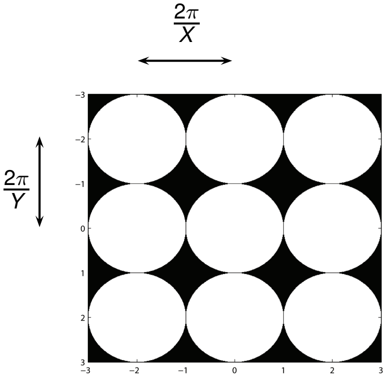

No aliasing in sampled spectrum for $X = Y \leq \frac{\pi}{R}$

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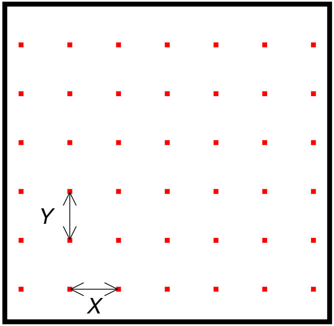

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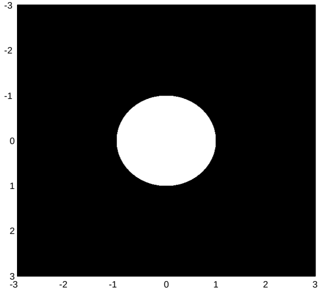

Perfect recovery of original spectrum

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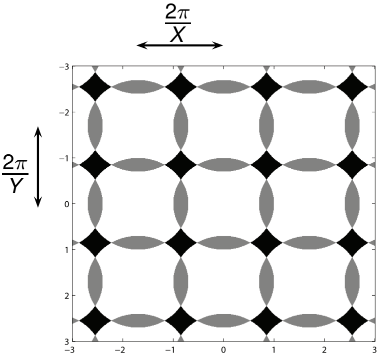
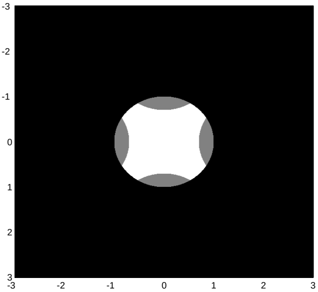


Aliased sampled spectrum



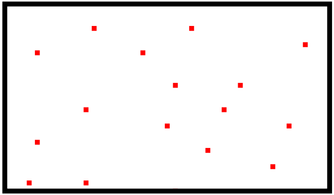
Perfect recovery impossible

- Lattice should be fine enough $\equiv$ Nyquist criterion in $\mathbb{R}^d$

Classical sampling vs Mobile sampling

Classical sampling

Static sensors record field values at points



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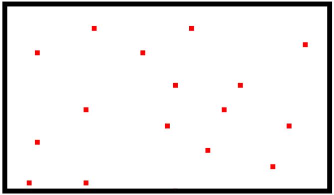
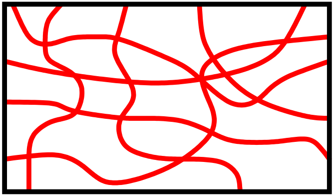


Mobile sampling

Mobile sensors record field values on trajectories



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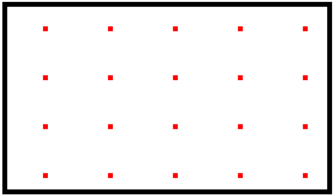
Static sensors record field values at points (e.g. a lattice)



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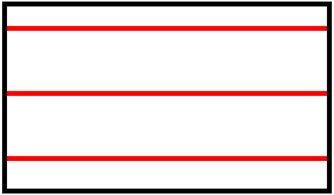

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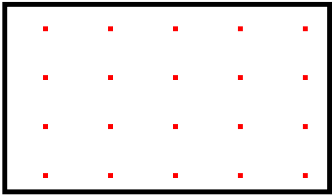

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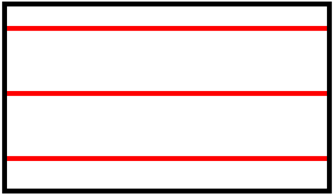

Focus on time-invariant fields

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$$p = \{p_i : i \in \mathbb{I}\}$$

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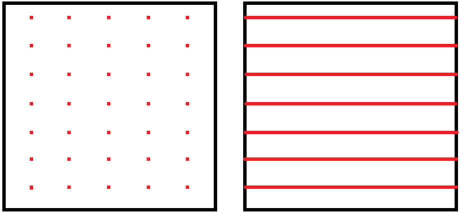

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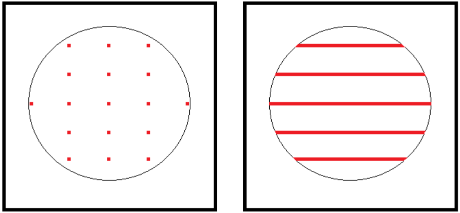

Sampling trajectories for bandlimited fields

- $\mathcal{N}_\Omega$: collection of Nyquist trajectory sets p for fields f bandlimited to Ω

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  - Regularity conditions

Main contributions

- Examples of trajectory sets in $\mathcal{N}_\Omega$
  - Some new results

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- Examples of trajectory sets in $\mathcal{N}_\Omega$
  - Some new results
- Designing trajectory-sets that are minimal in path density
  - New formulation
  - Optimality results from restricted classes

Related work

- Scanning trajectories for MRI
  - Trajectories in Fourier space indicate how to vary magnetic field in time

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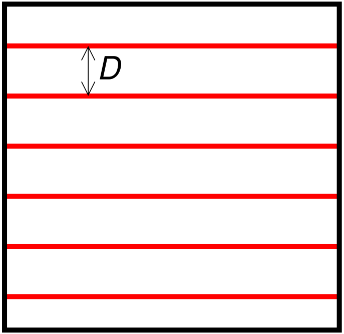
- Reconstructing bandlimited fields from readings on circles [Tewfik, Levy, Willsky '88], [Myridis, Chamzas '98] and spirals [Benedetto, Wu '00]

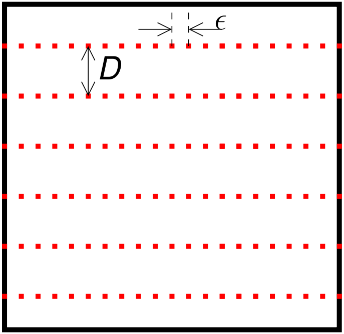
Related work

- Scanning trajectories for MRI
  - Trajectories in Fourier space indicate how to vary magnetic field in time
- Reconstructing bandlimited fields from readings on circles [Tewfik, Levy, Willsky '88], [Myridis, Chamzas '98] and spirals [Benedetto, Wu '00]
- Adaptive path-planning in mobile sensor networks etc.

A Uniform trajectory set in $\mathbb{R}^2$



Proof - Uniform set



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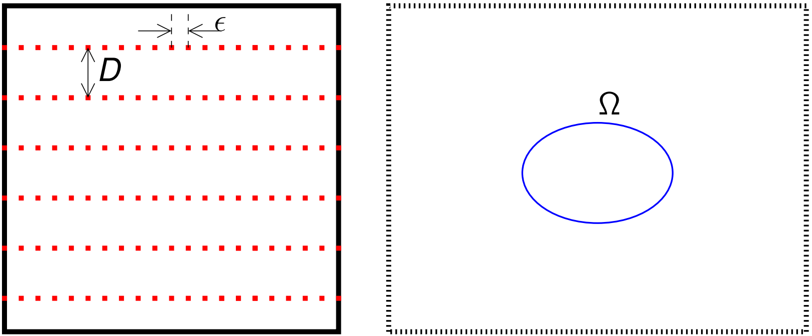
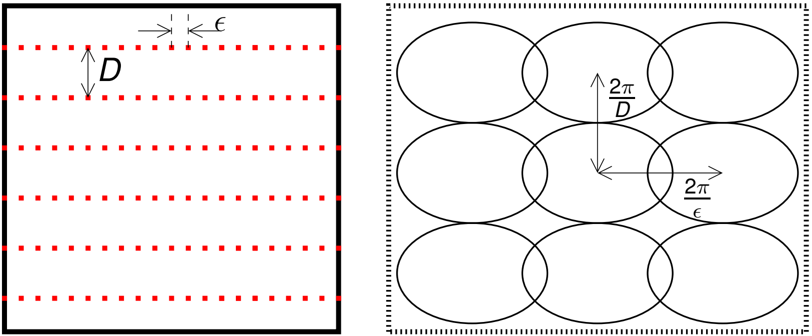


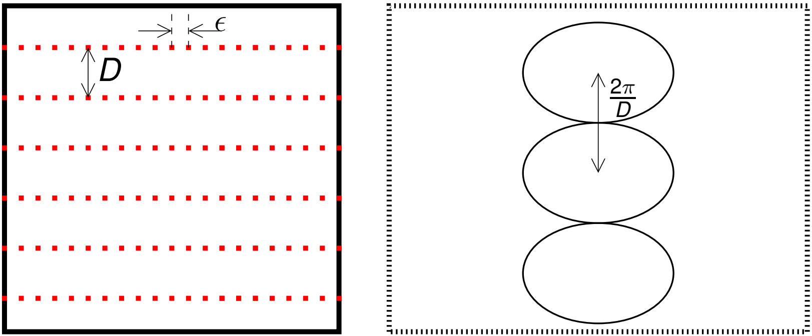
Original field spectrum

Proof - Uniform set



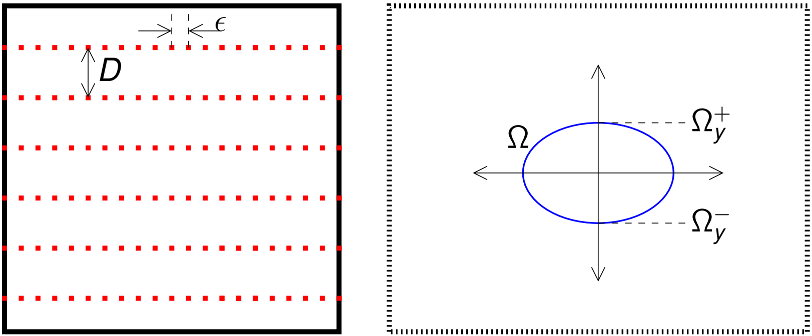
Sampled field spectrum

Proof - Uniform set



Sampled field spectrum as $\epsilon \to 0$

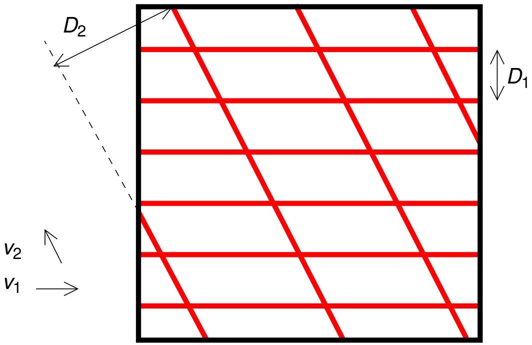
Proof - Uniform set



Perfect recovery provided $D \leq \frac{2\pi}{\Omega_y^+ - \Omega_y^-}$

i.e. $p \in \mathcal{N}_\Omega$

Union of Uniform sets in $\mathbb{R}^2$



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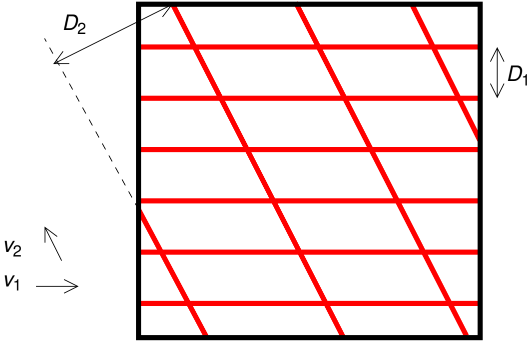
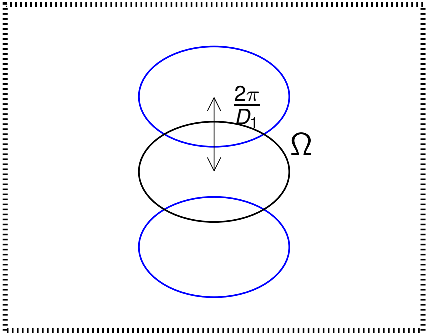


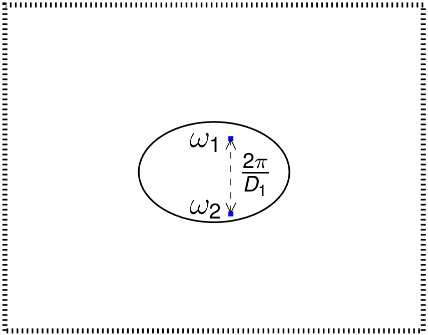
- Can identify exact conditions on D_i, v_i to ensure $p \in \mathcal{N}_\Omega$

Proof intuition - union of uniform sets



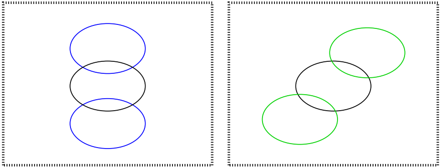
Aliased spectrum from first uniform set

Proof intuition - union of uniform sets



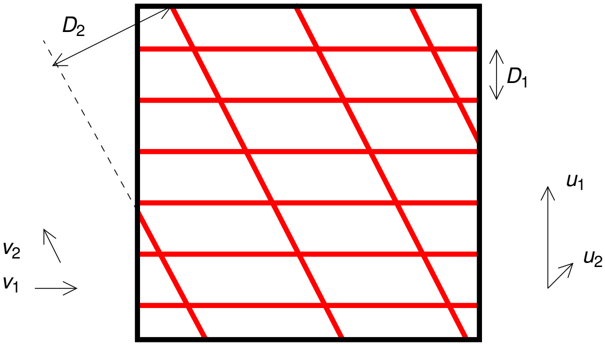
Aliased reconstruction: $\hat{\mathcal{F}}(\omega_1) = \mathcal{F}(\omega_1) + \mathcal{F}(\omega_2)$

Proof intuition - union of uniform sets



Linear equations solvable under condition

Union of regular parallel trajectories for $\mathbb{R}^2$



- For each i define

$$u_i = \frac{2\pi v_i^{\perp}}{D_i}$$

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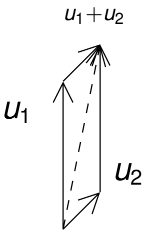

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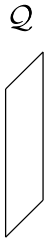

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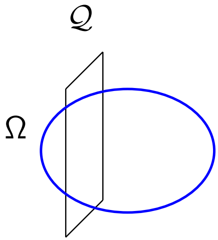
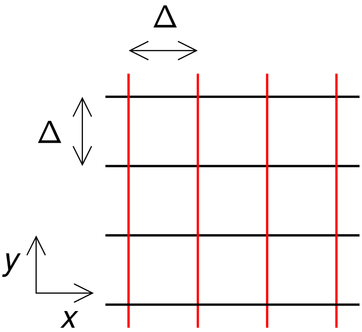


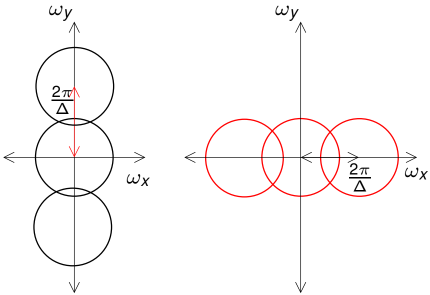
Theorem

For convex, compact Ω we have $p \in \mathcal{N}_\Omega$ if and only if $\mathcal{Q}$ is not contained within Ω or its translates

Example 1: Orthogonal trajectories and Isotropic field



Orthogonal sets of trajectories



Sampled spectra from the two sets

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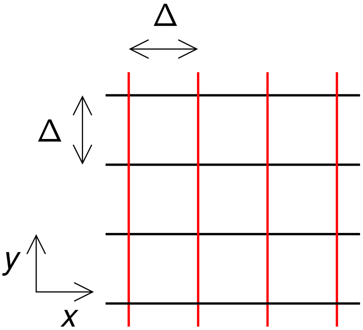

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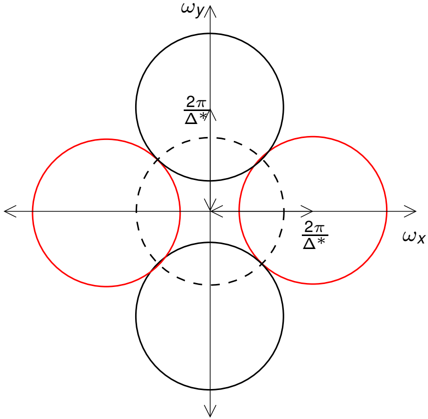
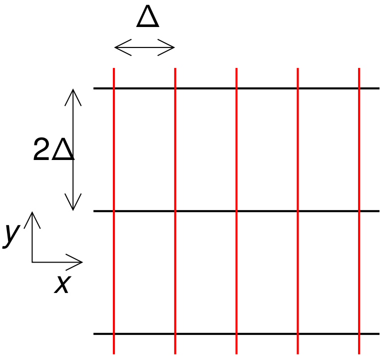


Critical sampling

Example 2: Non-isotropic field



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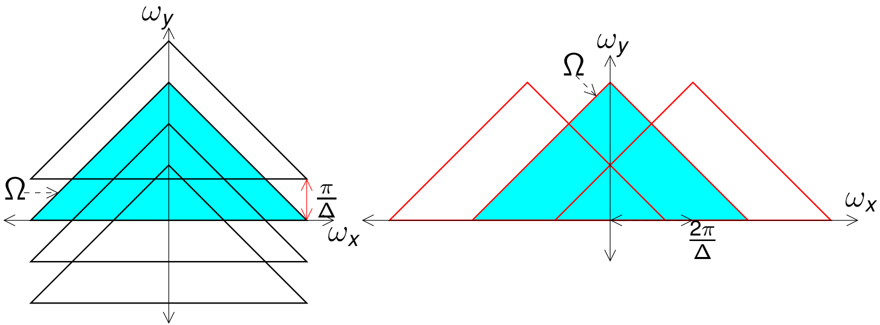

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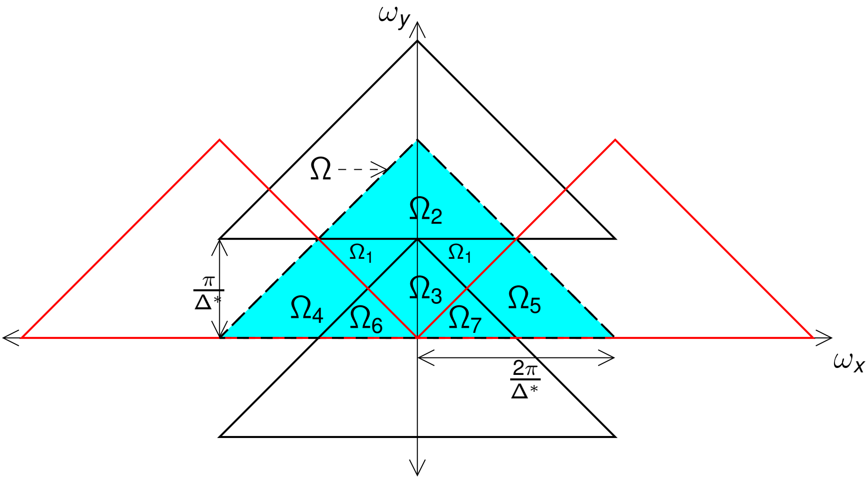
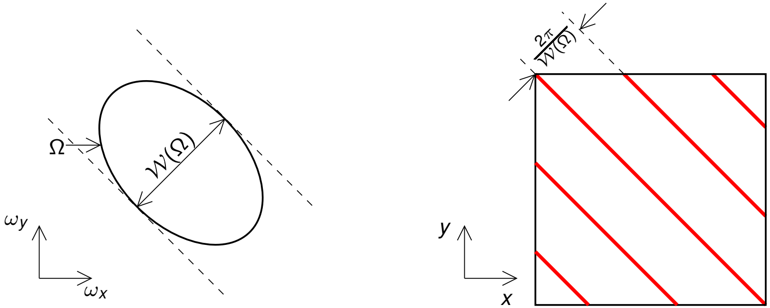


Critical sampling

Optimality of a Uniform set in $\mathbb{R}^2$

- Assume Ω is a convex set



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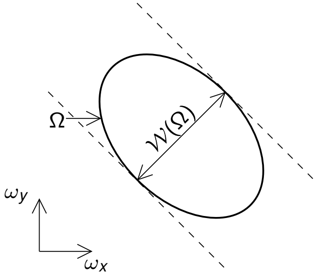

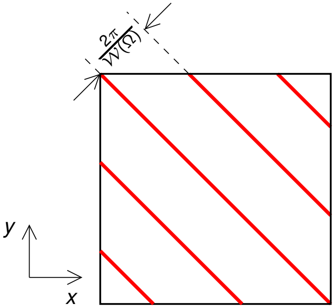

- Optimal Uniform set p is $\perp$ direction in which Ω is narrowest

Optimality of a Uniform set in $\mathbb{R}^2$

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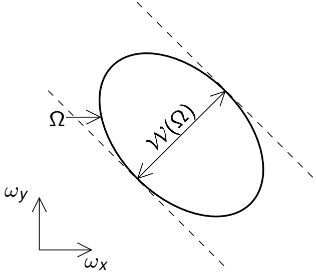
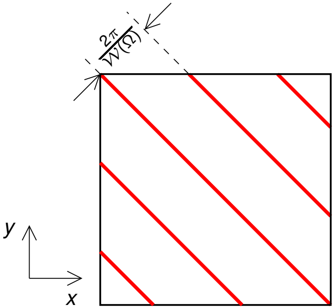




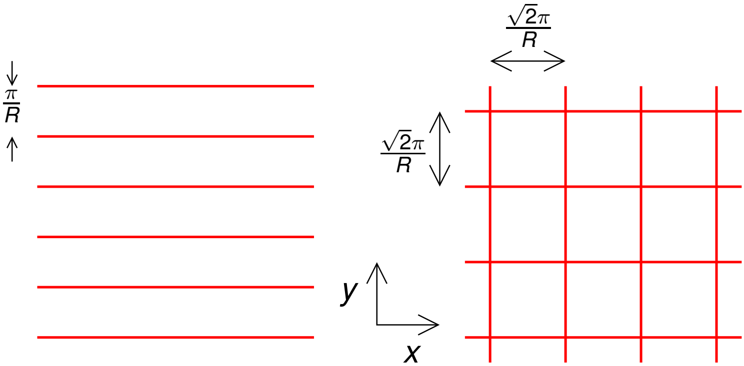
- Optimal Uniform set p is $\perp$ direction in which Ω is narrowest

Theorem

For any union of uniform sets $q \in \mathcal{N}_\Omega$, we have

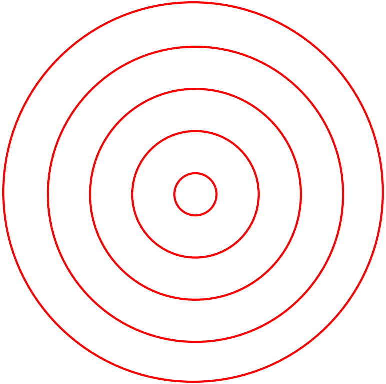
$$\ell(q) \geq \ell(p)$$

Implications for isotropic fields



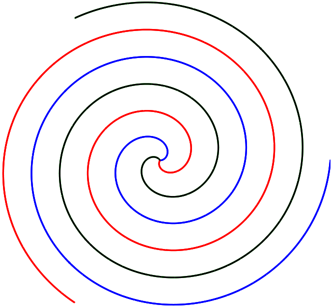
Optimal uniform set has lower path density than any union of orthogonal uniform sets

Other trajectory configurations



Equispaced concentric circles

Other trajectory configurations



Interleaved spirals

Uniform sets for $\mathbb{R}^3$

- Assume $\Omega \subset \mathbb{R}^3$ is convex and symmetric

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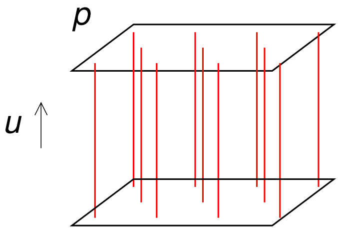

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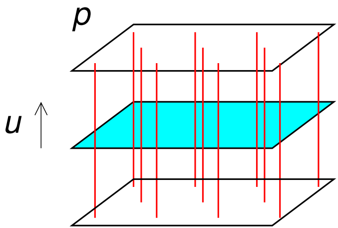

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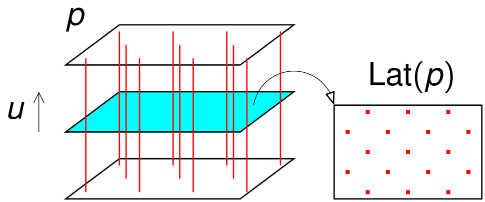

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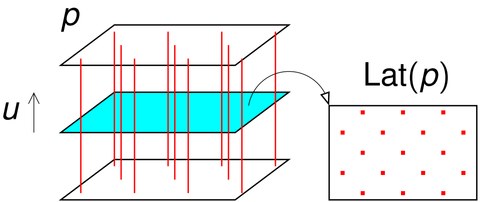
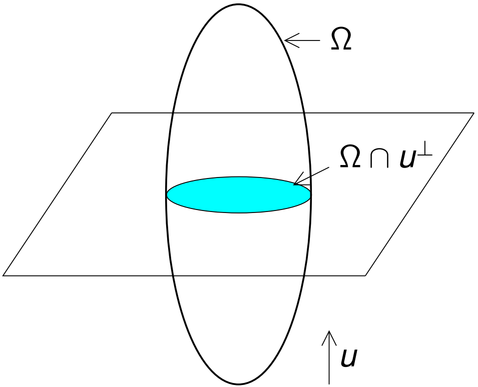


Theorem

$p \in \mathcal{N}_\Omega$ iff Lat(p) forms sampling lattice for $\Omega \cap u^\perp$. Furthermore

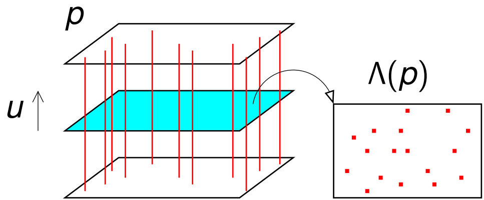
$$\ell(p) = \text{Sampling density}(\text{Lat}(p))$$

Section of a set



Non-uniform parallel trajectory sets for $\mathbb{R}^d$

- Consider non-uniform parallel trajectory sets



- If p is homogenous (i.e., $\Lambda(p)$ has equal density about every point), then

$$\ell(p) = \text{Sampling density}(\Lambda(p))$$

Optimal parallel trajectory sets for $\mathbb{R}^d$

Theorem

For Ω compact, convex, and symmetric let

$$u^* = \underset{u \in \mathbb{R}^d : \|u\| = 1}{\arg\min} \; |\Omega \cap u^\perp|$$

Then the optimal parallel trajectory set p^* is parallel to u^* and

$$\ell(p^*) = \frac{|\Omega \cap u^{*\perp}|}{(2\pi)^{d-1}}.$$

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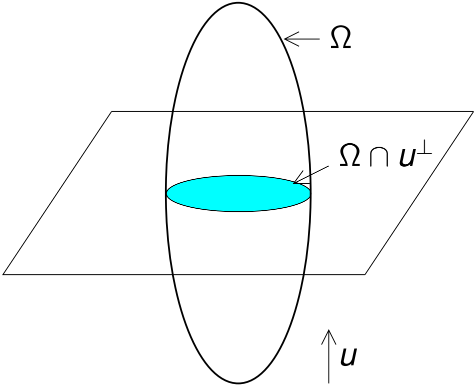
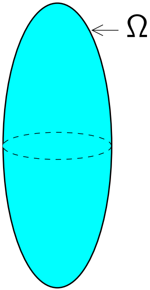
- I.e., minimum path density $\propto$ minimum section through the origin

Optimal parallel trajectory sets for $\mathbb{R}^d$



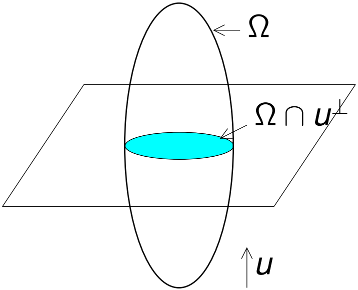
Classical sampling vs Mobile sampling

Classical sampling



Minimum sampling density
$\propto \mathrm{Vol}(\Omega)$ [Landau '67s]

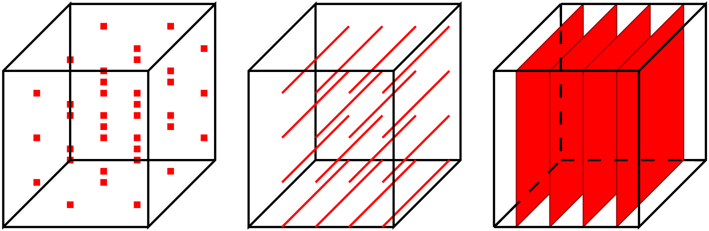
Sampling on parallel lines



Minimum path density
$\propto$ Min. section(Ω)

Extensions

- Generalize trajectories further to manifolds



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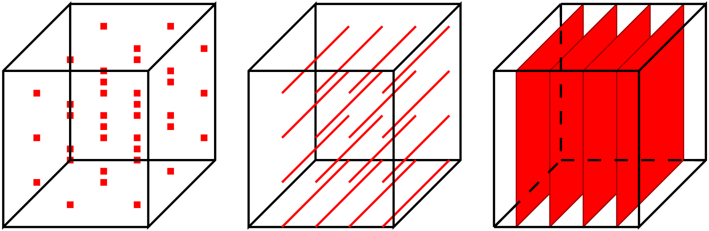

- Sampling on arbitrary curves: Ill-posed but can be fixed

Extensions

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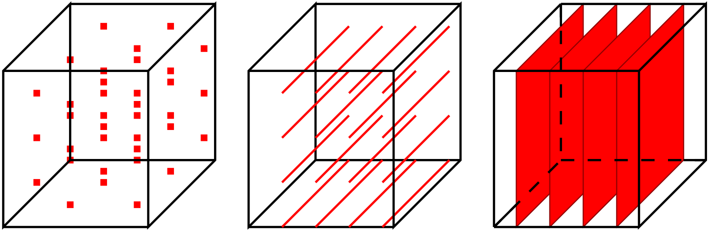

- Sampling on arbitrary curves: Ill-posed but can be fixed
- Time-varying bandlimited fields e.g. spatial audio fields
  - Reduce sensor density by increasing temporal sampling rate

Outline of the talk

1. Sampling Trajectories for Mobile Sampling
   - Classical sampling vs mobile sampling
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   - Optimal parallel trajectories
2. Spatial Anti-aliasing via Mobile Sensing
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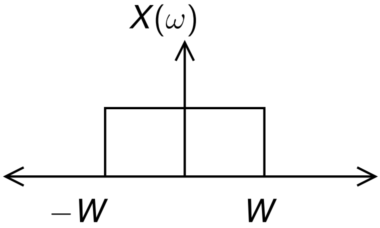
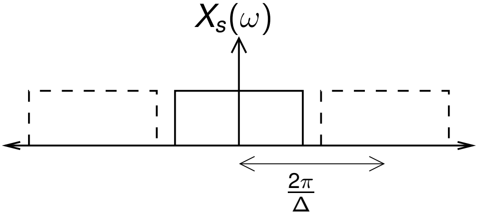
Anti-aliasing in one dimension



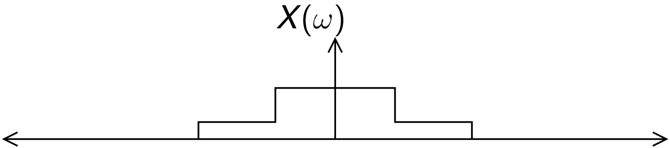
Bandlimited signal spectrum

Anti-aliasing in one dimension



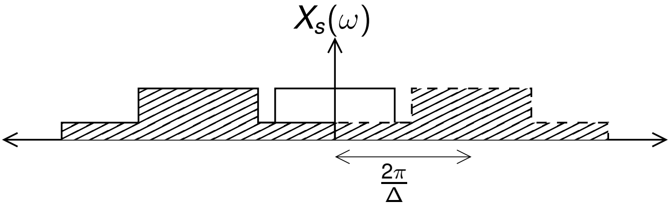
Sampled spectrum

Anti-aliasing in one dimension



Imperfectly bandlimited signal

Anti-aliasing in one dimension



Sampled spectrum

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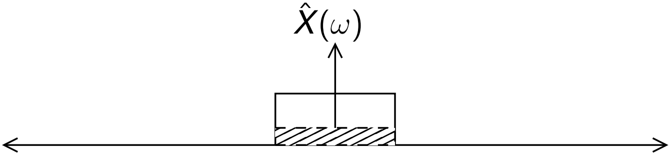
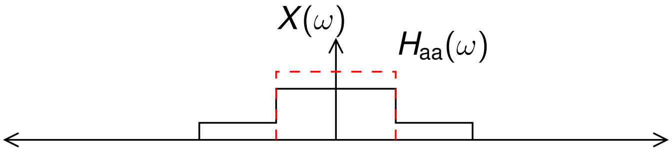
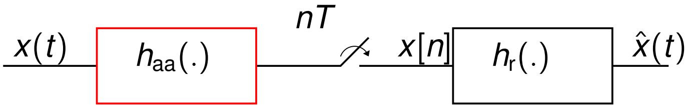


Aliasing in reconstructed spectrum

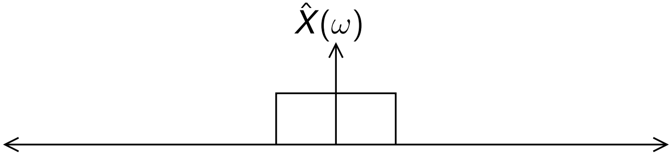
Anti-aliasing in one dimension

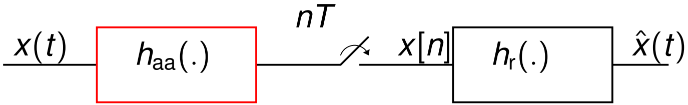




Anti-aliasing filtering

Anti-aliasing in one dimension





No aliasing in reconstruction

Spatial anti-aliasing

- Impossible with static sensors
  - Cannot integrate over continuous space

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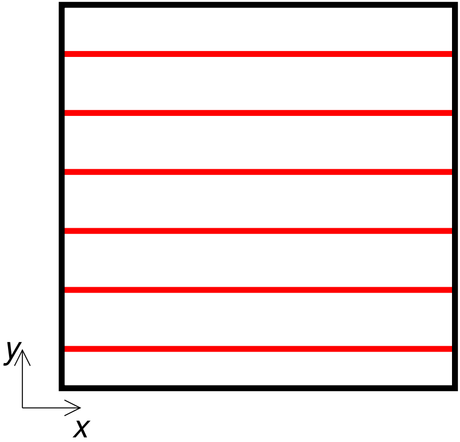
- Mobile sensor sees field as function of time
  - $s(t) = f(r(t))$
  - If constant velocity then $s(t)$ is bandlimited
- In presence of noise time-domain filtering can suppress spatial aliasing along direction of motion

Spatial anti-aliasing filter



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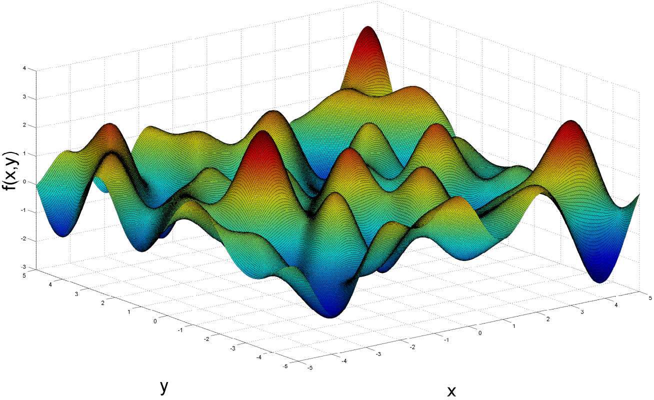

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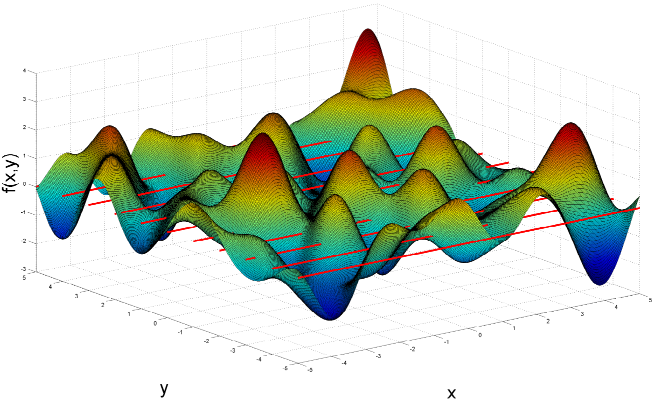

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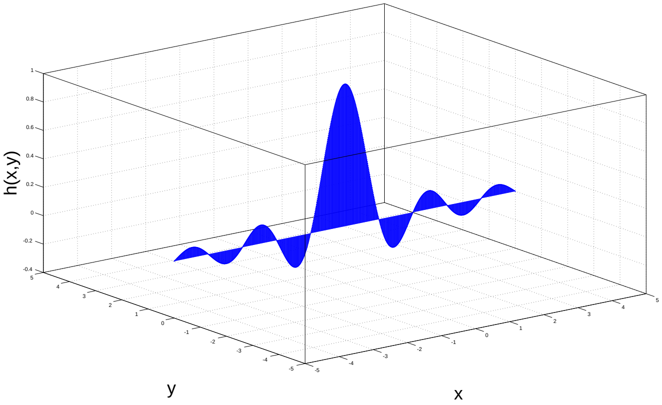

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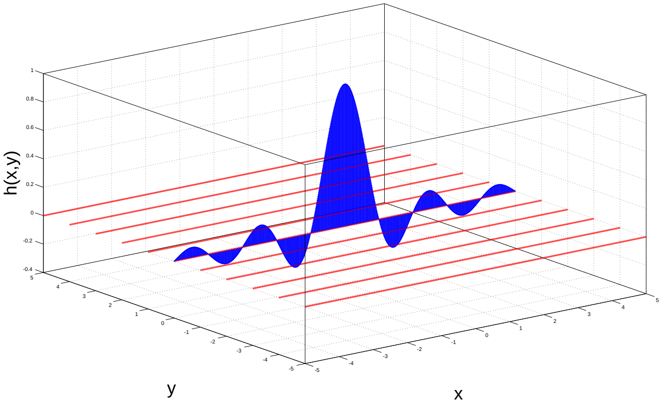

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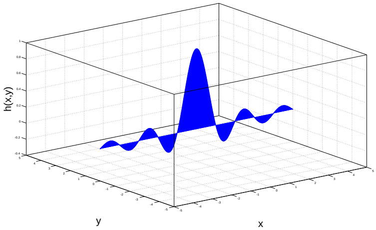
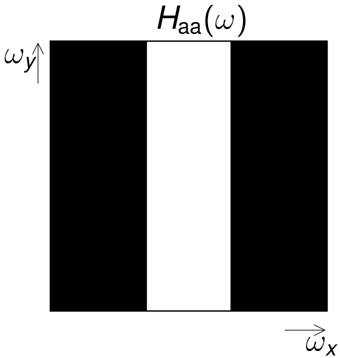
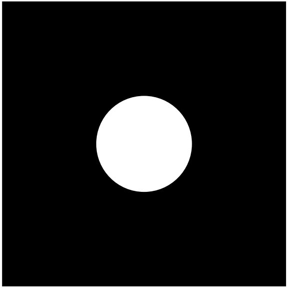


$\Longrightarrow$

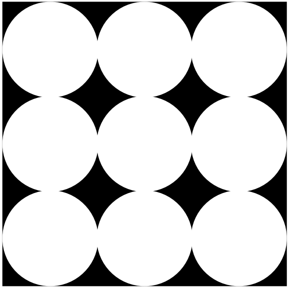


Spatial anti-aliasing illustrated

Static sampling



Original spectrum



Sampled spectrum

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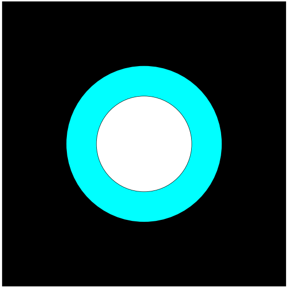
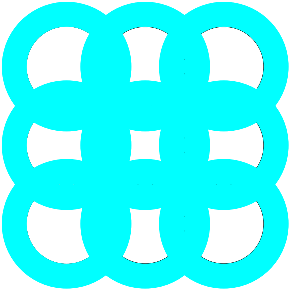


Original spectrum with noise



Sampled spectrum aliased in all directions

Spatial anti-aliasing illustrated

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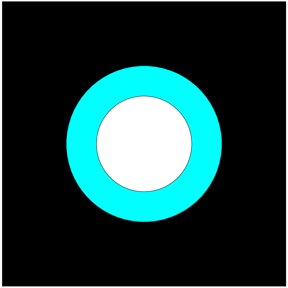

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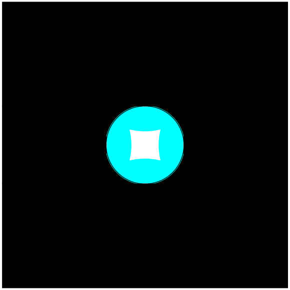
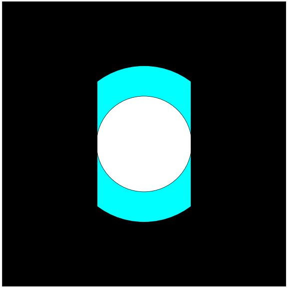


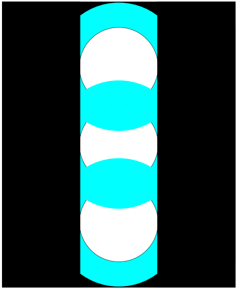
Reconstructed spectrum aliased in all directions

Spatial anti-aliasing illustrated

Mobile sampling



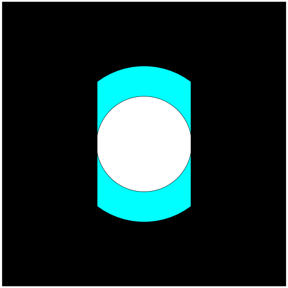
Spatially filtered field with noise



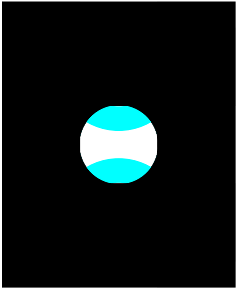
Sampled spectrum aliased only in two directions

Spatial anti-aliasing illustrated

Mobile sampling

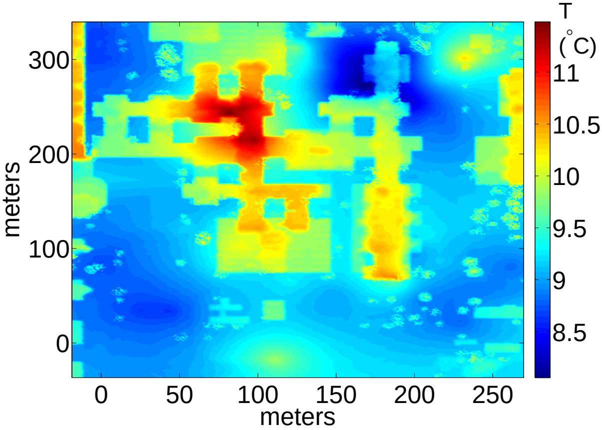


Spatially filtered field with noise



Reconstructed spectrum aliased in two directions

An example: Campus temperature



Performance comparison

Table : Reconstruction errors

Data type	Static sensing	Mobile sensing
Temperature	0.53%	0.45%
BL in noise	9.9%	1.5%

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- Improvement in SNR can be quantified analytically
- Conclusion: Mobile sensing enables spatial anti-aliasing filtering

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- Personal data being collected at unprecedented levels

Unique in the crowd

- Anonymized user data is not anonymous given auxiliary information, e.g., Netflix prize dataset [Narayanan, Shmatikov 2008]

Unique in the crowd

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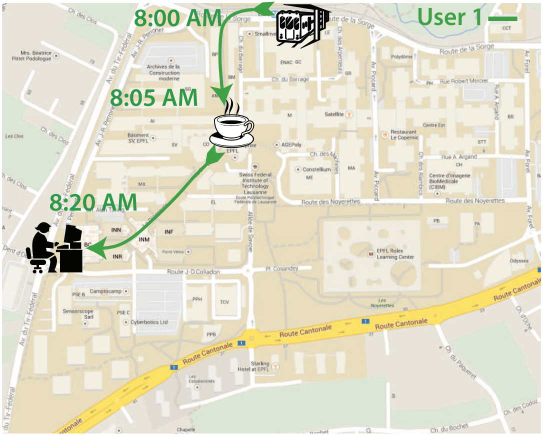
- Users are uniquely identifiable from small set of observations
  - 87% Americans uniquely identified given ZIP code, birthdate, and sex [Sweeney 2000]
  - 95% of mobile users in a country are uniquely identified from four spatio-temporal points [Montjoye et al 2013]

De-anonymization



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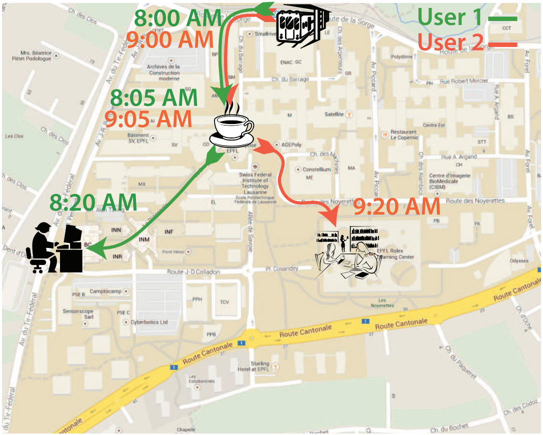

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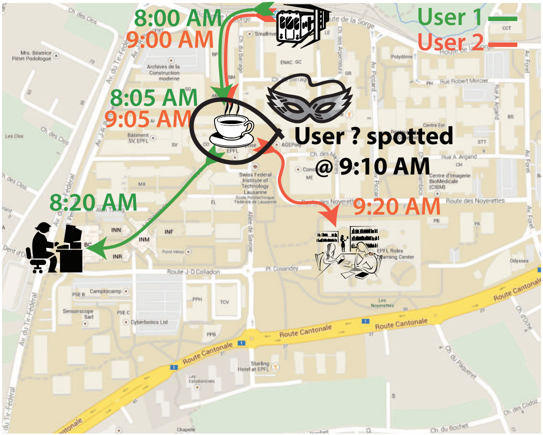

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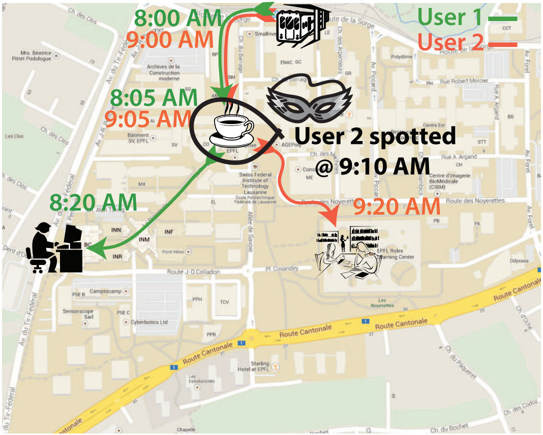

- Uniqueness of trajectories ⇒ Easily de-anonymized

Privacy of Statistics

- Sometimes statistics of data are sufficient, e.g.,
  - Fraction of time spent in particular locations or websites (useful for ad-placements, infrastructure planning)
  - # visits to particular restaurants (for popularity surveys)
  - # tweets/blog comments on a particular topic/ containing a particular word (for targeted ads)

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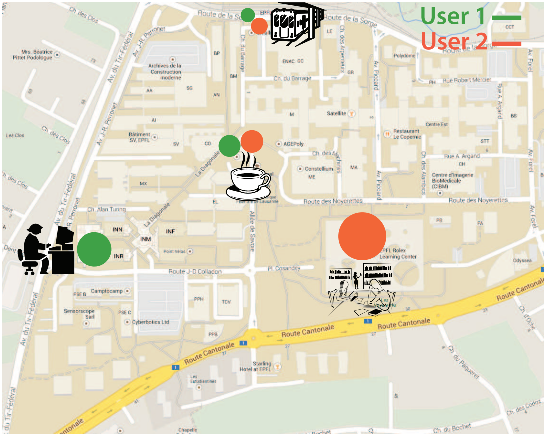
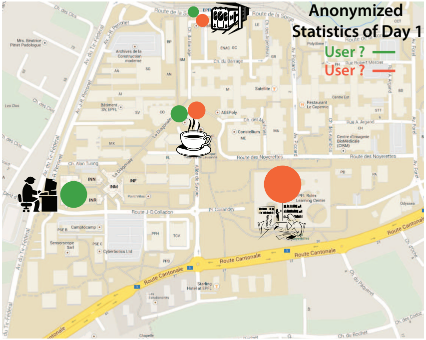
  - Focus on histograms
  - How unique are user location statistics?

Example: Location Histograms



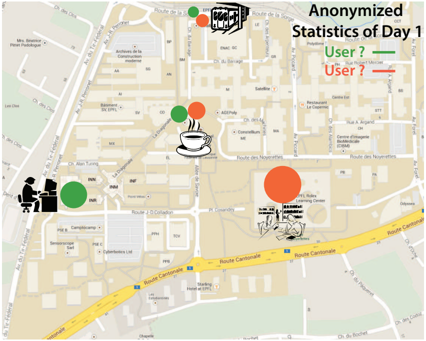
De-anonymizing as Matching

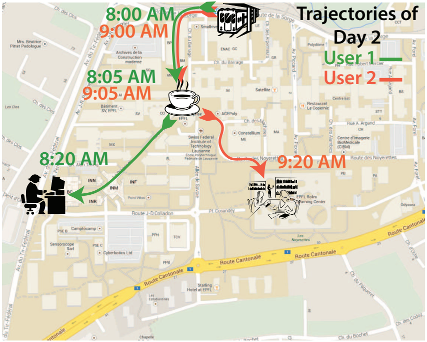
- Given: Anonymized Statistics of K users



De-anonymizing as Matching

- Given: Anonymized Statistics of K users' data and Auxiliary Information about independently generated data





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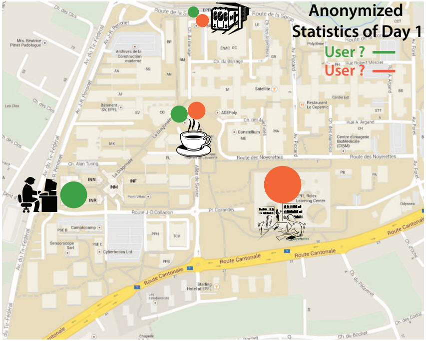

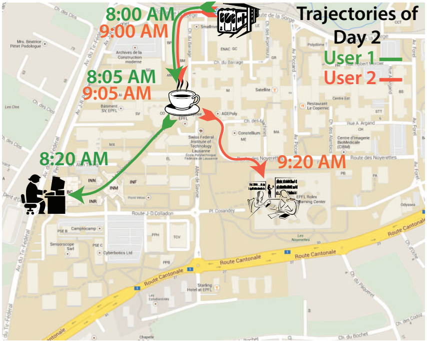

- Task: Match Auxiliary Information to the correct Anonymized Statistics

Data model and notation

- Let $x_{\pi(i)}$ and y_i denote length-n data vectors of user i

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- User's data: i.i.d. according to unknown law from finite alphabet Z
- Let p_{x_i} be empirical distribution of vector x_i:

$$p_{x_i}(z) = \frac{1}{n} \sum_{j=1}^{n} \mathcal{I}\{x_i^j = z\}, \quad z \in \mathrm{Z}$$

where

$$x_i = (x_i^1, x_i^2, \ldots, x_i^n)$$

Problem statement

- Statistics revealed: (Anonymized) Empirical distribution of users' x data streams, i.e., the set

$$\{p_{x_1}, p_{x_2}, \ldots, p_{x_K}\}$$

is revealed, but not the user identities, i.e., π is unknown

Problem statement

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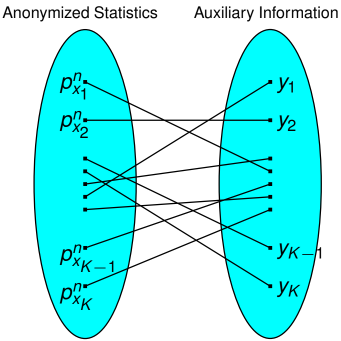
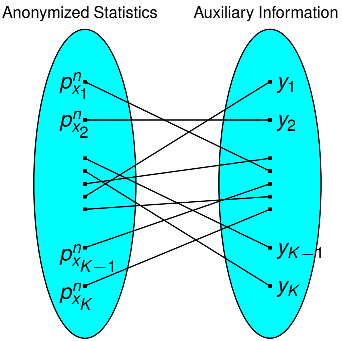
- Auxiliary information: y data streams of the users, as well as the identities
  - Note: y_i's are independent of x_i's

Matching Problem



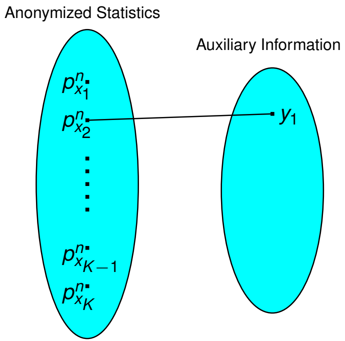
- All strings $\{x_i\}$ and $\{y_i\}$ have equal lengths

Matching Problem



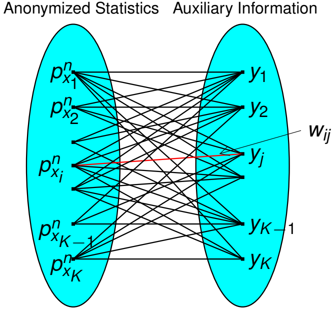
- All strings $\{x_i\}$ and $\{y_i\}$ have equal lengths
- Can be viewed as hypothesis testing problem with $M = K!$ composite hypotheses

Matching Problem



- Related problem: only one y string given; studied by Gutman ('89)

Potential solution: "ML"



$$w_{ij} = D(p^n_{x_i} \| \tfrac{1}{2}(p^n_{x_i} + p^n_{y_j})) + D(p^n_{y_j} \| \tfrac{1}{2}(p^n_{x_i} + p^n_{y_j}))$$

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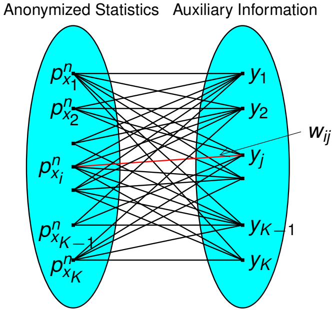

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  - $\Rightarrow$ easy to compute

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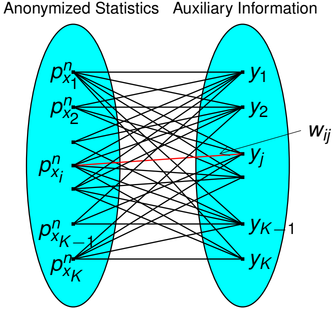

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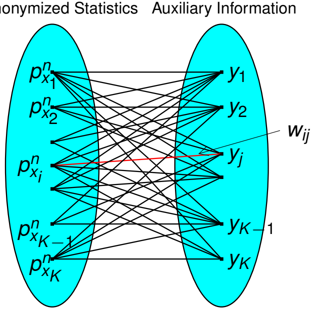
- ML permutation $\Leftrightarrow$ Min-Weight Matching
  - $\Rightarrow$ easy to compute
  - Is this optimal?

Potential solution: “ML”



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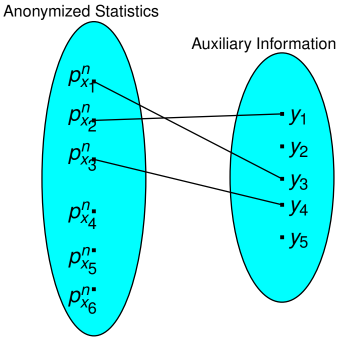
- ML permutation $\Leftrightarrow$ Min-Weight Matching
  - $\Rightarrow$ easy to compute
  - A slight variant of ML is optimal!

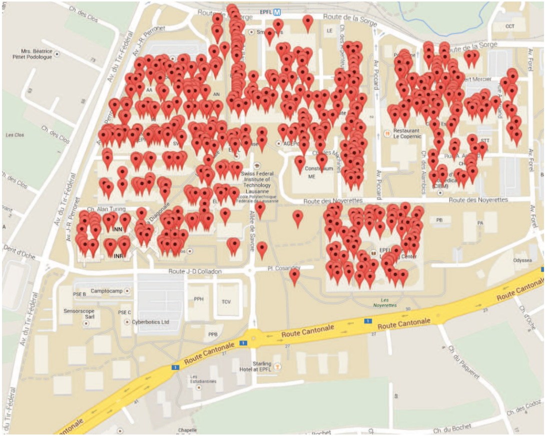
Generalizations

- Different sets of distinct users observed in two days
  - Can be handled provided # common users known



Experiment: Mobility traces on EPFL campus

Obtained from Wi-Fi connections



Dataset

- Anonymized mobility traces of $K \approx 1000$ users on EPFL campus measured on Mondays for two weeks
  - x_i in first week and y_i in second week

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    - x_i in first week and y_i in second week
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- Data-length $n = 28800$ seconds (8 hours)

Results

Dataset	# users (K)	Matching K users in second week (fraction of correct matches)
Mondays	1154	52.9%

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- Info on more users on second day $\Rightarrow$ Higher accuracy

Results

Dataset	# users (K)	Matching K users in second week (fraction of correct matches)	Matching 1 random user in second week (prob of success)
Mondays	1154	52.9%	44.5%
Mondays and Tuesdays	1047	70.5%	53.5%

- Info on more users on second day ⇒ Higher accuracy
- More days ⇒ Higher accuracy
- Conclusion: Simple anonymization is not effective

Outline of the talk

1. Sampling Trajectories for Mobile Sampling
   - Classical sampling vs mobile sampling
   - Sampling trajectories
   - Optimal parallel trajectories
2. Spatial Anti-aliasing via Mobile Sensing
3. Privacy of Mobility traces
4. Recap

Recap

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  - Notions of path density and optimal trajectories

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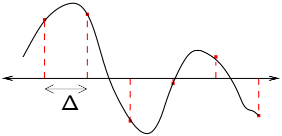
  - Uniform set better than unions of Uniform sets for $\mathbb{R}^2$
  - Optimal design of parallel trajectory sets for $\mathbb{R}^d$
- Spatial anti-aliasing via time-domain filtering
- Mobility statistics: Simple anonymization is ineffective
  - Optimal de-anonymization strategy can be identified

A new sampling theory



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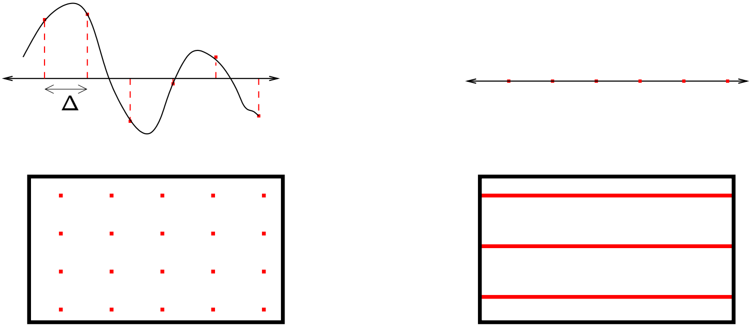

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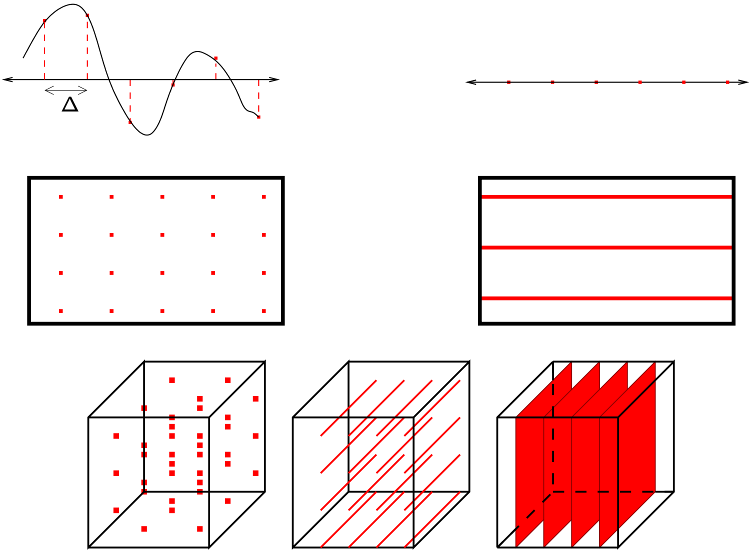

Thank You!

Questions?

Relevant publications

Mobile Sensing

JU, M. Vetterli, "*Sampling High-Dimensional Bandlimited Fields on Low-Dimensional Manifolds*" IEEE Trans. Inf. Theory, 2013

JU, M. Vetterli, "*Sampling and Reconstruction of Spatial Fields using Mobile Sensors*" IEEE Trans. on Signal Process., 2013

JU, M. Vetterli, "*On Optimal Sampling Trajectories for Mobile Sensing* " in Proc. of SampTA, 2013

K. Gröchenig, JU, M. Vetterli, J. L. Romero, "*On Minimal Trajectories for Mobile Sampling of Bandlimited Fields* " in prep., 2013

Privacy of Mobile Traces

JU, F. M. Naini, "*De-anonymizing Private Data by Matching Statistics*" Allerton conference, 2013.