The Impact of Observation and Action Errors on Informational Cascades

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Joint work with Tho Le & Randall Berry, Northwestern University

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CSP Seminar
November 6, 2014
Anecdote


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  - 15 weeks on *NYTimes* bestseller list
  - *Bloomberg Businessweek* bestseller list
  - ~250K copies sold by 2012

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Audience greatly influenced by NYTimes’ ratings of book

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Motivation

E-commerce, online reviews, collaborative filtering
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- Future customers can use this information to make their decisions/purchases.
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- Agent $i$’s payoff, $\pi_i$:
  - Action $A_i$
    - $N$: payoff $\pi_i = 0$
    - $Y$: payoff
      - $\pi_i = -\frac{1}{2}$ if $V = 0$
      - $\pi_i = +\frac{1}{2}$ if $V = 1$
Information Structure

• Agent \( i \ (i = 1, 2, \ldots) \) receives i.i.d. *private signal*, \( S_i \)
• Agent $i$ ($i = 1, 2, ...$) receives i.i.d. *private signal*, $S_i$
• Obtained from $V$ via a BSC$(1 - p)$

Diagram:

![Binary Symmetric Channel (BSC) Diagram]
Information Structure

- Agent $i$ ($i = 1, 2, ...$) receives i.i.d. private signal, $S_i$
- Obtained from $V$ via a BSC($1 - p$)

$$V \xrightarrow{0, 1-p} L \xrightarrow{p} S_i \xleftarrow{1-p, p} H$$

- Assume $0.5 < p < 1$: Private signal is informative, but non-revealing
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- Agent \(i \geq 2\) observes actions \(A_1, \ldots, A_{i-1}\) in addition to \(S_i\)
  
  *Database provides this information*
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$$
\begin{array}{c}
V \\
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0 \quad 1-p \\
1-p \quad p
\end{array}
\end{array}
\rightarrow
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- Denote the information set as $I_i = \{S_i, A_1, ..., A_{i-1}\}$
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  V & \rightarrow & S_i \\
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  1 & \overset{1-p}{\longrightarrow} & H \\
  & \overset{p}{\longrightarrow} & \\
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- Agent $i \geq 2$ observes actions $A_1, ..., A_{i-1}$ in addition to $S_i$
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- Denote the information set as $l_i = \{S_i, A_1, ..., A_{i-1}\}$
- Distribution of value and signals are *common knowledge*. 
Bayesian Rational Agents

• Suppose each agent seeks to maximize her expected pay-off.
  • Given her information set
Bayesian Rational Agents

• Suppose each agent seeks to maximize her expected pay-off.
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• Without any information:
  • Expected payoff $E[\pi_i] = 0$ since $P[V=1] = P[V=0] = \frac{1}{2}$
  • With only private signal:
    • Update posterior probability: $\Pr(V=G|S_i=H) = \Pr(V=B|S_i=L) = p > 0$.5
    • Optimal Action: Buy if and only if $S_i = H$.
  • Pay-off: $E[\pi_i] = \frac{1}{2}(2p - \frac{1}{2}) + \frac{1}{2}(0) = \frac{2}{4}p > 0$
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    - Follow own signal if $\Pr[V = 1|I_i] = \frac{1}{2}$

Can now iteratively calculate the actions of each agent for a given realization of $V$ and $\{S_i\}$. 


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BHW’92 Analysis

• First agent always follows their own signal.

\footnote{Here assume they always follow signal in this case.}
BHW’92 Analysis

- First agent always follows their own signal.
- Consider second agent.

\[I_3 = \{H, N, N\} \text{ or } \{L, Y, Y\}.\]

In these cases, optimal action is to “follow the crowd.”

Subsequent agents? \(^2\) Here assume they always follow signal in this case.
BHW’92 Analysis

- First agent always follows their own signal.
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  - Two possibilities:
    - Observation and signal match.
    - Observation and signal differ.
- Third agent?
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- BHW’92, Banerjee’92, Welch’92: Agents eventually exhibit herding
- BHW’92: *herding* as soon as $|\# Y’s - \# N’s| = 2$ in the history.

  Once herding starts, all agents follow suit.
Do real people herd?

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  - Two urns with mix of red and blue balls.
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  - Experiment is repeated, each time the urn is chosen randomly.
  - Students with correct guess will be rewarded after the experiment
  - Result: About 80% of the cases the students copy guesses.
Discussion

Why does herding happen? OR When can learning occur?

• Discrete feedback from agents is not rich enough
  • Cover1969, SmithSorensen2000: reporting posterior beliefs better
  • Cover1969, Hellman thesis: Can reduce to finite memory of display

• Likelihood ratios of private signals bounded
  • SmithSorensen2000, Sorensen2000 thesis: if unbounded, then learning occurs
  • Bayes update plus threshold rule may not be optimal
    • Cover1969: different rule with finite memory display

• Zhang et al. 2013: different sequence of thresholds gives learning

• Information structure reinforces actions
  • Acemoglu et al. 2011: changing set of past agents sampled gives learning even with bounded likelihoods

Why should strategic users follow any of these remedial schemes?
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Noisy Observations

• Introduce i.i.d. observation errors
Noisy Observations

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  - Actions are recorded on *common database* via another $\text{BSC}(\epsilon)$, $0 < \epsilon < 0.5$

\[ A_i \quad \xrightarrow{\begin{array}{c} \epsilon \\ 1 - \epsilon \end{array}} \quad O_i \]

- Information set is now $I_i = \{S_i, O_1, \ldots, O_{i-1}\}$
- Objective: Study the effects of such errors on BHW model
- Note: With noisy observations are less reliable
- Does herding still occur?
- How does probability of wrong herding change?
- Can parameters be changed to improve things?
Noisy Observations

- Introduce i.i.d. observation errors
  - Actions are recorded on *common database* via another BSC($\epsilon$), $0 < \epsilon < 0.5$

  \[
  \begin{array}{c}
  0 \\
  1 \\
  \end{array}
  \xrightarrow{\epsilon} \begin{array}{c}
  0 \\
  1 \\
  \end{array}
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\epsilon & 0 & 1 - \epsilon \\
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<table>
<thead>
<tr>
<th>Noiseless Model $\epsilon = 0$</th>
<th>Noisy Model $\epsilon &gt; 0$</th>
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**Herding in noiseless and noisy models**

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Agent $n$ herding iff $|\# Y' - \# N'| \geq k$ and $\epsilon < \epsilon^*(k+1, p)$ for some integer $k \geq 2$
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| Posterior Probability       | $P[V = 1|S_i, A_1, ..., A_{i-1}]$ | $P[V = 1|S_i, O_1, ..., O_{i-1}]$ |
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Model inherits many behaviors of noiseless model ([BHW'92], $\epsilon = 0$)
Summary of herding property

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  - Eventually herding happens (in finite time)
Markov chain viewpoint

• Assume $V = 1$ and $\epsilon^*(k, p) \leq \epsilon < \epsilon^*(k + 1, p)$
Markov chain viewpoint

- Assume $V = 1$ and $\epsilon^*(k, p) \leq \epsilon < \epsilon^*(k + 1, p)$
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![Diagram showing transitions between states]

- Agent 1 starts at state 0
- $a = P[One more Y added] = (1 - \epsilon)p + \epsilon(1 - p) > 0.5$, decreasing in $\epsilon$, increasing in $p$
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```
1  1-a  1-a
-k  -k+1  \cdots
```

```
1-a  1-a  1-a
-1  0  1  \cdots
```

```
1-a
k-1  k
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Absorbing state $k$: herd $Y$, Absorbing state $-k$: herd $N$
Markov Chain viewpoint (continued)

- Can exactly calculate expected payoff $E[\pi_i]$ & probability of wrong (correct) herding for any agent $i$
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  - $\mathbb{P}[\text{correct}_{i-1}] = \sum_{n=1}^{i-1} \mathbb{P}[\text{agent } n \text{ is the first to hit } k]$
Can exactly calculate expected payoff $E[\pi_i]$ & probability of wrong (correct) herding for any agent $i$

- $E[\pi_i]$ (MC with rewards)
- $\mathbb{P}[\text{wrong}_{i-1}] = \sum_{n=1}^{i-1} \mathbb{P}[\text{agent n is the first to hit } - k]$
- $\mathbb{P}[\text{correct}_{i-1}] = \sum_{n=1}^{i-1} \mathbb{P}[\text{agent n is the first to hit } k]$
- First-time hitting probabilities: Use probability generating function method [Feller’68]
Results

- Payoff for agents is non-decreasing in $i$ & at least $F = \frac{2p-1}{4} > 0$

![Graph of limiting wrong herding probability]

![Graph of limiting payoff $\Pi(\epsilon)$]

Limiting wrong herding probability

Limiting payoff $\Pi(\epsilon) = \lim_{i \to \infty} E[\pi_i]$
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    $F = \Pi(\epsilon^*(k + 1, p)^-) \leq \Pi(\epsilon^*(k + 1, p)^+)$
- There exists a range where increasing noise improves performance!!!
Results for an arbitrary agent $i$

Similar ordering holds for every user’s payoff & probability of wrong herding

- Discontinuities and jumps at the same thresholds
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Individual payoff for signal quality $p=0.70$
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![Graph showing individual payoff for signal quality $p=0.70$]

- For given level of noise, adding more noise may not improve all agents pay-offs.
Extension: Quasi-rational agents

• Real-world agents not always rational
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Limiting payoff, $p = 0.70$

Limiting payoff, $p = 0.80$
Conclusions

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  • For $\epsilon^*(k, p) \leq \epsilon < \epsilon^*(k + 1, p)$, require $|\#Y's - \#N's| \geq k$ to trigger herding.
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- With noisy observations, sometimes it is better to increase the noise
  - Probability of wrong herding decreases
  - Asymptotic individual expected welfare increases
  - Average social welfare increases
Future directions

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  - Combination with Sgroi’02 (guinea pigs)
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  - Investment in private signal when facing high wrong herding probability
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- Achieve learning with agents incentivized to participate
References


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Thank you!