Pricing Electricity through a Stochastic Non-Convex Market-Clearing Model

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What

Nonconvex pricing

Nonconvex pricing

Day-ahead price (dual variable)

Balancing price (dual variable)

$$\underset{x}{\text{Minimize}} \quad \text{expected production cost}(x)$$

subject to:

$$\text{supply} = \text{demand} : \ \lambda_{nt} \ \text{(price)}$$

$$\text{other constraints}$$

$$\text{adjustments in supply} = \text{adjustments in demand} : \ \lambda_{nt\omega} \ \text{(price)}$$

$$\text{other constraints}$$

This requires continuity

Nonconvex pricing

1. Current industry practice
2. An unorthodox approach
3. Examples
4. Conclusions
5. Reading

Current industry practice

1. Solve a MILP problem to clear the market & compute the optimal value of the binary variables.
2. Obtain an LP problem from the MILP problem by fixing the binary variable to their optimal values, solve it, & compute marginal prices (dual variables).

Current industry practice

3. Use these prices to pay producers and charge consumers.

4. If a producer does not recover cost, assign to it the minimum uplift required to recover cost, and socialize such uplift.

Current industry practice

Pool auction: computing binary variables

MILP

$$
\left.
\begin{aligned}
&\underset{x,y}{\text{Minimize}} \quad c^{\mathrm{T}} \begin{bmatrix} x \\ y \end{bmatrix} && \text{(1a)}\\
&\text{s. t.} \quad A \begin{bmatrix} x \\ y \end{bmatrix} \geq b && \text{(1b)}\\
&x \geq 0, x \in \mathbb{R}^{n}, y \in \mathbb{B}^{o}\\
&c \in \mathbb{R}^{n+o}, b \in \mathbb{R}^{m}, A \in \mathbb{R}^{m \times (n+o)}
\end{aligned}
\right\} \; y^{*}
$$

Note that $A \begin{bmatrix} x \\ y \end{bmatrix} \geq b$ includes $y \leq 1$

Current industry practice
Pool auction: computing productions & prices

LP

$$\underset{x}{\text{Minimize}} \quad c^{\mathrm{T}} \begin{bmatrix} x \\ y^* \end{bmatrix} \tag{2a}$$

s. t.

$$A \begin{bmatrix} x \\ y^* \end{bmatrix} \geq b : \quad \mu \tag{2b}$$

$$x \geq 0, x \in \mathbb{R}^n, y^* \in \mathbb{B}^o$$

x^*, μ^*

Current industry practice
Pool auction: cost recovery?

$$R_i(x_i^*, y_i^*, \mu^*) - C_i(x_i^*, y_i^*) \geq 0 \quad ?$$

Yes: OK

No: $\text{uplift}_i = C_i(x_i^*, y_i^*) - R_i(x_i^*, y_i^*, \mu^*)$

Current industry practice
Unorthodox approach: relaxation

LP

$$\underset{x,y}{\text{Minimize}} \quad c^{\mathrm{T}} \begin{bmatrix} x \\ y \end{bmatrix} \tag{3a}$$

s. t.

$$A \begin{bmatrix} x \\ y \end{bmatrix} \geq b : \quad \mu \tag{3b}$$

$$x \geq 0, x \in \mathbb{R}^n, y \geq 0, y \in \mathbb{R}^o$$

Relaxation

Note that $A \begin{bmatrix} x \\ y \end{bmatrix} \geq b$ includes $y \leq 1$

Current industry practice
Unorthodox approach: dual

LP

$$\underset{\mu}{\text{Maximize}} \quad b^{\mathrm{T}}\mu \qquad (4a)$$

s. t.

$$A^{\mathrm{T}}\mu \leq c \qquad (4b)$$

$$\mu \geq 0, \mu \in \mathbb{R}^m$$

$$\left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \text{Dual}$$

Current industry practice
Unorthodox approach: primal-dual

LP

$$
\left.
\begin{array}{ll}
\displaystyle \underset{x,y,\mu}{\text{Minimize}} \quad c^{\mathrm{T}} \begin{bmatrix} x \\ y \end{bmatrix} - b^{\mathrm{T}} \mu & \text{(5a)} \\
\text{s. t.} & \\
A \begin{bmatrix} x \\ y \end{bmatrix} \geq b & \text{(5b)} \\
A^{\mathrm{T}} \mu \leq c & \text{(5c)} \\
x \geq 0, y \geq 0, \mu \geq 0 &
\end{array}
\right\} \text{Primal-Dual}
$$

Current industry practice
Unorthodox approach: primal-dual

LP

$$\underset{x,y,\mu}{\text{Minimize}} \quad \overbrace{c^{\mathrm{T}} \begin{bmatrix} x \\ y \end{bmatrix} - b^{\mathrm{T}} \mu}^{\text{Duality gap}} \tag{5a}$$

s. t.

$$A \begin{bmatrix} x \\ y \end{bmatrix} \geq b \tag{5b}$$

$$A^{\mathrm{T}} \mu \leq c \tag{5c}$$

$$x \geq 0, y \geq 0, \mu \geq 0$$

$$\left.\begin{matrix} \\ \\ \\ \\ \\ \\ \end{matrix}\right\} \text{Primal-Dual}$$

Current industry practice
Unorthodox approach: primal-dual

Problem (5) above allows including additional constraints involving both primal & dual variables.

This is done at the cost of not achieving a zero duality gap and not being fully equivalent to the original problem.

Current industry practice

Unorthodox approach: primal-dual + integrality

MILP

$$\underset{x,y,\mu}{\text{Minimize}} \quad c^{\mathrm{T}} \begin{bmatrix} x \\ y \end{bmatrix} - b^{\mathrm{T}} \mu \tag{6a}$$

s. t.

$$A \begin{bmatrix} x \\ y \end{bmatrix} \geq b \tag{6b}$$

$$A^{\mathrm{T}} \mu \leq c \tag{6c}$$

$$x \geq 0, y \geq 0, \mu \geq 0$$

$$y \in \mathbb{B}^{o} \tag{6d}$$

Primal-Dual
+
Integrality

Current industry practice
Unorthodox approach: + integrality + cost recovery

$$\underset{x,y,\mu}{\text{Minimize}} \quad c^{\mathrm{T}} \begin{bmatrix} x \\ y \end{bmatrix} - b^{\mathrm{T}} \mu \tag{7a}$$

s. t.

$$A \begin{bmatrix} x \\ y \end{bmatrix} \geq b \tag{7b}$$

$$A^{\mathrm{T}} \mu \leq c \tag{7c}$$

$$x \geq 0, y \geq 0, \mu \geq 0$$

$$y \in \mathbb{B}^{o} \tag{7d}$$

$$R_i(x_i, y_i, \mu) - C_i(x_i, y_i) \geq 0 \quad \forall i \tag{7e}$$

MINLP

Primal-Dual

$+$

Integrality

$+$

Cost recovery

Current industry practice
Unorthodox approach

MINLP

$$\underset{x,y,\mu}{\text{Minimize}} \quad c^{\mathrm{T}} \begin{bmatrix} x \\ y \end{bmatrix} - b^{\mathrm{T}}\mu \tag{7a}$$

s. t.

$$A \begin{bmatrix} x \\ y \end{bmatrix} \geq b \tag{7b}$$

$$A^{\mathrm{T}}\mu \leq c \tag{7c}$$

$$x \geq 0, y \geq 0, \mu \geq 0$$

$$y \in \mathbb{B}^{o} \tag{7d}$$

$$R_i(x_i, y_i, \mu) - C_i(x_i, y_i) \geq 0 \quad \forall i \tag{7e}$$

x^*, y^*, μ^*

Duality gap $\neq 0$

Current industry practice
Unorthodox approach: primal-dual

The solution of (7) is as close as possible to that of the original problem: the duality gap is minimum.

Problem (7) guarantees that both primal & dual constraints are satisfied.

Nonconvex pricing in an electricity pool

Primal-dual

$$
\begin{aligned}
&\underset{\Xi_p, \Xi_d}{\text{Minimize}} \quad \text{Primal o.f.} - \text{Dual o.f.} && (10a)\\
&\text{subject to:}\\
&\quad \text{Primal constraints} && (10b)\\
&\quad \text{Dual constraints} && (10c)\\
&\quad \color{red}{\text{Integrality constraints}} && (10d)\\
&\quad \color{red}{\text{Cost recovery constraints}} && (10e)
\end{aligned}
$$

Both stochastic and deterministic

Nonconvex pricing in an electricity pool

The aim is to obtain a set of uniform revenue-adequate prices λ_{nt} (day-ahead market) and $\lambda_{nt\omega}$ (real-time market).

In other words, to provide appropriate incentives to the producers by ensuring that, if dispatched, they would not experience losses.

Nonconvex pricing in an electricity pool

Cost recovery is enforced only at the day-ahead market stage:

$$\sum_t \left((\lambda_{nt} - C_i) P_{it} - C_{it}^{\mathrm{SU}} \right) \geq 0 \quad \forall i$$

Nonconvex pricing in an electricity pool

Cost recovery is enforced in expectation:

$$\sum_{t} \Big[(\lambda_{nt} - C_i) P_{it} - C_{it}^{\mathrm{SU}}$$
$$+ \sum_{\omega} \pi_{\omega} (\lambda_{nt\omega}/\pi_{\omega} - C_i)(r_{it\omega}^{\mathrm{U}} - r_{it\omega}^{\mathrm{D}}) \Big] \geq 0, \forall i$$

Nonconvex pricing in an electricity pool

Cost recovery is enforced per scenario:

$$\sum_t \Big[(\lambda_{nt} - C_i)P_{it} - C_{it}^{\mathrm{SU}}$$

$$+ (\lambda_{nt\omega}/\pi_\omega - C_i)(r_{it\omega}^{\mathrm{U}} - r_{it\omega}^{\mathrm{D}})\Big] \geq 0, \forall \omega, \forall i$$

Nonconvex pricing in an electricity pool

These constraints ensure the nonnegativity of the profit of each producer.

Note that the problem above, including these nonlinear constraints, is a mixed integer nonlinear programming problem (MINLP).

Nonconvex pricing in an electricity pool

MINLPs are in general hard to solve, and no off-the-shelf solver is available to guarantee convergence or optimality.

For computational tractability, these constraints can be (approximatelly) linearized.

Acronyms

Con Conventional Method - No Uplift

U Conventional Method - With Uplift

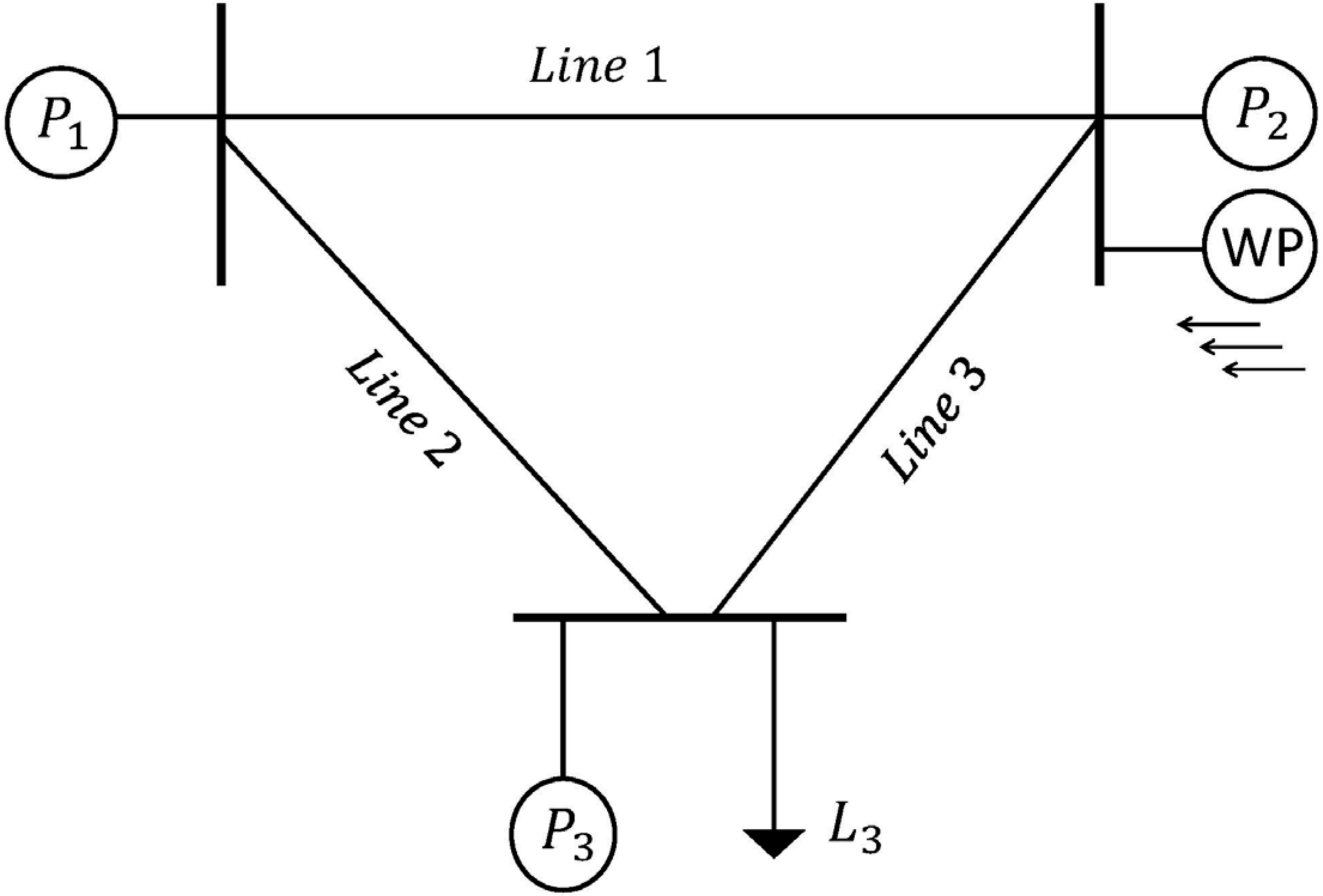
CR Pricing approach with cost recovery at the day-ahead market stage.

AR Pricing approach with average cost recovery.

SR Pricing approach with cost recovery per scenario.

Example

Example: Data



Inelastic demand

Two period

No congestion

Example: Data

Table 1: Data of generating units.

Unit	K_i^{SU}	C_i	$P_i^{\max}$	$P_i^{\min}$
1	101.1	20.03	95	10
2	103.2	50.06	100	10
3	2001.06	100.01	105	10

Table 2: Wind scenarios and Load profile [MW]

Period	High	Low	L_3
t_1	59	13	110
t_2	111	17	280

Example: Results

Table 3: Day-ahead energy prices [$/MWh]

method	λ_{t_1}	λ_{t_2}
Con	20.03	70.046
CR	33.85	120.02
AR	33.85	102.9
SR	33.85	105.4

Example: Results

Table 4: Consumer payment, expected cost and duality gap in [$]

	CR	AR	SR	U
Consumer payment	37328.9	33134.1	32538.1	25315
Expected cost	13044.5	13084.46	13084.46	13044.5
Gap	2091.86	1528.8	1607.44	–

Case Study

RTS Data

Table 1: Characteristics of the Generating Units

	U_{76}	U_{50}	U_{155}	U_{50}	U_{197}	U_{50}	U_{400}
Node	2	7	15, 18	15	21	22	23
$P_i^{\max}$	76	50	155	50	197	50	400
$P_i^{\min}$	15	15	55	15	69	15	100
C_i^{SU}	400	100	320	100	300	100	1000
C_i	13.89	0	10.68	0	11.09	0	5.53

RTS Data

Table 2: Total demand in [MW]

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
441.1	481	482	483	490	1021.6	1132	1097
t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}
960.5	910.2	910	941.2	943	960	970	1031
t_{17}	t_{18}	t_{19}	t_{20}	t_{21}	t_{22}	t_{23}	t_{24}
1123	1130	1112	1101	998	930.1	780	440

Table 3: Demand location

Demand	D_1	D_2	D_3	D_4	D_5
Node	1	4	13	14	20
Share %	33.5	18.9	14.9	16.2	16.5

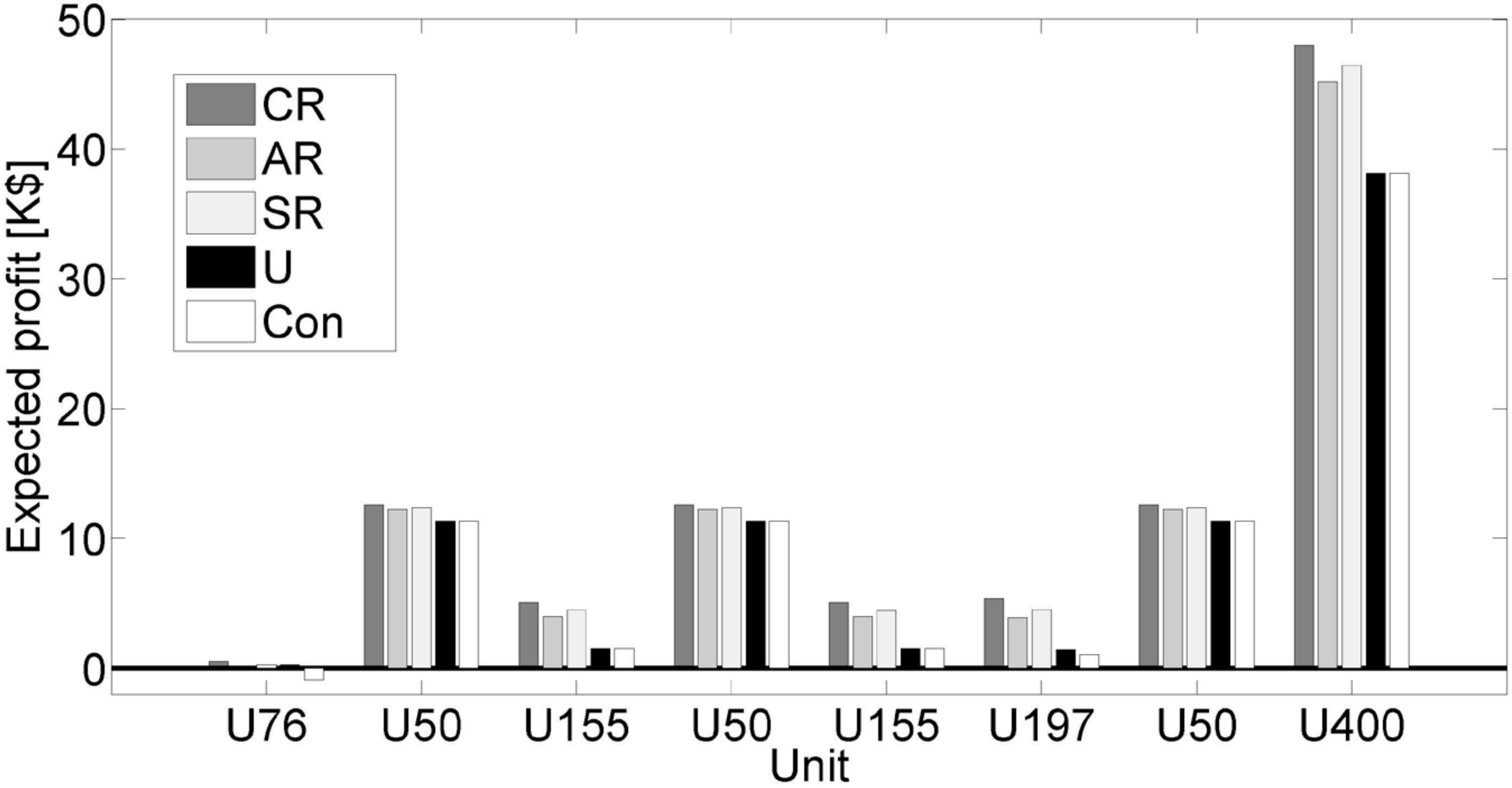
RTS Data

Table 4: Dimension of the proposed models

	CR	AR	SR	U
No. of continuous variables	93368	96968	96968	27600
No. of integer variables	1560	5160	5160	1176
No. of total variables	94928	102128	102128	28776
No. of constraints	95361	108561	108585	65384
Computation time (s)	22705	14231	1624	57

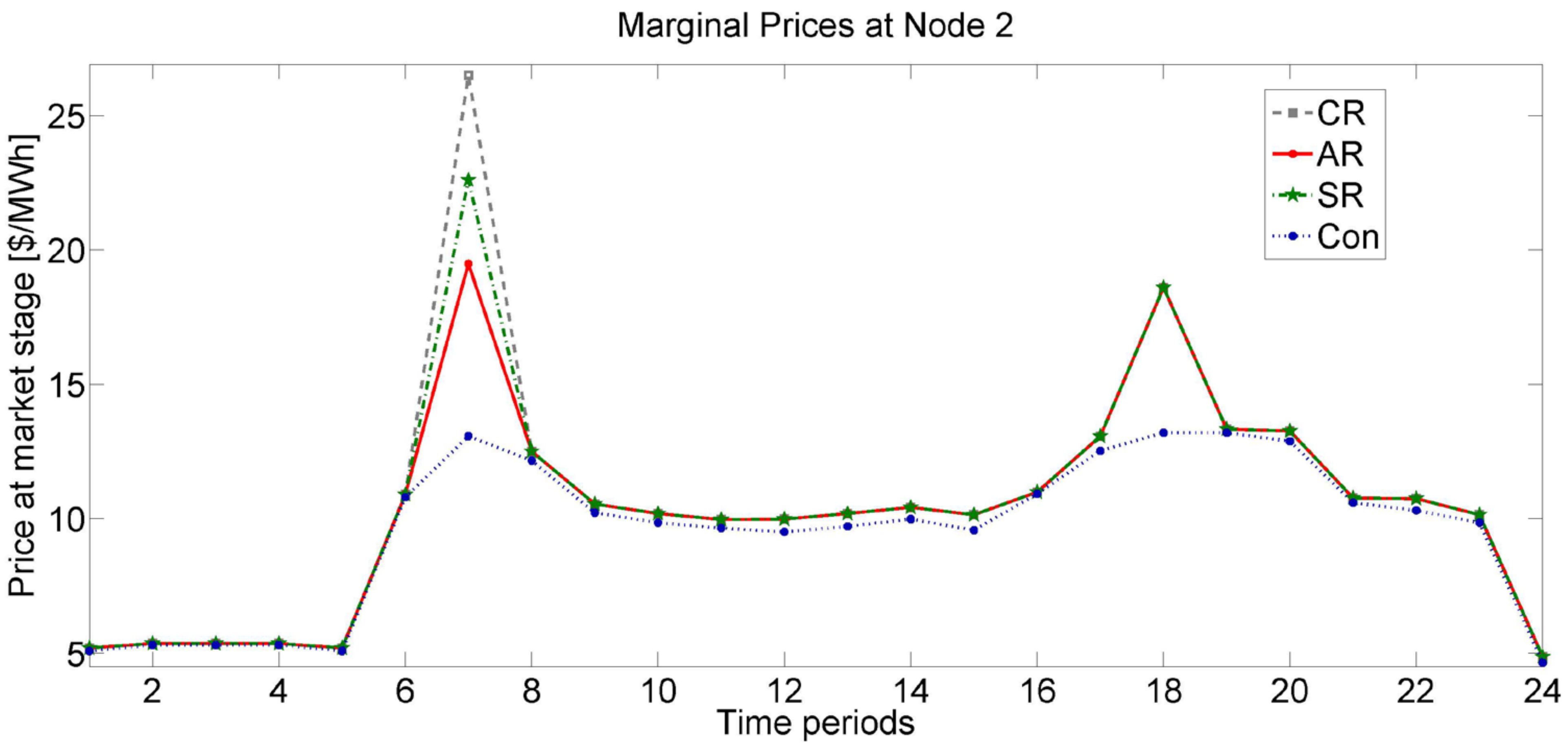
RTS Results

Figure 1: Expected Profit (RTS system).



RTS Results

Figure 2: Energy prices at node 2 under different approaches (RTS system).



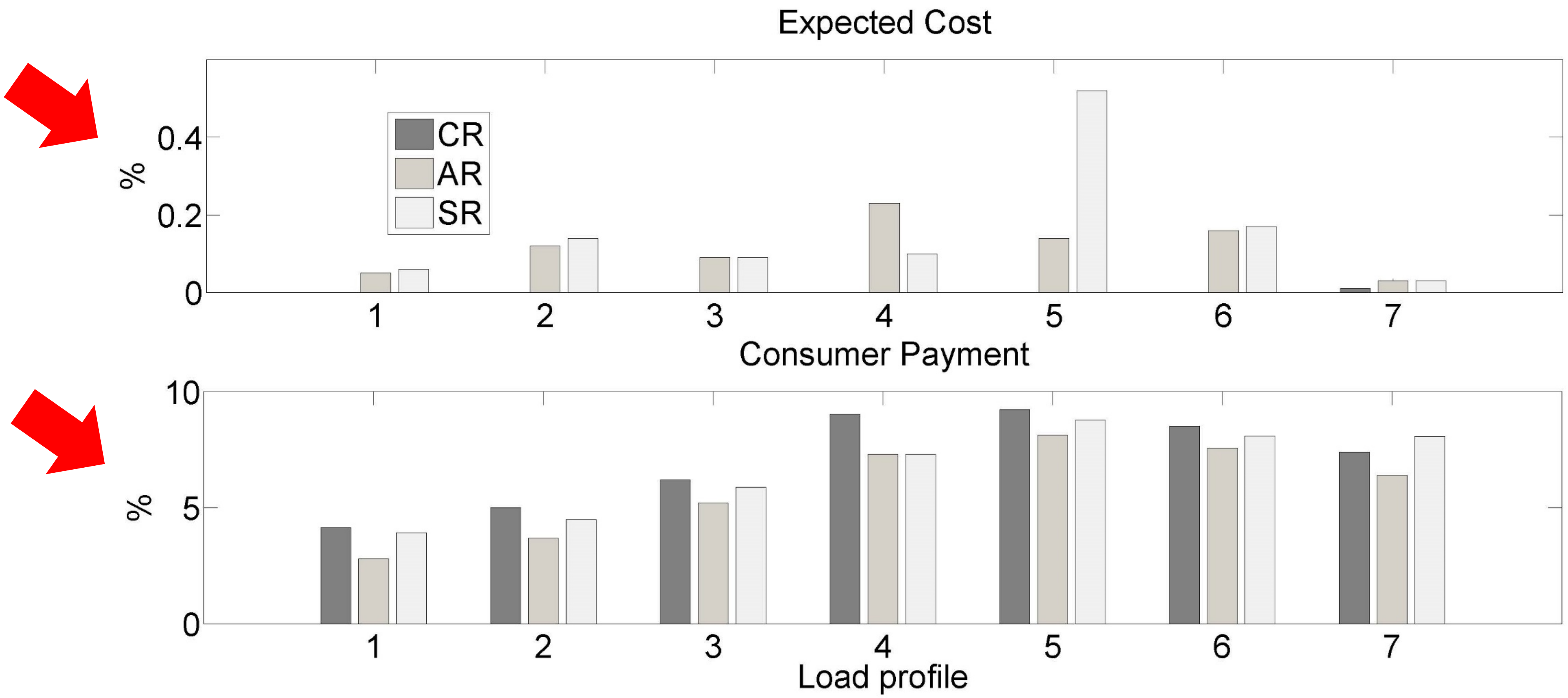
RTS Results

Table 5: Consumer payment, expected cost and duality gap for the RTS system [$].

	CR	AR	SR	U
Consumers payment	2.42e+05	2.34e+05	2.38e+05	2.17e+05
Expected cost	127,066	127,169	127,153	127,066
Gap	288.24	384.80	371.10	–

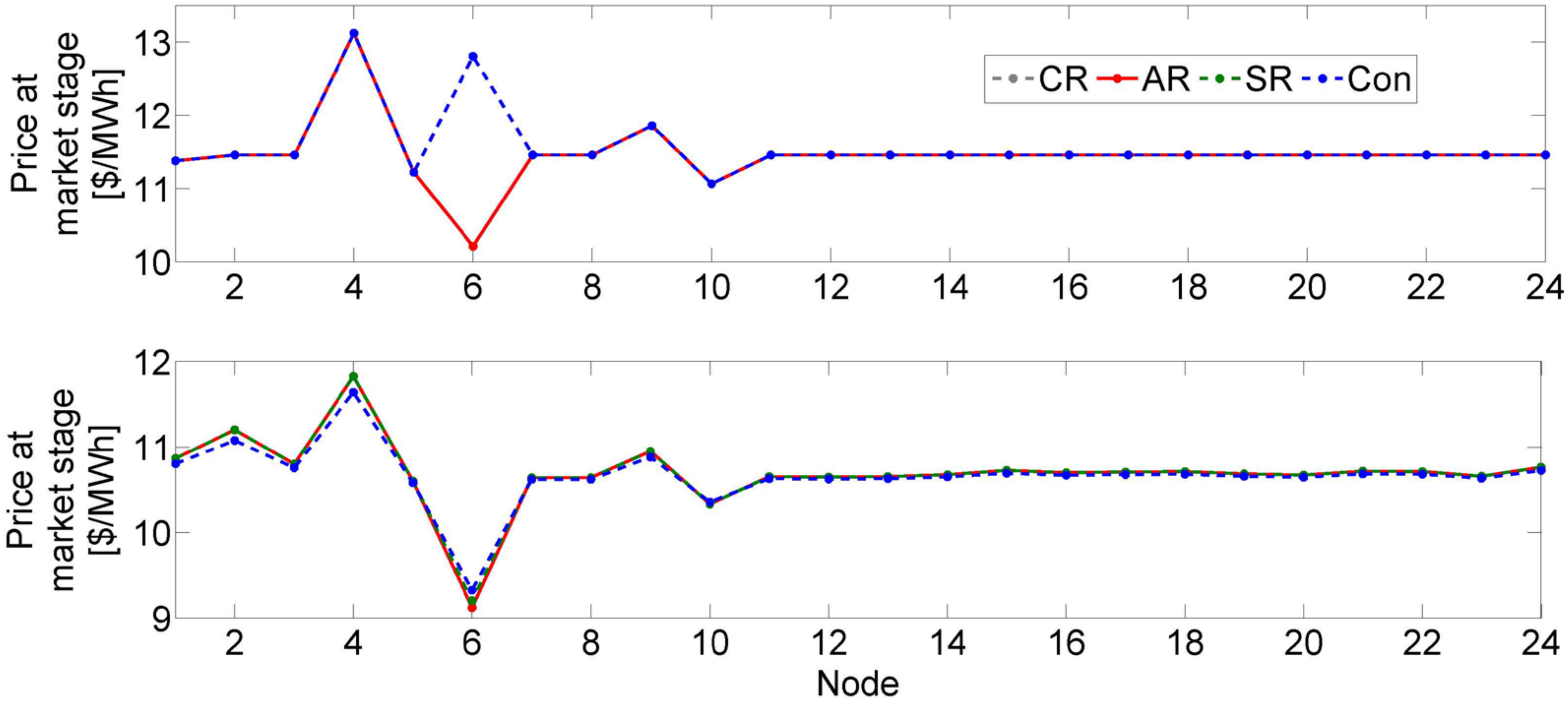
RTS Results

Figure 3: Cost increase in percent (with respect to original problem), and consumer payment increase (with respect to payment from the uplift method) in percent for different load profiles (RTS system)



RTS Results - congestion

Figure 4: LMPs at t_{18} (top) and t_{21} (bottom) obtained by the different approaches.



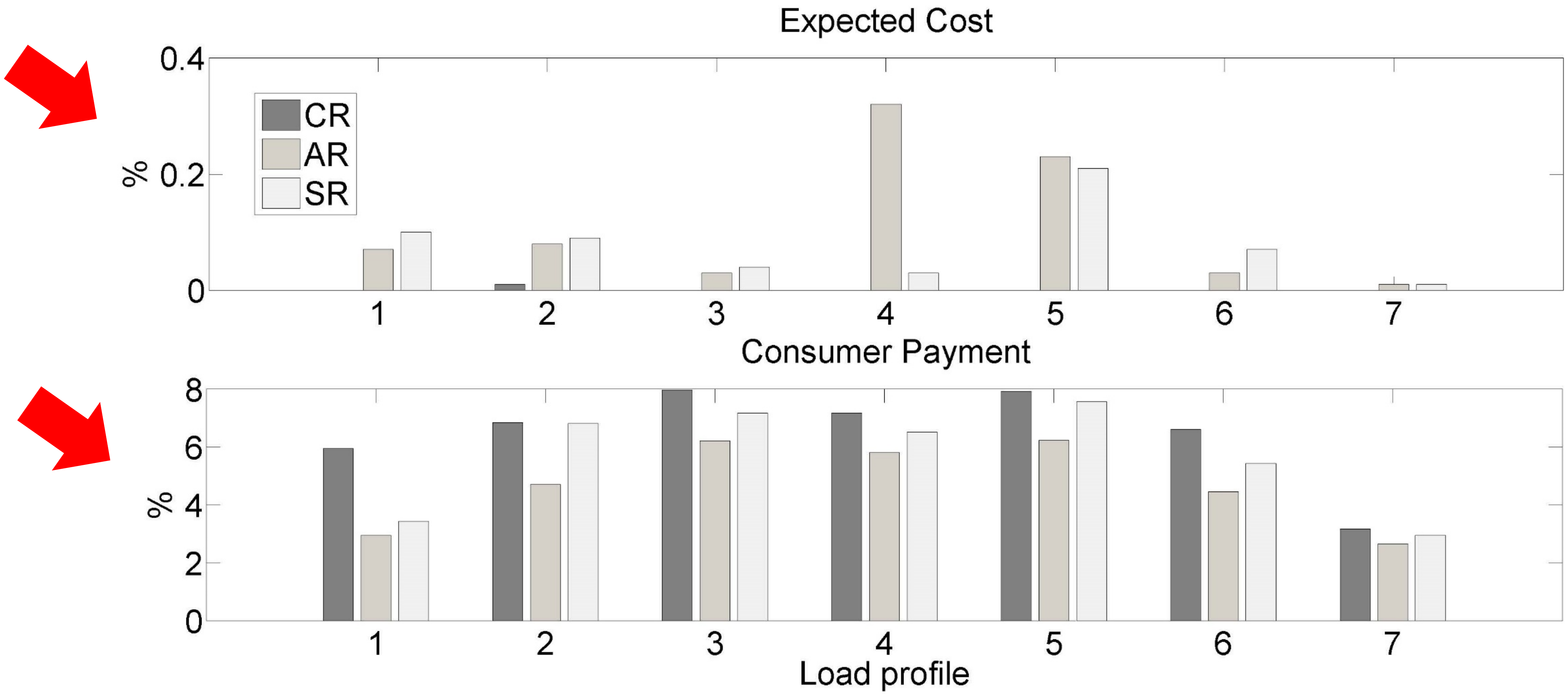
RTS Results - congestion

Table 7: Consumer payment, expected cost and duality gap for the RTS system with congestion [$].

	CR	AR	SR	U
Consumers payment	286903.5	285342.1	286100.8	277298.1
Expected cost	170,257	170,271	170,273	170,257
Gap	211.58	174.27	230.91	–

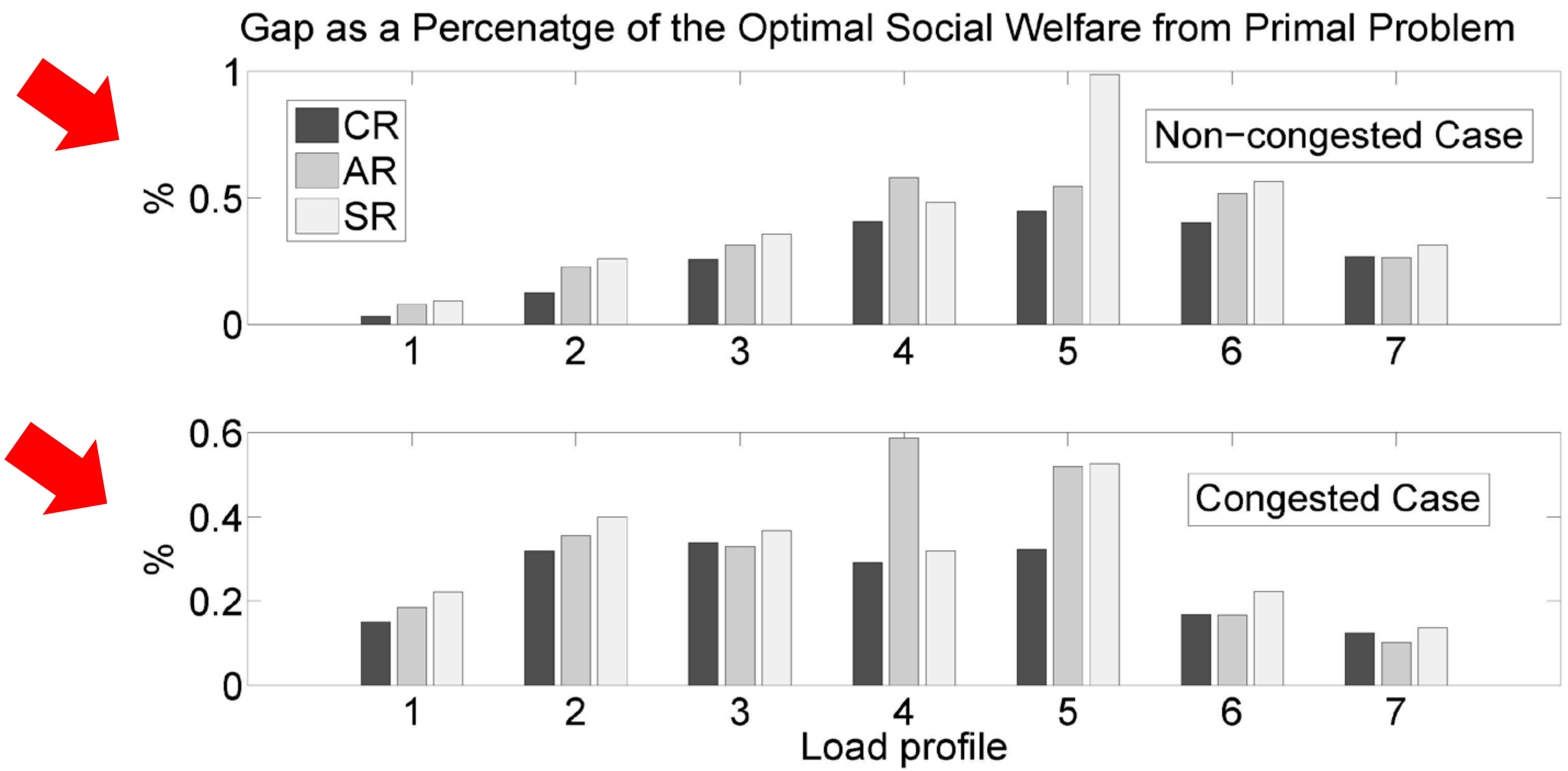
RTS Results - congestion

Figure 5: Cost increase in percent (with respect to original problem), and consumer payment (with respect to payment from the uplift method) in percent for different load profiles (RTS congestion case)



RTS Results - congestion

Figure 6: Social welfare gap as a pecentage of the optimal social welfare obtained from the primal problem for different load profiles



Concluding remarks

Concluding remarks
Good proposal!

Support market outcomes (no producer willing to leave)

Slightly deviates from marginal prices if integrality is relaxed

No computational overburden!

No non-uniform uplifts!

Concluding remarks
"Drawback"

Resulting prices not in the demand curve, but same with uplifts!

Self-scheduling profits might be higher for some producers... lost opportunity profits.

Reading

Ruiz, C.; Conejo, A.J.; Gabriel, S.A., "Pricing Non-Convexities in an Electricity Pool," Power Systems, IEEE Transactions on, vol.27, no.3, pp.1334-1342, Aug. 2012.
http://dx.doi.org/10.1109/TPWRS.2012.2184562

F. Abbaspourtorbati; A. Conejo; J. Wang; R. Cherkaoui, "Pricing Electricity through a Stochastic Non-Convex Market-Clearing Model," in IEEE Transactions on Power Systems, in press.
http://dx.doi.org/10.1109/TPWRS.2016.2569533

thank you