DISC SCATTERING AT EDGE-ON INCIDENCE

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Abstract

As part of a general study of the back scattering from thin plates illuminated by a plane wave at glancing incidence, measurements have been made of the surface and far fields of circular discs. Some results of the surface field probing are presented and interpreted in terms of optics and creeping wave components analogous to those appropriate to the surface of a cylinder. Analytic forms for the components are then determined, and an expression for the back scattered field deduced therefrom.

The result is compared with measured far field data for electrically thin discs, and the effects of increasing the electrical thickness, either uniformly or non-uniformly, are also discussed.

1. Introduction

Many types of radar targets have one or more component structures which are substantially plane and thin compared with their lateral dimensions. An example of such a structure is a wing or fin of an airborne vehicle. It is obvious that this will provide a large contribution to the radar echo (or backscatter) when viewed normal to its planar surface, and that in comparison the contribution is small at other aspects; but it is not so well appreciated that when the structure is viewed edge-on the return can still be large enough to dominate the scattering behavior of the overall target, particularly when the electric vector is in its plane and the wavelength is comparable to its lateral dimensions.
Unfortunately, the standard methods for radar cross section estimation fail in the case of a thin plate at edge-on incidence. Geometrical and physical optics predict zero backscatter, and the geometrical theory of diffraction, which can be so effective for a plate of simple configuration near normal incidence (see, for example, Ross, 1966), breaks down near glancing incidence due to failures in the expressions for the diffraction coefficients. Indeed, the only 'plates' for which accurate formulae for the edge-on scattering are available are the half plane and ribbon, and since the scattering from any finite plate is critically affected by the front-to-rear coupling afforded by the side edges, even the solution for a ribbon (see Fialkovsky, 1966, for an asymptotic expansion uniform in angle) provides no basis for predicting how a finite plate will scatter.

On the experimental side, not much information is available on the edge-on scattering characteristics of plates. Hey and Senior (1958) have provided some data for a rectangular plate of width $2\lambda$ and a triangular plate of vertex angle $30^\circ$ showing the variation in cross section as a function of the electrical length of the plate. The frequency was maintained constant, thereby preserving the same electrical thickness of the plate, and it was shown that a simple theory based only on the surface field characteristics for a half plane was inadequate to explain the results. A more complete set of data for a rectangular plate of width $\lambda$ has been presented by Ross (1966) who also gives a purely empirical formula to fit the data; and for a disc, several sets of data for somewhat restricted ranges of $a/\lambda$ have been published, most noticeably by Hey et al (1956) and by Ryan and Peters (1968).

In order to better understand the effect of wing shape and thickness on the edge-on scattering from aircraft wings, a rather comprehensive series of measurements was performed using plates of various configurations: circular, triangular, rectangular, star-shaped, etc. In each experiment the frequency (electrical thickness of plate) was held constant while the lateral dimensions of
the plate were varied, and from the data it was apparent that a theoretical interpretation akin to that provided by the geometrical theory of diffraction in the case of more general bodies (including plates near normal incidence) could also be appropriate here, but with such modifications as were demanded by the small electrical thickness of the plates. Thus, specular contributions from edges could be discerned with magnitudes in accordance with 'optics' estimates based on edge scattering from a half plane, as could the contributions from traveling waves supported by longitudinal edges (with reflection coefficients determined by the nature of the rear corner), and from a form of creeping wave along edges which were curved. By appropriate combination of these essentially local contributions, most of the edge-on scattering characteristics of plates of fairly general configuration can be estimated with reasonable accuracy even for lateral dimensions as small as the wavelength.

Probably the most simple plate configuration with which to demonstrate this local scattering is a disc. Extensive measurements of the edge-on scattering have been made for wide ranges of $a/\lambda$, where $a$ is the radius, and for discs of different thicknesses up to $\lambda/4$. In addition, selected surface field probing has been carried out to determine the nature of the current distribution. Such surface field measurements have been a key feature of our study. We here describe some of the data that have been obtained for electrically thin discs; show how they relate to the concept of local scattering, and deduce therefrom the form of the dominant contributors to the surface field; and from these contributors, compute the back scattering behavior for electrically thin discs. The results are in excellent agreement with the measured far field data. Some comments on the role played by the electrical thickness of the disc are also given.
2. Surface Field Characteristics

The primary motivation for the general study of plates at edge-on incidence was the investigation of the dependence of the back scattering cross section on the geometry and electrical dimensions of the plate. The initial measurements were of the back scattering cross sections of plates of various configurations as the plates were systematically reduced in size, their thickness being held constant, but whilst such data usually provided some indication of the nature of the dominant contributors to the return, it was often desirable to resort to surface field probing to verify these sources and/or to determine the magnitude of their contributions. This was true, in particular, for discs, and it is convenient to begin this account with an examination of the field induced in the surface.

A series of six circular discs with radii ranging from 4 to 10 inches were cut from 1/32 inch thick aluminum sheet. Each was mounted on top of a styrofoam support and illuminated at grazing incidence by a horizontally polarized wave at a frequency of 3.0 GHz (see Knott et al, 1965, for a description of the measurement facility). The circumferential component of the current density \( J_\phi \) was then measured at the center of the rim using a small loop probe whose position was varied from \( \phi = 0 \) (the front of the disc) to \( \phi = \pi \) (the rear) and on back to the front again (the geometry is shown in Fig. 1). Due to a slight asymmetry in the probe response, there was a detectable difference between the readings taken at corresponding points on the two edges of the disc; but the differences averaged less than 0.2 dB and the subsequent discussion is based on the averages of the readings for the left and right sides of the rim. It was also observed that the measured values varied by as much as \( \pm 0.3 \) dB between the upper and lower edge, and care was therefore taken to maintain the loop in the center of the rim.

A typical curve of \( |J_\phi| \) versus \( \phi \) produced from measurements made every \( 2-1/2^\circ \) in \( \phi \) is that shown in Fig. 2 for a disc 11-1/4" in diameter.
(ka = 9, where k = 2π/λ is the free space propagation constant). Normalization is with respect to the incident field level (0 dB), but because of the integrating properties of the probe about the rim in a radial plane, the scale is relative as regards the absolute value of the current. The most striking feature of the curve is the regular oscillation of the current amplitude about a mean which, for small φ, is almost constant, then rises slowly to a maximum about 2.5 dB higher at φ = 90°, and thereafter decreases with increasing φ. The oscillations have a rather uniform period. Then amplitude increases markedly with increasing φ, and on the rear half (π/2 < φ ≤ π) of the disc at least, the behavior of the rim current is quite similar to that of the surface field on a circular cylinder (King and Wu, 1959). On the shadowed portion of a cylinder the field is attributable to creeping waves which are born in the vicinity of the shadow boundary and proceed around the circumference leaking off energy in the tangential direction as they go. As a result of this leakage, each wave is exponentially attenuated at a rate proportional to the distance travelled. About each shadow boundary there is a transition region of angular width O{(ka)^{-2/3}} within which the creeping wave form of the shadow field goes over into the optics form characteristic of the illuminated portion of the body, but even as φ approaches zero the creeping wave born near φ = 3π/2 is still apparent as an interference with the more dominant optics component.

It would appear that the same type of wave is present on a disc, and to confirm the creeping wave character of the rim current over the rear portion, a piece of absorbing material was wrapped around the rim near to the shadow boundary at φ = 3π/2. The aim was to suppress the wave originating there, leaving only the one born at φ = π/2 and proceeding in the direction of increasing φ. The measured data for π/2 ≤ φ ≤ π are indicated by the broken line in Fig. 2, and though there are still some small and irregular oscillations, doubtless due to the imperfections of the absorber, the exponential nature of the residual field is evident.
For a thin disc the analogue of the optics component on a circular cylinder is the current induced near the edge of a half plane at glancing incidence. Values for this component can be deduced from the measured disc data by taking the mean through the oscillations, and an examination of Fig. 2 shows that forward of the transition region (\( \phi < \pi/4 \), say) the mean curve does display the \( \sqrt{\sec \phi} \) dependence predicted by the half plane current (Hey and Senior, 1958). To complete the identification, a comparison of the magnitudes of the disc and half plane currents is required. Because the current parallel to the edge of a half plane is infinite at the edge, whereas the probe integrates over a small but finite area, it was necessary to probe the half plane current itself. This was done using a large (2' x 4') plate with absorber placed round the rear and side edges to suppress the effect of these terminations. When the plate was illuminated at glancing incidence in a plane normal to the edge, the current at the edge was found to be within 0.2 dB of that indicated by the mean disc current at \( \phi = 0 \).

Further confirmation of the agreement between the optics component of the disc current and the half plane current was obtained by probe measurements on the upper surfaces of the disc and plate along a trajectory perpendicular to the front edge. The results are shown in Fig. 3 along with the
theoretical curve for the half-plane current. The two sets of measured data are in agreement out to more than a wavelength from the edges, at which point the current amplitudes are almost 20 dB below the values that they had there. Apart from the distance $0 < x < 0.1 \lambda$, where the probe integration is significant, the measured data follow the theoretical curve extremely closely.

To determine the extent to which all of the disc currents were concentrated in the immediate vicinity of the rim, measurements were now made along two perpendicular diameters of the upper surface of a disc 12" in diameter ($ka = 9.6$). Along the diameter from front to rear, the component measured was that parallel to the front edge and, hence, to the incident electric vector, whereas along the other diameter (connecting the 'shadow boundary' points on the rim) the perpendicular component was probed. The results are presented in Fig. 4. Each component resembles a half plane current near to the edge that it parallels and, in the case of the side-to-side trajectory, the component is down by 10 dB at a distance of 0.6 $\lambda$ from the rim and continues to decrease to almost zero at the center of the disc with evidence of an exponential fall-off in addition to the $x^{-1/2}$ behavior. Along the front-to-rear trajectory the component is somewhat larger and near to the back there is evidence of an interference between the wave propagating away from the front edge and the more localized rear-edge current; but even with this component the current is still substantially confined to the vicinity of the rim, suggesting that a knowledge of the current here may be adequate for predicting the scattering behavior of a disc edge-on (see Section 4).
3. Numerical Estimates of the Surface Field Components

Data curves for the rim current amplitude $|J_\phi|$ such as that shown in Fig. 2 were obtained for six discs having $ka=6.4$, 9.0, 9.6, 10.0, 11.2 and 16.0 and from them it is possible to estimate the analytic form of the creeping wave component. The procedure that was adopted is as follows. For each set of data, a curve representing the mean through each oscillation was determined from the levels of the maxima and minima, and the data renormalized relative to the mean value at $\phi = 0$. Taking henceforth the renormalized data, the current on that portion of the rim beyond the transition region ($\phi > 3\pi/4$, say) can be written as:

$$J_\phi = A e^{i(ka+\psi)} \left\{ e^{(i\gamma_1 - \gamma_2)(3\pi/2 - \phi)} + e^{(i\gamma_1 - \gamma_2)(\phi - \pi/2)} \right\}, \tag{1}$$

where $A$, $\psi$, $\gamma_1$, and $\gamma_2$ are real, and the first term in braces is associated with the creeping wave born at $\phi = 3\pi/2$, whereas the second term is associated with that born at $\phi = \pi/2$. The first wave must preserve its form into the region $\phi < \pi/4$ (say) where it interferes with the optics component (represented by the mean curve), and curves such as the solid one in Fig. 2 therefore give information about the amplitude of this wave over an effective range $0 < \phi < 3\pi/4$.

From the levels of the maxima and minima relative to those of the mean curve at the corresponding points, a sequence of values of the creeping wave amplitude are obtained at discrete values of $\phi$. When plotted on a logarithmic scale, these reveal a linear dependence on $\phi$ as predicted by Eq. (1), and by means of a least squares fit, $A$ and $\gamma_2$ can be deduced. Thus, for $ka = 9$, it is found that $A = 2.1$ and $\gamma_2 = 0.90$. The locations of the maxima and minima also specify $\gamma_1$, but since we must now confine attention to the range $3\pi/4 \lesssim \phi \leq \pi$, the accuracy of its determination is somewhat less. For $ka = 9$, $\gamma_1 \approx 0.94, ka$.

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*M. K. S. units are employed and a time factor $e^{-iat}$ suppressed.*
This procedure was carried through for each of the six sets of current data, and by comparing the resulting values of $A$, $\gamma_2$ and $\gamma_1$, their dependence on $ka$ was separated out. It was found, for example, that $\gamma_2$ was proportional to $(ka)^{1/3}$ and a least squares fit then gave $\gamma_2 = 0.43 (ka)^{1/3}$. The values for $\gamma_1$ were somewhat more scattered, but did suggest that $\gamma_1 - ka = -\alpha (ka)^{1/3}$, with the best fit corresponding to $\alpha = 0.25$. Since the apparent phase velocity of the creeping wave relative to the velocity of light is $ka/\gamma_1$, the velocity exceeds unity in contrast to the case for a creeping wave on a cylinder.

The only quantity in Eq. (1) that is still to be determined is the uniform phase $\psi$, and though the measurements of the rim current were of the amplitude alone, $\psi$ can be found by reference to the known phase of the half plane current representing the optics component. For small values of $\phi$ the oscillations in $|J_\phi|$ are primarily due to interference between the optics component and the creeping wave which originated at $\phi = 3\pi/2$, and since the phase of the latter relative to the former is

$$\gamma_1 \left( \frac{3\pi}{2} - \phi \right) + \psi + ka - \frac{\pi}{4}$$

with $\gamma_1$ known, an inspection of these oscillations suffices to determine $\psi$. The result obtained is $\psi \sim \pi/3$ independently of $ka$, and the specification of the creeping wave component of the rim current is now complete.

There are two comments that should be made before leaving this discussion. Because of the finite amount of data available for analysis, it cannot be claimed that the values deduced for the parameters in Eq. (1) are without possibility of error. Though the $ka$ dependences arrived at are almost certainly correct, and the numerical constants in the expressions for $\gamma_2$ and $A$ are also believed reliable, the values for $\alpha$ and, hence, $\psi$ are somewhat more uncertain. We remark, however, that any substantial error in either $\alpha$ or $\psi$ would have a marked effect on the locations of the maxima and minima in the back scattering cross section as a function of $ka$, and as we shall see, the values given provide excellent agreement with measured data for the cross section. We also note that our analytic
approximation of the rim current does not cover the entire 0 to \( \pi \) range in \( \phi \).
Within the transition region about \( \phi = \pi /2 \), only that portion of the rim current represented by a wave traveling in the negative \( \phi \) direction (the creeping wave originating at \( \phi = 3\pi /2 \)) has been approximated, but this is adequate for the derivation of the back scattered field.

4.\, \textbf{Backscattered Field}

Let us choose a Cartesian coordinate system \((x, y, z)\) such that the disc lies in the xy plane with its front at the origin of the coordinates. The incident field is a plane wave at glancing incidence with its electric vector parallel to the disc, and can be taken as
\[
\mathbf{E} = Y \mathbf{e}^{i k x}, \quad \mathbf{H} = Y \mathbf{e}^{i k x},
\]
where \( Y = 1/Z \) is the intrinsic admittance of free space. The scattered field can be expressed as an integral over the currents induced in the surfaces of the disc, and, in particular, the far zone back scattered field is
\[
\mathbf{E}^s = Y S e^{i k R/k R}, \quad \mathbf{H}^s = -2 Y S e^{i k R/k R},
\]
in which
\[
S = i k^2 \int_0^a \int_0^\pi e^{i k (r \cos \phi)} Z J_y rdr d\phi, \quad (2)
\]
where \( r, \phi \) are polar coordinates in the plane of the disc, and symmetry has been invoked to confine the integration to one half of the upper surface. In terms of \( S \) the backscattering cross section is
\[
\sigma = \frac{\lambda^2}{\pi} \left| S \right|^2. \quad (3)
\]
The optics contribution to \( S \) originates in the vicinity of the front edge of the disc, and our expectation, supported by the surface field measurements, is that this can be estimated using the local approximation to \( J_y \) represented by the half plane current. For small \( \phi \),
\[ ZJ_y \sim \sqrt{\frac{2 \cos \phi}{\pi k (a-r)}} e^{i k (a-r \cos \phi) + \frac{\pi}{4}} \]

(Hey and Senior, 1958) and thus

\[ S^{\text{op}} \sim \frac{k}{\pi} \sqrt{\frac{2k}{\pi}} e^{\frac{3i \pi}{4}} \int_0^a \int_0^a \frac{\cos \phi}{a-r} e^{2ika(1-\cos \phi) + 2ika(r-a) \cos \phi} r \, dr \, d\phi, \tag{4} \]

where the \( \phi \) integration extends from zero to some small positive value. The radial integral can be reduced to a Fresnel integral whose asymptotic approximation for large \( ka \) represents the contribution from the immediate vicinity of the front edge of the disc. Substituting this into Eq. (4) produces an \( \phi \) integral amenable to saddle point approximation with saddle point at \( \phi = 0 \), from which we have

\[ S^{\text{op}} \sim \frac{1}{2} \sqrt{\frac{ka}{\pi}} (1 - \frac{i}{4ka}) e^{-3i \frac{\pi}{4}}. \tag{5} \]

The leading term in (5) is identical to that obtainable from the geometrical theory of diffraction (Ryan and Peters, 1969), and the small correction represented by the second term is similar to that found for a circular cylinder or sphere (see, for example, Senior, 1965).

The calculation of the creeping wave contribution to \( S \) is somewhat more involved. Resolution of the current component \( J_y \) gives

\[ J_y = \cos \phi J_\phi + \sin \phi J_\rho, \]

but since \( J_\phi \propto (a-r)^{-1/2} \) as \( r \to a \) whereas \( J_\rho \propto (a-r)^{1/2} \) by virtue of the edge condition (Van Bladel, 1964), we expect \( J_\phi \) to provide the dominant contribution to \( S^{\text{cw}} \) in spite of the location of the appropriate saddle point near to (but not at) \( \phi = 7/2 \). The analytical form of \( J_\phi \) in the immediate vicinity of the edge was discussed in Section 3, and bearing in mind that only the creeping wave traveling
in the negative \( \phi \) direction is required for an asymptotic estimate of the integral in (2), we have

\[
S^{cw} \omega \frac{ik}{\pi} \sqrt{\frac{2}{\pi}} \int_0^a \int f(a, a-r) e^{ik(a-r\cos \phi)} \frac{r \cos \phi}{\sqrt{a-r}} \, dr \, d\phi,
\]

where the \( \phi \) integration is confined to a neighborhood of \( \phi = \pi/2 \) and, from Section 3,

\[
f(a, 0) = 1.04(ka)^{1/3} \exp \left\{ ika + \frac{\pi}{3} + ika \left( 1 - \frac{1}{2} (ka)^{-2/3} e^{-i \pi/3} \right) \left( \frac{3\pi}{2} - \phi \right) \right\}.
\]

The smooth continuation of the known rim behavior, \( f(a, 0) \), somewhat away from the actual rim is necessary for a correct evaluation of the integral. Unfortunately, the surface field data does not indicate the form of this continuation, but a reasonable assumption consistent with the concept of a wave motion that is primarily circulatory is

\[
f(a, a-r)=f(a, 0)\exp \left\{ -ika-r \left( 1 - \frac{1}{4} (ka)^{-2/3} \right) \left( \frac{3\pi}{2} - \phi \right) + i\gamma(k(a-r)) \right\}.
\]

valid for \( \frac{a}{r} - 1 \ll 1 \). The second term in the exponent provides some degree of inward radial propagation, and \( \gamma \) is presumed to have a small negative imaginary part implying an increasing attenuation as \( r \) decreases.

With the above expression for \( f(a, a-r) \) inserted into Eq. (6), the \( \phi \) integral is found to have saddle points at

\[
\phi = \frac{\pi}{2} \pm (ka)^{-1/3} \sqrt{\frac{1}{2} \left( 1 - i \frac{\sqrt{3}}{r} \right)}
\]

but only for the first of these (with the upper sign in (9)) can the path be deformed to pass through it. The result appears as the difference of two Airy functions, and when the remaining \( r \) integral is evaluated by the saddle point method, we have
\[ S^{cw} \propto B(ka)^{2/3} e^{2i\gamma - 3i\pi/4} e^{i\pi ka(1 - \frac{1}{2}(ka)^{-2/3} e^{-i\pi/3})} \]  

with
\[ B = (1 - \gamma/\pi)^{-1/2} e^{i0.068\pi}. \]

It is worth noting that many different assumptions about the nature of the continuation of the creeping wave away from the rim of the disc all produce the same general formula for \( S^{cw} \) apart from slight variations in the multiplicative constant \( B \).

The asymptotic expression for the back scattered far field amplitude of a thin disc at glancing incidence is therefore
\[ S = \left[ \frac{1}{2} \sqrt{\frac{ka}{\pi}} (1 - \frac{1}{4ka}) B(ka)^{2/3} \exp \left\{ 2i\gamma + i\pi ka \left( 1 - \frac{1}{2}(ka)^{-2/3} e^{-i\pi/3} \right) \right\} \right] e^{-3i\pi/4}. \]

To see how this compares with experiment, we show in Fig. 5 values of \( |S| \) deduced from measured data for \( \sigma/\lambda^2 \) for a disc 0.0027 \( \lambda \) in thickness at 77 values of \( ka \) spanning the range 4.27 \( \leq ka \leq 13.57 \), and for a disc 0.0036\( \lambda \) in thickness at 32 values of \( ka \) in the range 0.49 \( \leq ka \leq 6.28 \). The latter set of data is taken from Hey et al (1956) and the agreement between the two sets confirms that the small difference in disc thickness has no effect. Superimposed on Fig. 5 is the curve computed from Eq. (12) with the trial value \( \gamma = 0.8\pi \), and it is observed that the theoretical formula reproduces all the significant features of the measured behavior. Notwithstanding the fact that the formula is an asymptotic one derived under the assumption \( ka \gg 1 \), the agreement between theory and experiment remains excellent even down to \( ka \approx 0.8 \), and in this respect Eq. (12) is analogous to the theoretical far field amplitude for a sphere (Senior, 1965) whose form it resembles in many ways.

5. Concluding Remarks

Although the above derivation of an analytic form for the back scattering cross section has been based on a numerical investigation of the fields induced in
the surface of the disc and is therefore subject to all the uncertainties of such a procedure, the results obtained are consistent with all of the available surface and far field data for thin discs. In addition, the scattering mechanism implied is directly analogous to that for a sphere or cylinder, and the simplicity of the formula for $S$ provides some hope of a rigorous derivation by asymptotic techniques applied to the exact modal solution (Meixner and Andrejewski, 1950; Flammer, 1953) for a disc at glancing incidence. We hope to explore this in the near future.

As mentioned in the Introduction, the back scattering cross sections were also measured for a number of other sets of discs whose thicknesses ranged up to $0.25\lambda$, and it may be of interest to note the effect that increasing the electrical thickness, $t/\lambda$, has on the scattering. In all cases the edges of the discs were essentially square.

An immediate observation is that for each value of $t/\lambda$, the back scattering cross section shows the same regular oscillation as a function of $ka$ as that provided by Eq. (12), suggesting that the nature of the primary contributors to $S$ is not affected by $t/\lambda$, and allowing us to estimate their magnitudes from an examination of the maxima and minima in the back scattering curves. It is found that the magnitude of the creeping wave contribution decreases somewhat with increasing $t/\lambda$, but the effect is very slight. Indeed, no change at all can be detected for $t/\lambda < 0.01$, and for larger $t/\lambda$ the fractional change in $|S_{cw}|$ appears independent of $ka$, amounting to only about 10 percent for $t/\lambda = 0.25$. Not too surprisingly, the effect on the optics component is more pronounced, and $|S_{op}|$ increases substantially with increasing $t/\lambda$. Again, however, the fractional change is independent of $ka$, at least to a first order, and the results could be accounted for by a factor $(1+\pi t/\lambda)$ multiplying the expression for $|S_{op}|$ appropriate to a thin disc. Increasing $t/\lambda$ also produces a slight shift in the locations of the maxima and minima towards the smaller values of $ka$. The shift is consistent with a decrease in $\arg (S_{op})$ by an amount which is an
increasing function of $t/\lambda$ and is as much as $\pi/2$ for $t/\lambda \approx 0.15$, but the data is insufficient to pinpoint its functional dependence.

In addition to these measurements of the edge-on backscattering from discs of uniform thickness $t$, a more limited series of experiments was carried out with discs shaped like 'flying saucers' in that their thicknesses at the center were substantially greater than at the rim. The results provided a striking demonstration of the dominant role played by the edge currents. Increasing the central thickness to as much as $\lambda/4$ whilst keeping the edge thickness small had no effect on the scattering, and the cross section remained the same as for a disc of uniform thickness equal to the saucer thickness at the rim. It would therefore seem reasonable that we could estimate the scattering from a disc of non-uniform thickness by using the above modifications to the thin disc expression with the parameter $t$ determined by the thickness of the edge.

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References


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Legends for Figures

1. The geometry.

2. Rim current amplitude for a disc of radius $1.432\lambda$: (—) bare and (---) with absorber on opposite side of rim.

3. Amplitude of current (O O) parallel to edge of a disc, radius $1.528\lambda$, as function of distance $x/\lambda$ from edge, compared with that (x x) on simulated half plane. The solid line shows the theoretical half plane current.

4. Absolute amplitudes of current components parallel to edge of disc $1.528\lambda$ in radius on traverses from front to rear (O O) and side to side (x x).

5. Comparison of $|S|$ predicted by Eq. (12) with $\gamma = 0.8\pi$ and measured data for discs $0.0036\lambda$ in thickness (O O) (Hey et al, 1958) and discs $0.0027\lambda$ in thickness (x x).