TRANSIENT DIFFRACTION AND SCATTERING

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The propagation of time dependent fields in dispersive regions is a problem which has attracted more attention of late and a number of asymptotic methods have been developed (for example, by Felsen and his co-workers) for analyzing the propagation process in dispersive media which are homogeneous, or inhomogeneous with either abrupt or gradual transitions, or even time-varying.

A sub-class of the above general problem is that in which we have some scatterer immersed in a homogeneous, non-dispersive medium—call it free space—and illuminated by some time-dependent field. We wish to find the scattered signal, and here the only dispersive element in our system is the scatterer itself. This is about the simplest transient problem that we can conceive of, but it is one that has become rather important in the last few years. One of the main reasons for this is the development and practical implementation of systems capable of radiating and receiving very broad band pulses. Such pulses range from modulated high frequency carriers with a time duration of a nsec or less, all the way to video pulses whose spectra extend down to very low frequencies indeed. It is obvious that the scattered pulse will contain a great deal of information about the target and the objective of many of these systems—whether they be laboratory ones in which we are merely illuminating a piece of material, or field systems in which we are looking at the ground to find objects buried in it, or systems in which we are illuminating a body in space—is to obtain information about the scatterer: its size, shape, material, etc. In other words, the transient is used as a diagnostic tool.
The problem of calculating the transient return and, to some extent, using it to extract information about the target, will be the main focus of my talk. I want to survey some of the work of the last few years, much of it so recent that only now is it beginning to filter into the published literature. It will turn out that in the digital area at least we are at somewhat of a turning point, and I will try to indicate some of the new directions that are being thought about.

Let's first set up the problem we have to deal with. Suppose we have some time dependent illumination which, for simplicity, I write as

$$\mathbf{F_i} = \hat{\mathbf{r}} \mathbf{F_i}(t - z/c) \quad \text{\(F_i\) real}$$

This falls on a body which I shall assume to be of finite dimensions and perfectly conducting, though neither assumption is essential for most of my talk. The field is scattered and at some point in space a distance \(r\) from the body, the time dependent signal that appears is

$$\mathbf{E} = \mathbf{F}(t - r/c) \quad \text{\(F\) real}$$

where I have taken just one component of the field. The task is to find the transient response \(\mathbf{F}(t)\), and since the mapping from the incident to the scattered field is a linear time-invariant one,

$$\mathbf{F}(t) = \mathbf{F_i}(t) \ast \mathbf{F_i}(t)$$

where \(\ast\) denotes the convolution operation and \(\mathbf{F_i}(t)\) is the impulse response, i.e. the response of the body to an impulse or delta function excitation. Alternatively, we can work in the frequency domain and write

$$\mathbf{F}(t) = \mathcal{F}^{-1} \{ \mathbf{F_i}(\omega)g(\omega) \}$$

where \(\mathcal{F}^{-1}\) is the inverse Fourier transform, \(\mathbf{F_i}(\omega)\) is the spectrum (FT) of the illuminating pulse and \(g(\omega)\) is the transfer function (or frequency response) of the system.
Before going further I should remark that in practice it is not the field in space which is the true observable, but rather the current that appears across the terminals of some receiving antenna. Thus, the practically significant mapping is that which takes place between the terminals of the transmitting and receiving antennas, leading to distortions of the space waveforms by virtue of the inherently dispersive properties of the antennas. I hope to say a few words later on about this intrinsic involvement of the antennas in any true transient situation, and about some of the antenna aspects per se, but for the moment let us just ignore the antennas and concentrate on the scattering by the body.

The transfer function \(g(\omega)\) is just the response of the target to a true time harmonic plane wave of circular frequency \(\omega\) and if the point of observation \(\mathbf{r}\) is in the far field of the scatterer, \(g(\omega)\) is (apart from the range factor \(1/\mathbf{r}\)) just the complex far field amplitude customarily measured or calculated in a cw scattering situation. If we knew \(g(\omega)\) for all real \(\omega\), we could obviously find the impulse response \(F(\mathbf{r}, t)\), and vice-versa, and each provides a complete characterization of the scattering behavior for the appropriate directions of incidence and scattering.

In addition to the impulse illumination leading to the impulse response of the target, there are two other idealized forms of transient illumination that are of special interest. They are [Slide 2]:

1) unit step function

\[
F^t(t) = U(t) \rightarrow F(t) = F^t_U(t)
\]

and because

\[
U(t) = \int_0^t \delta(\mathbf{r})d\mathbf{r} ,
\]

we have

\[
F^t_U(\mathbf{r}) = \int_0^t F^t(\mathbf{r})d\mathbf{r} .
\]
2) ramp function

\[ F^I(t) = R(t) = tU(t) \quad \rightarrow \quad F(t) = F_R(t) \]

and because

\[ R(t) = \int_0^t u(t)dt , \quad \text{we have} \]

\[ F_R(t) = \left( \int_0^t F_I(t)dt \right)^2 . \]

Thus, the ramp response will be a smoother function of time than the step response, which in turn is a smoother function than the impulse response. As regards the transform pairs, obviously

\[ F_I(t) \leftrightarrow g(\omega) \]

\[ F_U(t) \leftrightarrow \frac{g(\omega)}{-i\omega} \]

\[ F_R(t) \leftrightarrow \frac{g(\omega)}{(-i\omega)^2} \]

and since \( g(\omega) = O(\omega^2) \) for small \( \omega \) by virtue of the low frequency expansion for a finite body, even \( g(\omega)/(-i\omega)^2 \) is analytic in a neighborhood of \( \omega = 0 \). More importantly, it falls off more rapidly than \( g(\omega) \) does at high frequencies. This reduces the range of frequencies for which we must know \( g(\omega) \) in any practical computation of the transient behavior, and this is one of the main reasons for concentrating on the ramp (or step) response rather than the impulse response.

I should make one other point here. If we have to resort to a numerical determination of the inverse Fourier transform—and this is usually the case—what
we shall be using is the fast (or finite) Fourier transform (FFT) procedure in which the frequency response is sampled at discrete equally-spaced frequencies. Thus, the actual target illumination which is treated is that of infinite train of pulses with the repetition rate sufficiently low (and, hence, the frequency samples sufficiently close) to avoid any aliasing or overlapping of the transients from the individual pulses in the train. It follows that an incident step function is actually simulated by a train of square pulses, and a ramp function by a periodic sawtooth.

Unfortunately, there are very few bodies for which \( g(\omega) \) is known for all frequencies and which would enable us to find the characteristic responses we have talked about. In many cases we may know a high frequency approximation to \( g(\omega) \) and this would allow us to find the initial time behavior. We will certainly know some low frequency features of \( g(\omega) \), and from these we can deduce the large time behavior—and also arrive at certain constraints on the impulse response in the form of moment conditions (Kennaugh and Moffatt, 1966). Nevertheless, there is one body where we do know \( g(\omega) \) for all \( \omega \). This is our old faithful, the sphere, for which we have the Mie series expression for \( g(\omega) \). It is nowadays a straightforward task to compute the series at all frequencies required and the next slide [Slide 3] shows the impulse, step and ramp responses for backscattering by a metallic sphere. Here, \( \tau = 2a/c \) is the transit time for one sphere diameter and \( t \) is measured from the time of onset. \( F_1 \) contains the expected impulse (shown by an arrow) at \( t = 0 \), and we notice the peak corresponding to the creeping wave centered on the time \( t = (1 + \frac{\pi}{2})\tau \). The pulse is spread because of the dispersive character of this contributor. As expected, the step response is smoother, and the ramp response is smoother still. But none of them is a particularly complicated curve and there are those who claim that all three are simpler and more intelligible than the frequency domain curve.

There are, of course, a few other cases where we do have an analytical expression—usually an infinite series—for \( g(\omega) \). Thus, Hodge (1971) has used Andrewjewski's formulation for a disk to compute the impulse response for axial backscattering, and I might also mention the work of Schafer (1968) who has computed and analyzed the actual current distribution on a circular cylinder when illuminated by an impulse or step function.
Once we have the impulse response of a target, we can trivially obtain the transient response for any form of pulse illumination. The next slide shows the backscattering from a metallic sphere when illuminated by a cosine-modulated carrier, and we see here the main attribute of such illuminations, the ability to resolve individual portions of the target, as evidenced in this case by the time separation of the specular and creeping wave contributions. Even more dramatic is what happens when this same pulse falls on a dielectric sphere. A large number of individual contributors are now resolved and these can be identified, at least tentatively, from a consideration of their onset times. Such a result certainly gives considerable insight into the sources of the scattering phenomena, and we have only to look at the frequency response to see one advantage of the time domain.

This pinpoints one of the main uses of transient illumination: to discriminate between adjacent targets and to resolve individual portions of a single one. In many practical cases the bandwidth of the illumination, though large, may still be only a small fraction of a high central (or carrier) frequency. We may then be able to approximate the frequency response using high frequency asymptotics, leading to a transient response consisting of a number of reflected pulses, each attributable to an individual surface singularity and separated in time. Quite often the dispersive effect of each singularity over the bandwidth of the illuminating pulse can be neglected, so that each reflected pulse is just a scaled replica of the incident one. Measured data of this form can be used to confirm and extend our knowledge of high frequency techniques. Some examples of this are the experimental work of Hong et al (1968) on oblique backscattering from thin wires where the results provide excellent support for Ufimtsev's theory for cw scattering; the many studies that have been made on cones with various terminations, fully supporting Keller's theory; and the more recent measurements of Liang et al (1971) on sphere-disk-cone combinations in which the multiple interactions were resolved.
If the transient return is to represent a true discriminant for a particular target, the illuminating pulse should contain frequencies which span not only the high frequency region, but also the resonance and low frequency regions as well. The main challenge in transient work at the moment is in this case. For want of a better word, I will call it video pulse illumination. Since for most bodies of practical interest we do not have an analytical expression for the frequency response valid for all of the frequencies required, a lot of the work in recent years has been directed at computational procedures; that is, the direct digital determination of the transient response from a consideration of the appropriate integral equation. This is the central theme of my talk.

There are two basic ways we can go about the task of computing the transient: via the frequency response or directly in the time domain. The calculation of the frequency response by what is often called the moment method (Harrington, 1968) is now rather standard. For solid bodies it has been found convenient to use the H-field integral equation; for wires to use the E-field equation (but see Mittra, 1972), and for hybrid bodies (surfaces with wires sticking out), to use a combination of both (Albertsen et al, 1972). There are many variations, subtleties and levels of sophistication in the various methods that have been developed, but all the methods have certain features in common: the sampling of the unknown over the surface of the body, the conversion of the integral equation to a simultaneous set of linear algebraic equations for the sampled values, the inversion of the resulting matrix to yield these values, followed by an integration to give the field or transfer function at a point in space. For reasonably accurate results, it has been found necessary to sample at between 4 and 8 points per linear wavelength for the H-field equation, and at somewhere between 6 and 20 points per linear wavelength for wires. Miller et al (1971) have recently examined the affects of such sampling on accuracy. To construct, then, the transient response for some pulse illumination, we must repeat this process at a sequence of discrete frequencies spanning the appropriate frequency band and apply an FFT routine.
Now since it is the time domain behavior that is of concern, it would seem more logical to work in the time domain throughout: in other words, not develop the frequency response per se. The last 4 years have seen considerable progress in the development of time domain integral equation procedures. In 1968 Bennett and Weeks and Sayre and Harrington presented results based respectively on the H-field equation for a cylindrical body and the E-field equation for a thin wire. The attractiveness of the time domain equations is evident from the next slide [Slide 7] where I show the H-field equation for a [3]al body. T is the retarded time and you will notice that the current \( \mathbf{J} \) at a space-time point is made up of a direct incident field contribution and one from all other points but at earlier times—and these last are either zero or have previously been computed. We can therefore develop a solution by stepping in space and time and avoid the matrix inversion which is inherent in the frequency domain. This type of approach has been widely studied in the last few years, particularly by Bennett and his associates using the H-field version here, and by Miller and his associates using the E-field equation for wire scatterers. Hybrid techniques have also been developed. Three months ago, Bennett and Miller gave a combined survey paper and as part of this they examined the relative merits of the time versus frequency domain approaches as regards the computer times involved in finding transient responses. What they did was to break down each computation into its component parts and estimate the computer time taken. Each was expressed as a function of the number of sampling points, where the numerical constants are both computer and algorithm dependent. They also performed the same transient calculations by the two methods and compared the computer times for different body sizes. I will show just one of their comparisons [Slide 8]. This is for a Gaussian pulse incident on a particular [3]al body, and the computation is of the transient response in a number of bistatic directions. The abscissa is \( c/\lambda \), where \( c \) is the circumference of the body and \( \lambda \) is the smallest wavelength of interest in getting the numerical solution. The ordinate is the time in secs. on a CDC 6600 computer, which is one of the fastest available. There are two main conclusions to be drawn:
(i) the direct time domain procedure does not show the clear advantage in
economy that was hoped for it. Indeed, for this calculation it took almost
3 times as long as the freq. domain one. In other cases, e.g. wire
calculations, the time domain method is slightly better, but there is
still not much difference between the computer times required;
(ii) both methods take a fantastic amount of computer time: of order 3 hours
for a body \( \sim 8\lambda \) in circumference.

At £1000 per hour (commercial rate in the U.S.) for this computer, you have to
want the transient response pretty desperately to follow this route, and at the end
of it you still have only the results for a particular body with a particular transient
illumination.

With the completion of these numerical programs, a little of the theoretical
excitement has disappeared and there is also some disappointment at the enormous
costs involved. Nevertheless, at this moment such programs do provide our only
means of obtaining numerically rigorous transient information for nontrivial
shapes, and since they are in frequent use, it is important that we try to refine
these programs and, hopefully, make them more economical. As regards the
time domain equations, some program optimisation is certainly feasible and
economies could be made by taking into account any symmetries that may exist in
the problem (Bennett and Auckenthaler, 1971). There is also the possibility of
incorporating analytical approximations within the context of the digital program.
As regards the frequency domain route, such amalgamation of integral equation
methods with analytical approximations, for example, physical optics or Ufimtsev's
fringe theory, is being actively pursued, particularly by people at the Northrop
Corporation in California, but the results obtained so far are rather marginal.
There is, however, one other approach in the frequency domain that may offer con-
siderable economies when \( c/\lambda \) is large. This is based on the so-called k-space
formulation developed by Bojarski (1971) and permits a digital solution by interactive
techniques (rather than matrix inversion) in the space frequency domain. The
method certainly looks promising and could make it quite advantageous to seek
the transient response via the frequency response.
In spite of all the transient responses that have been computed, they don't seem to have added much to our understanding of transient phenomena. Perhaps because of this, there is a need to explore in more detail the use of analytical approximations and also to develop numerical methods that are more physically based. One such numerical method which is exciting considerable interest at the moment is the "singularity expansion" method. A brief description was recently given by Marin and Latham (1972) and a more complete discussion by Baum (1972). The case of a metallic sphere provides a simple illustration of the method. If this is illuminated by a plane time harmonic wave, the solution is given by the Mie series. In the complex frequency plane each term in the series has a finite number of first order poles at the zeros of the spherical Hankel functions, and in the time domain each pole contributes a damped sinusoid to the transient response. For any given pulse illumination, we have only to compute the excitation strength of each of the sinusoids to arrive at a synthesis of the response. The main attractiveness of this method—apart from its simplicity of concept—is that in many cases only a few of these sinusoids seem to be required to get accurate results. Thus for a sphere excited by a step function, only about 8 of these poles are needed to synthesize the response to a high degree of accuracy. This is the vital feature.

The pole locations are just the natural (complex) frequencies of oscillation and since the only body additional to the sphere for which they are known is the prolate spheroid (Page and Adams, 1938; Page, 1944), the method would be no more than an interesting novelty were it not for the fact that it is now feasible to locate the poles numerically. In a paper soon to be published, Tesche (1972) has computed these pole locations for a thin wire and, hence, deduced the step response. He starts with the Pocklington integral equation for plane wave illumination of the wire and uses basically the moment method to reduce it to a system of linear algebraic equations in the sampled values of the current. He then uses an iterative scheme to compute the zeros of the determinant of the
of the system matrix, corresponding to the poles of the inverse operator. This proves to be relatively straightforward, though it should be realized that each iteration is equivalent to the re-solution of the integral equation for a new complex frequency of excitation. For a wire, as for a sphere, these poles lie on certain trajectories or layers. This is illustrated in the next slide [Slide 9]. Since the poles form complex conjugate pairs, I show only one quadrant of the complex ω plane. These particular results are for a wire of diameter:length ratio d/L = 0.01, but Tesche has investigated the displacement of these poles as d/L increases to 0.1. Having found the poles, one can then compute the residue for a given transient illumination on the assumption that the poles are first order ones; and for a step excitation, Tesche has computed the transient currents at various stations along the wire. I show in the next slide [Slide 10] some results at a point which is L/4 from the nearer end for a pulse incident at 30° to axial. The top curve is that obtained by the straightforward moment method followed by Fourier inversion. The next 3 are the results from this singularity expansion method and using 2, 6 and 10 poles respectively. All of the poles used are from the first layer—those in subsequent layers have such large imaginary parts that their contributions are insignificant. The agreement is rather remarkable, even at the shortest times and for as few as 6 poles. Moreover, the time involved in the computer solution including the pole determination is less than 1/2 that required by the direct method; and once the poles have been located, they are applicable for any direction and type of illumination.

Attractive as this method seems—and it is now being applied to other geometries—there are many fundamental questions yet to be answered. It has been shown that for a perfectly conducting body of finite extent, the only singularities of the inverse operator are poles. But what if the body is not perfectly conducting? Can branch points occur if the body is placed in a parallel plate region? There is some evidence that they do. Under what conditions (if any) do other than simple poles occur? How would higher order poles be treated numerically? And so on.
The accuracy required in a transient calculation depends very much on the purpose for which the results are to be used. In some cases we may require a precise knowledge of the ripple content or of the crossing times, and we may then be forced to use a numerical method of the type that I have described. But in other cases we may be only interested in, say, the initial rise times, or the decay rates at large times and a high or low frequency approximation to the transfer function could then suffice. And in still other cases, particularly those where we are seeking to interpret the transient response of a target for diagnostic purposes, we could be satisfied with even a crude analytical approximation to the transient return provided it was explicitly tied to the geometry of the target. With the increasing use of video pulses in systems applications, it is important to develop a better feel and understanding for the nature of the responses, as well as some analytical method to approximate them. Although I am not aware of any breakthroughs in the area in the last few years, a good example of what I am talking about here is some recent work at the Ohio State University. It is based on physical optics [Slide 11]. As many of you are aware, the physical optics approximation to the transfer function in backscattering can be written as

\[
g(\omega) = \frac{k_0}{2\pi cr} \int_{z=0}^{\text{sh. bdry}} e^{-\frac{2i\omega z}{c}} \frac{d}{dz} A(z) dz
\]

\[
= -\frac{1}{4\pi r} \int_{z=0}^{\text{sh. bdry}} e^{-\frac{2i\omega z}{c}} \frac{d^2}{dz^2} A(z) dz
\]

where the \(z\) axis is the direction of incidence and \(A(z)\) is the projection of the surface area on a plane perpendicular to this. It then follows immediately that for a ramp excitation, the p.o. approximation to the response is simply

\[
F_R(t) = -\frac{1}{2\pi cr} A(z)
\]

where \(z = ct/2\), i.e. the cutting plane moves at half the wave velocity. Admittedly
this is only a crude approximation, but from measured data for the ramp response of targets taken from three mutually perpendicular directions, Young and his co-workers (1971) at Ohio State have used this simple formula to reconstruct the target geometry with a surprising degree of accuracy.

The mention of measurements of transient responses leads naturally to the question of the antennas used to radiate and receive these transient fields. Because of the lack of standardised and reproducible transient antennas it is not at the moment possible to compare measured data taken at two different places: indeed, there is a complete lack of antenna standards per se in the transient area. The whole of present antenna terminology, e.g. gain, directivity, pattern function, aperture area, etc., is rooted in the concept of interference and is valid only at a single frequency or in a narrow band of frequencies. Certainly these concepts are inapplicable when we are talking about video pulses, and if we are to establish the framework for the technology which is now developing, it would seem that some international organisation such as URSI should concern itself with the definition of standards appropriate for transient antennas. I don't want to imply that no work at all has yet been done in this area; for example, Polk (1960) has considered aperture antennas, and there is also the work of Feng and Cheng (1964) and Foster and Tai (1971), but much still remains to be done before we can define standards which are meaningful.

As regards the design of antennas for use in transient work, what we would ideally like is some geometry which would radiate undistorted the transient that was applied across the terminals of the antenna, in other words, to have an entirely flat transfer function. We should also admit that a step function applied to the terminals is unrealistic and consider instead some pulse like a double exponential that we might hope to generate. There is some activity at the moment in the design of antennas for transient work: conceptual studies and also theoretical and computational work on the loading of biconical and wire antennas to minimize the ringing produced by the reflection of the current pulses at the ends (e.g. Sengupta and Liu, 1972), but this is still a wide-open field.
References for Transient Diffraction and Scattering


C. E. Baum (1972), "Electromagnetic transient interaction with objects with emphasis on finite size objects, and some aspects of transient pulse production," US URSI Spring Meeting, Washington, D. C.


\[ E^i = \hat{x} F^i(t - z/c) \]

\[ E^S = F(t - r/c) \]

To find

\[ F(t) = F^i(t) \ast F^i(t) \]

\[ = \mathcal{F}^{-1} \left\{ F^i(\omega)g(\omega) \right\} \]
Unit Step:

\[ F^I(t) = U(t) \]

\[ \implies F(t) = F_U(t) = \int_0^t F^I(t) \, dt \]

Ramp:

\[ F^I(t) = tU(t) \equiv R(t) \]

\[ \implies F(t) = F_R(t) = \left( \int_0^t t^2 \right) F^I(t) \, dt \]

Transform Pairs:

\[ F^I(t) \leftrightarrow g(\omega) \]

\[ F_U(t) \leftrightarrow \frac{g(\omega)}{-i\omega} \]

\[ F_R(t) \leftrightarrow \frac{g(\omega)}{(-i\omega)^2} \]
Characteristic backscattered waveforms of a conducting sphere, (Young, et al, 1969)
Short pulse response of conducting sphere ($\tau = 2a$, $T = 25a$, $a/\lambda_c = 1$), (Reinhardt, 1966)
Short pulse response of dielectric sphere, \( n = 2.5 \), \( \omega = 0.1 \), \( T = 0.5 \), \( \lambda = 0.01 \) (after Stein, 1968).
Backscatter cross section of lossy dielectric sphere with
\( n = 2.5 \quad 10.01 \), (Rheinstein, 1968)
Time domain integral equation:

\[ \mathbf{J}(\mathbf{r}, t) = 2\hat{n} \times \mathbf{H}^i(\mathbf{r}, t) + \frac{1}{2\pi} \int_{S} \hat{n} \times \left\{ \left[ \frac{1}{R} \mathbf{J}(\mathbf{r}', \tau) + \frac{1}{Rc} \frac{\partial}{\partial \tau} \mathbf{J}(\mathbf{r}', \tau) \right] \bigg|_{\tau = t - R/c} \right\} \mathbf{dS}' \]

(Bennett and Weeks, 1968), where

- \( S \) is the surface of the scatterer
- \( \mathbf{r} \) is the field point (on the surface)
- \( \mathbf{r}' \) is the integration point

\( R = |\mathbf{r} - \mathbf{r}'| \), \( \hat{R} = (\mathbf{r} - \mathbf{r}')/R \).
THIN WIRE, $\theta^i = 30^\circ$ (Tesche, 1972)
Physical optics, backscattering

\[ g(\omega) = \frac{i\omega}{2\pi c r} \int_{z=0}^{s.b.} e^{\frac{2i\omega z}{c}} \frac{d}{dz} A(z) dz \]

\[ = -\frac{1}{4\pi r} \int_{z=0}^{s.b.} e^{\frac{2i\omega z}{c}} \frac{d^2}{dz^2} A(z) dz \]

\[ \Rightarrow F_R(t) = -\frac{1}{2\pi c r} A(z) \quad (z = ct/2) \]