RAYLEIGH SCATTERING BY DIELECTRIC BODIES*

Purpose: To compute the tensor elements specifying the low frequency (Rayleigh) scattering of a plane electromagnetic wave by a homogeneous dielectric body with axial symmetry.

Language: Fortran IV

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Description: When a plane electromagnetic wave is incident on a homogeneous dielectric body immersed in free space, the leading (Rayleigh) term in the low frequency expansion of the far zone scattered field can be expressed in terms of the electric and magnetic polarizability tensors \( \bar{\epsilon} \) and \( \bar{\mu} \). These are functions of the relative permittivity \( \varepsilon_r \) and relative permeability \( \mu_r \), respectively, and of the geometry of the body, but are independent of the excitation. If the body is symmetric about the \( x_3 \) axis of a Cartesian coordinate system \( x_1, x_2, x_3 \), the tensors diagonalize and \( P_{12} = P_{11}, M_{22} = M_{11} \). As shown in [1], \( \bar{\epsilon} \) and \( \bar{\mu} \) are particular cases of a general polarizability tensor \( \bar{\chi}(\tau) \) with \( \bar{\epsilon} = \bar{\chi}(\varepsilon_r) \), \( \bar{\mu} = -\bar{\chi}(\mu_r) \). The tensor elements are weighted integrals of potential functions which are obtained by solution of the corresponding integral equation by the moment method. The program computes \( X_{11}(\tau) \) and \( X_{33}(\tau) \), corresponding to the transverse and axial components of the dipole moments respectively, for any complex \( \tau \).

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and is a modified version of an analogous program [2.3] for perfect conductivity \((\varepsilon_r = \infty, \mu_r = 0)\).

Although it is difficult to state the highest frequency at which the Rayleigh scattered field is applicable, a reasonable criterion is that the free space and interior wavelengths should both exceed the maximum linear dimensions of the body by a factor of 10 or more.

The body is described by its profile in the \(z=\chi, \rho\) plane of a cylindrical coordinate system. The profile is assumed to be composed of up to 15 straight line or circular arc segments. The former are described by their end points in the \(z, \rho\) plane, while the latter are described by their end points, the angles subtended at their centers of curvature and their concavities. Each segment is uniformly subdivided into cells and the total number \(N\) of cells across the entire profile must not exceed 102. For many shapes however, a much smaller value of \(N\) is adequate. Concave (re-entrant) shapes can be treated, as well as multiply connected ones, e.g., a torus. Bodies having two or more disjoint portions can also be considered provided their material parameters are the same and they have a common axis of symmetry.

An input data set consists of one control card and a number of segment specification cards, one for each segment of the profile. The control card lists:

- **SEGMENTS:** number \((\leq 15)\) of segments
- **EXCLUSION:** a decimal fraction involved in the computation of a self cell; defaults to 0.001
- **TAU:** complex material parameter
- **PRINT:** prints \((PRINT = 1)\) or suppresses \((PRINT = 0)\) the two potentials obtained by solution of the integral equation; defaults to 0
A segment specification card lists:

**CELLS:** number of sampling cells on the segment

**TYPE KEY:** type of segment: 1 circular arc, concave up; 2 circular arc, concave down; 3 straight line

**VOLUME SENSE:** + on blank: additive volume;
-: subtractive volume

**Z-COORD.**

**ENDPOINTS:** self-explanatory

**RHO-COORD.**

**ENDPOINTS:** self-explanatory

**THETA(\text{DEG.}):** angle subtended by a circular arc at its center of curvature

As an illustration, consider a sphere of radius 0.5 and $\tau = 2+i$ with $N = 20$. The corresponding profile is a semicircle in the $z, \rho$ plane of radius 0.5 centered (say) at $(0.5,0)$. The print out is as follows:

```
>***** PROGRAM DIELCOM *****

> SEGMENTS     = 1
> EXCLUSION    = 0.0010
> TAU          = 0.2000000E+01 +0.1000000E+01

>SEGMENT # 1:
> CELLS        = 20
> TYPE KEY     = 1
> VOLUME SENSE = +
> 1ST END POINT = ( 0.0 , 0.0 )
> 2ND END POINT = ( 1.000000 , 0.0 )
> THETA (DEG)  = 180.00000

>COMPUTED RESULTS;
> VOLUME       = 0.52360
> X11/V        = 0.88155 +J 0.52941
> X33/V        = 0.88297 +J 0.52928
```
The known (exact) results are $X_{11}/V = X_{33}/V = 0.88235... + 10.52941...$. In the sample run the imaginary part of $X_{11}/V$ was computed with least accuracy (0.2 percent). The c.p.u. time for the Amdahl 470/V8 computer at the University of Michigan was 0.14 s. Increasing $N$ to 30 reduced the errors by about a factor 2, but at the expense of a factor 2 increase in the c.p.u. time.

References


COMPUTER PROGRAM: DIELCOM

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1. Introduction

The program solves two integral equations and then computes the two independent elements $X_{11}$ and $X_{33}$ of the general polarizability tensor $\mathbf{X}$ for a homogeneous dielectric body whose material parameter $\tau$, representing either the relative permittivity or relative permeability, may be complex. The body is assumed symmetric about the $x_3$ (=z) axis of a Cartesian coordinate system $x_1, x_2, x_3$. The theoretical background and the definition of the tensor $\mathbf{X}$ are given in [1], and the program itself is a modified version of one previously developed [2] for a perfectly conducting body. The numerical procedures will be discussed and the various subroutines presented along with a flow chart of the program.

2. Formulation

2.1 Body Description

It is convenient to introduce a cylindrical polar coordinate system $\rho, \phi, z$ whose $z$ axis is the axis of symmetry of the body. The profile of the body in the $\rho, z$ plane is defined as $\rho = \rho(z)$, and is constructed of segments which may be linear arcs or circular arcs of either concavity.
Distance along the profile is measured by the variable $s$, and the
tangent at any point forms the angle $\alpha$ with the $z$ axis: $-\pi < \alpha \leq \pi$.

The volume of rotation of a linear arc (Type 3) segment whose
endpoints in the $z, \rho$ plane are $(z_1, \rho_1), (z_2, \rho_2)$ is

$$
\delta V = \int_{z_1}^{z_2} \pi \rho^2(z) \, dz
$$

$$
= \frac{\pi}{2} (z_2 - z_1) (\rho^2_2 + \rho^2_1 + \rho_2 \rho_1) \ . \quad (1)
$$

For a circular arc which subtends an angle $\theta$ at its center $(z_o, \rho_o)$ and is
of radius $a$, let

$$
\xi = \begin{cases} 
-1 & \text{if segment is concave down (Type 1)} \\
1 & \text{if segment is concave up (Type 2)} 
\end{cases}
$$

Then

$$
\delta V = (z_2 - z_1) \left\{ \rho_o^2 + a^2 - \frac{1}{3} (u^2_1 + u u_2 + u^2_2) \right\}
$$

$$
+ \rho_o \left\{ u_2 (\rho_2 - \rho_o) - u_1 (\rho_1 - \rho_o) - \xi \delta a^2 \theta \right\} \ . \quad (2)
$$

where

$$
d = \frac{z_2 - z_1}{|z_2 - z_1|}
$$

and

$$
u_2 = z - z_2, \quad u_1 = z - z_1, \quad u_0 = z - z_o .
$$
2.2 Integral Equations

Given the profile \( \rho = \rho(z) \) and the volume \( V \), quantities \( m \) and \( m_1 \) are defined as

\[
m = m(z, \rho, z', \rho') = \frac{4\rho \rho'}{(\rho - \rho')^2 + (z - z')^2}
\]

and

\[
m_1 = 1 - m,
\]

where \((z, \rho)\) and \((z', \rho')\) are distinct points on the profile. The unprimed variables denote the observation point and the primed variables the point of integration. Note that \( 0 \leq m, m_1 \leq 1 \).

If \( K(m) \) and \( E(m) \) are the complete elliptic integrals of the first and second kinds respectively, let

\[
\Omega_0 = \frac{1}{4} \left( \frac{m}{\rho \rho'} \right)^{3/2} \left\{ K(m) + 2mK'(m) \right\}
\]

\[
\Omega_1 = \left( \frac{m}{\rho \rho'} \right)^{3/2} \left\{ \left(1 - \frac{m}{2}\right) K'(m) - \frac{1}{4} K(m) \right\}
\]

\[
\Omega_2 = \frac{1}{m^2} \left( \frac{m}{\rho \rho'} \right)^{3/2} \left\{ 2m \left(1 - \frac{m}{2}\right)^2 K'(m) - \left(1 - \frac{m^2}{4}\right) K(m) + E(m) \right\}
\]

where \( K'(m) = (d/dm)K(m) \). For a body whose complex material parameter is \( \tau \), the integral equations to be solved are (see \([1,3]\))
\begin{align}
\frac{1 + \frac{\tau}{\tau}}{1 - \frac{\tau}{\tau}} W_1(s) &= 2\rho + \frac{1}{\pi} \int_{W_1(s')} \left\{ \rho \cos \alpha' \Omega_2 \right. \\
&\quad \left. + \left[ (z' - z)\sin \alpha' - \rho' \cos \alpha' \right] \Omega_0 \right\} \rho' \, ds' \\
\text{and} \\
\frac{1 + \frac{\tau}{\tau}}{1 - \frac{\tau}{\tau}} W_3(s) &= 2z + \frac{1}{\pi} \int_{W_3(s')} \left\{ \rho \cos \alpha' \Omega_1 \right. \\
&\quad \left. + \left[ (z' - z)\sin \alpha' - \rho' \cos \alpha' \right] \Omega_0 \right\} \rho' \, ds' ,
\end{align}

where the integration is over the entire profile of the body and the bar across the integral sign implies a Cauchy principal value. In terms of the solutions,

\begin{align}
\frac{X_{11}}{V} &= -\frac{\pi}{V} \int \rho W_1(s') \cos \alpha' \, ds' \\
\frac{X_{33}}{V} &= -\frac{2\pi}{V} \int \rho W_3(s') \sin \alpha' \, ds' ,
\end{align}

where \( V \) is the volume of the body.

3. Numerical Solution of the Integral Equations

The numerical procedures involved in computing \( \frac{X_{33}}{V} \) are similar to those required for \( \frac{X_{11}}{V} \), and it is therefore sufficient to give the details only for the latter.
3.1 $X_{11}/V$ Computation

The primary task is the solution of the integral equation (8) for the function $W_1(s)$ and this entails the determination of a sequence of values $W_1^{(i)}$, $i = 1, 2, \ldots, N$, approximating $W_1(s)$ at the sampling points $s = s_i$ on the profile $\rho = \rho(z)$. For this purpose the profile is divided into $N$ cells $C_j$ of arc length $\Delta s_j$ and midpoints $s_i$ corresponding to the coordinates $(z_i, \rho_i)$. Within each cell we also define the points $s_{i-}$ and $s_{i+}$ where

$$
\begin{align*}
    s_{i-} &= s_i - \alpha_0 \Delta s_i \\
    s_{i+} &= s_i + \alpha_0 \Delta s_i
\end{align*}
$$

(12)

with the restriction $0 \leq \alpha_0 \leq 1/2$.

By assuming that $W_1(s)$ has the constant value $W_1^{(j)}$ over the $j$th cell, the integral on the right-hand side of (8) can be evaluated as a linear combination of the $W_1^{(j)}$ whose coefficients depend on the position $(z, \rho)$ of the field point, leading to a linear system of $N$ equations in $N$ unknowns, viz.

$$
- \sum_{j=1}^{N} W_1^{(j)} \int_{C_j} \rho_i \cos \alpha \hat{n}_2 + [(z' - z_i) \sin \alpha' - \rho' \cos \alpha'] \hat{n}_1 \rho' d\sigma' = 2\pi \rho_i - \pi \frac{1 + \tau}{1 - \tau} W_1^{(i)}
$$

(13)

for $i = 1, 2, \ldots, N$. Hence, the system to be solved is
where \( w_1 \) is a column vector with elements

\[
w_{1j} = W_1(j), \quad j = 1,2,\ldots,N,
\]

\( A \) is a square matrix with elements

\[
a_{ij} = - \int_{C_j} f(s') \rho' \, ds', \quad (i \neq j)
\]

and

\[
a_{ij} = - \int_{C_j} f(s') \rho' \, ds' + \pi \frac{1 + \tau}{1 - \tau} \quad (i = j)
\]

where

\[
f(s') = \rho_i \cos \alpha' \Omega_2 + [(z' - z_1) \sin \alpha' - \rho' \cos \alpha'] \Omega_1
\]

and \( b \) is a row vector with elements

\[
b_j = 2\pi \rho_j, \quad j = 1,2,\ldots,N.
\]

The integrations were performed using three point Gaussian quadrature for which the parameter \( \alpha_0 \) in (12) is \( \alpha_0 = 1/2 \sqrt{3/5} \).

For \( i \neq j \), we then have

\[
a_{ij} = - \left[ \frac{5}{18} \left\{ \rho_{j-} f(i,j-) + \rho_{j+} f(i,j+) \right\} + \frac{4}{9} f(i,j) \right] \Delta s_j
\]

for \( i,j = 1,2,\ldots,N \), in which
\[ f(i,j) = \rho_i \cos \alpha_j \Omega_2(i,j) + \left\{ (z_j - z_i) \sin \alpha_j - \rho_j \cos \alpha_j \right\} \Omega_1(i,j) \]

(18)

where the subscripts \( j^- \) and \( j^+ \) correspond to the points \( s_{j^-} \) and \( s_{j^+} \) of (12). The diagonal elements \( a_{i1} \) of the matrix \( A \) are

\[ a_{i1} = \pi \frac{1 + \tau}{1 - \tau} - \int_{s_i - (1/2)\Delta s_i}^{s_i + (1/2)\Delta s_i} \rho \, f(s') \, ds' \]

and require the numerical determination of a Cauchy principal value. As an approximation to this principal value, we remove from the cell \( C_i \) a slice defined by the interval \( (s_i - (1/2)\beta \Delta s_i, s_i + (1/2)\beta \Delta s_i) \) where \( \beta, 0 < \beta \leq 1 \), is the fractional exclusion; \( \beta = 0 \) implies no exclusion, i.e., that the principal value is not taken.

We now have

\[ a_{i1} = \pi \frac{1 + \tau}{1 - \tau} - \int_{s_i - (1/2)\Delta s_i}^{s_i - (\beta/2)\Delta s_i} \rho \, f(s') \, ds' - \int_{s_i + (1/2)\Delta s_i}^{s_i + (\beta/2)\Delta s_i} \rho \, f(s') \, ds' \]

(19)

and these integrals are also computed using three-point Gaussian quadrature. Defining
\[ s_{12} = s_1 - \frac{1}{4} (1 + \beta) \Delta s_i \]
\[ s_{11} = s_{12} - \frac{1}{2} \alpha \Delta s_i \]
\[ s_{13} = s_{12} + \frac{1}{2} \alpha \Delta s_i \]
\[ s_{15} = s_1 + \frac{1}{4} (1 + \beta) \Delta s_i \]
\[ s_{14} = s_{15} - \frac{1}{2} \alpha \Delta s_i \]
\[ s_{16} = s_{15} + \frac{1}{2} \alpha \Delta s_i \]

we obtain

\[ a_{ii} = \pi \left( \frac{1 + \tau}{1 - \tau} - \frac{1}{2} (1 - \beta) \right) \left[ \frac{5}{18} \left\{ \rho_{11} f(i, i_1) + \rho_{13} f(i, i_3) + \rho_{14} f(i, i_4) + \rho_{16} f(i, i_6) \right\} + \frac{4}{9} \left\{ \rho_{12} f(i, i_2) + \rho_{15} f(i, i_5) \right\} \right] \Delta s_i . \]  

Equations (17) through (21) completely describe a system of N linear equations in N unknowns \( \mathbf{W}_1^{(i)}, i = 1, 2, \ldots, N \). Having determined the sampled values \( \mathbf{W}_1^{(i)} = \mathbf{W}_1(s_i) \), \( \chi_{11} / V \) is computed from (10) by integration over each segment of the profile using a second order integration procedure (subroutine INTEG).
3.2 $X_{33}/V$ Computation

The procedure for solving (8) and then computing $X_{33}/V$ from (11) is about identical to that described above. The matrix equation for the sampled values $W_{3}(s) = W_{3}^{(i)}$ is $AW_{3} = b$ where

$$
W_{3j} = W_{3}(j), \quad j = 1, 2, \ldots, N, \quad b_{j} = 2\pi z_{j},
$$

and for $i, j = 1, 2, \ldots, N$ with $i \neq j$,

$$
a_{ij} = -\left[\frac{5}{18}\left(\rho_{j-}g(i,j-) + \rho_{j+}g(i,j+)ight) + \frac{4}{9}g(i,j)\right] \Delta s_{i},
$$

with

$$
g(i,j) = \rho_{1}\cos \alpha_{j} \Omega_{1}(i,j) + \left\{(z_{j-} - z_{j})\sin \alpha_{j} - \rho_{j}\cos \alpha_{j}\right\} \Omega_{0}(i,j).
$$

The same approximation as before is used to compute the diagonal elements $a_{ii}$ of the matrix $A$, and thus

$$
a_{ii} = \pi \frac{1 + \tau}{1 - \tau} - \frac{1}{2}(1 - \beta)\left[\frac{5}{18}\left\{\rho_{1}g(i,i_{1}) + \rho_{3}g(i,i_{3}) + \rho_{4}g(i,i_{4})

+ \rho_{6}g(i,i_{6})\right\} + \frac{4}{9}\left\{\rho_{2}g(i,i_{2}) + \rho_{5}g(i,i_{5})\right\}\right] \Delta s_{i},
$$

for $i = 1, 2, \ldots, N$. 
Equations (22) through (25) completely describe a system of $N$ linear equations in $N$ unknowns $w^{(j)}$, $j = 1, 2, \ldots, N$. Once the sampled values $w^{(j)} = w_3(s_j)$ are found, $X_{33}/V$ is computed from (11) by integration over each segment of the profile using a second order integration routine as before.

4. Computer Program

The program computes $X_{11}/V$ and $X_{33}/V$, and consists of a main program and six subroutines.

4.1 Data Set

A data set is made up of a control card and a number of segment specification cards, one for each segment (or sub-segment) of the profile.

**Control Card**

<table>
<thead>
<tr>
<th>Columns</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>Integer number of segments in the body (must be &lt;15).</td>
</tr>
<tr>
<td>6-17</td>
<td>The fraction exclusion $\beta$. If these columns are blank, $\beta$ defaults to 0.001.</td>
</tr>
<tr>
<td>18-41</td>
<td>Complex material parameter $\tau$.</td>
</tr>
<tr>
<td>42-46</td>
<td>Integer print key; 1: print $W_1$ and $W_3$ potential functions 0: do not print $W_1$ and $W_3$ (default is 0).</td>
</tr>
</tbody>
</table>
Segment Specification Card

<table>
<thead>
<tr>
<th>Columns</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>Integer (right justified): the number of sampling points or cells on the segment.</td>
</tr>
</tbody>
</table>
| 6-10    | Segment type key: (right justified)  
1: circular arc, concave down  
2: circular arc, concave up  
3: linear. |
| 11-13   | Volume sense: (single character, right justified)  
+ or blank: additive volume  
- : subtractive volume. |
| 14-25, 26-37 | Two real numbers: respectively, the end coordinates \( z_1 \) and \( z_2 \) of the segment. |
| 38-49, 50-61 | Two real numbers: respectively, the end coordinates \( p_1 \) and \( p_2 \) of the segment. |
| 62-73   | A real number: for circular arcs, the included angle in degrees. |

There are the following restrictions:

(i) the total number of segments must not exceed 15 and

(ii) the total number of cells over all segments must not exceed 102.

The profile is specified in the direction of increasing profile-length, beginning either at its left-hand intersection with the z-axis and ending at its right-hand intersection with the z-axis, or beginning and ending at its own left-most point. Re-entrant segments are permitted, allowing
$z_1 > z_2$, and the "body" may consist of two or more disjoint parts. For maximum accuracy in the $x_{33}$ computation the body should be approximately centered about the origin.
4.3 Subroutine DATA (IN, MX, MY, ZEP, RHOEP, THETA, B, VOL)

This subroutine is called once for each segment of the profile. From the input specification for the segment, DATA computes the (z,ρ) coordinates of the necessary sampling points on the profile, the quantities cos α and sin α at these points and the incremental volume of the segment.

Comments:

Stored in COMMON are the arrays RHO(102,9), Z(102,9), ARC(102), C(102,9) and S(102,9) which contain the numbers computed by DATA.

For the Ith cell, the subscripts (I,J) correspond to the points s_{ij} of (41) when 1 ≤ J ≤ 6. For J = 7,8,9, the subscripts (I,J) refer to the points s_{i-}, s_{i}, and s_{i+} respectively of (12).

4.4 Subroutine INTEG (V, NSEG, NUMPTS, SUM)

INTEG numerically integrates quadratic interpolating polynomials approximating the complex valued data on each segment of the profile. When the profile is composed of several segments, no interpolation is performed across segment boundaries. Hence, the integration is accurate even for disconnected segments, e.g., the circular arcs of two spheres.

Comments:

Stored in COMMON are the lengths ARC (I), I = 1,...,N required to compute the integral.
4.5 Subroutines CDLUD \((N, ADIM, A, IV)\) and CDBS \((N, TDIM, T, IV, B, X)\)

Used together, CDLUD and CDBS solve the linear system \(AX = B\). CDLUD performs an L-U decomposition of the \(N \times N\) matrix \(A\) and CDBS performs back-substitution. These routines are adapted from Forsythe and Moler [3].

4.6 Subroutine ELLI \((M1, K, E, KPR)\)

This computes the elliptic integrals \(K(m)\) and \(E(m)\) and the derivative \(K'(m)\) from their power series approximations [4].

4.7 Subroutine SETUP \((I, J, L, AXII1, AXI33)\)

This is essential in computing the linear systems. Specifically, SETUP, after calling ELLI, computes the quantities \(\Omega_0\) (Eq. 5), \(\Omega_1\) (Eq. 6), and \(\Omega_2\) (Eq. 7). These are then used to compute \(f(i,j)\) (AXII1) of Eq. (18) and \(g(i,j)\) (AXI33) of Eq. (24).

A listing of the entire program follows:
DIELCOM

The program DIELCOM computes the elements of the polarizability tensor for a rotationally symmetric homogeneous dielectric body. See "Radio Science", vol. 11 No. 5, pages 477-482, May 1976; Low-frequency scattering by a dielectric body, by T.B.A. Senior

For a discussion of a similar program (MINK) see The University of Michigan's Radiation Laboratory report, "The Numerical Solution of Low Frequency Scattering Problems", by Thomas B. A. Senior and David J. Ahlgren; report no. 013630-9-T.

LOGICAL DEVICE ASSIGNMENT

5: FORMATTED INPUT PARAMETERS (SEE INPUT FORMAT)
6: FORMATTED OUTPUT OF TENSOR ELEMENTS

INPUT FORMAT

CARD#1 FORMAT(I5,3E12.6,I5)
NSEQS NUMBER OF SEGMENTS IN THE BODY
(NSEQS MUST BE <=15)
FR FRACTIONAL EXCLUSION (BETA) USED WITH CELLS
TAU COMPLEX MATERIAL PARAMETER
PRINTW INTEGER PRINT KEY; PRINTW=1 PRINTS THE W POTENTIALS, PRINTW=0 DOES NOT (DEFAULT IS 0)

CARD#2 TO CARD#1+NS1+NS2 FORMAT(2I5,2X,A1,5E12.6)
NUMPTS NUMBER OF CELLS THIS SEGMENT IS TO BE DIVIDED UP INTO (THE SUM OF ALL NUMPTS MUST BE <=102)
ITYP TYPE OF SEGMENT; ITYP=1 CONCAVE DOWN SEGMENT,
ITYP=2 CONCAVE UP SEGMENT, ITYP=3 LINEAR
ISIGN CHARACTER CORRESPONDING TO THE VOLUME SENSE OF THIS SEGMENT; ISIGN='+' POSITIVE VOLUME,
ISIGN='-' NEGATIVE VOLUME (DEFAULT IS '+')
ZEP(1) Z CO-ORDINATE OF THE 1ST END POINT OF SEGMENT
ZEP(2) Z CO-ORDINATE OF THE 2ND END POINT OF SEGMENT
RHOEP(1) RHO CO-ORDINATE OF THE 1ST END POINT
RHOEP(2) RHO CO-ORDINATE OF THE 2ND END POINT
THETA ANGLE (IN DEGREES) SUBTENDED BY SEGMENT

COMPLEX AX11(102,102),AX33(102,102),X(102),TAU,POM,X11,X33
DIMENSION B(102),ZEP(2),RHOEP(2),ST1(6),ST3(6)
INTEGER NUMPTS(15),PLUS/*'++'/,BL/*' /,INDX(2),PRINTW
COMMON RHO(102,9),Z(102,9),ARC(102),C(102,9),S(102,9)
COMMON /SOL/ IPS(102)
DATA MIN, TWOPI, PI, W0, W1/\-' , 6.283185, 3.141593, .4444444, .2777777/

C
CCC READ, CHECK, AND WRITE INPUT VALUES

C
37 READ(5,34,END=999) NSEGS, FR, TAU, PRINTW
WRITE(6,4) NSEGS
IF(NSEGS .LE. 0 .OR. NSEGS .GT. 15) GO TO 990
IF(FR .LE. 0. OR. FR .GT. 1.) FR=.001
WRITE(6,5) FR
WRITE(6,150) TAU
M=0
V0=0.0

C
CCC READ AND WRITE INPUT PARAMETERS FOR EACH SEGMENT

C
DO 11 I=1, NSEGS
READ(5,12) NUMPTS(I), ITYP, ISIGN, ZEP, RHOEP, THETA
IF(ISIGN .EQ. 0) ISIGN=PLUS
WRITE(6,13) I, NUMPTS(I), ITYP, ISIGN, ZEP(1), RHOEP(1), ZEP(2),
& RHOEP(2)
IF(NUMPTS(I).LT.0 .OR. ITYP.LE.0 .OR. ITYP.GT.3) GO TO 990
IF(ITYP .NE. 3) WRITE(6,14) THETA
IF(NUMPTS(I).EQ.0) GOTO 11
N=M
THETA=PI*THETA/180.0
M=M+NUMPTS(I)
IF(M .GT. 102) GO TO 990
CALL DATA(ITYP, N, M, ZEP, RHOEP, THETA, FR, VOLINC)
IF(ISIGN .EQ. 0) VOLINC=-VOLINC
V0=V0+VOLINC
11 CONTINUE

C
CCC SET UP LINEAR SYSTEMS OF EQUATIONS TO BE SOLVED

C
DO 2 N=1, M
INDX(N)=N
AN=ARC(N)
TN=RHO(N, 8)

C
DO 3 L=N, M
IF(L .EQ. N) GO TO 82
AL=ARC(L)
TL=RHO(L, 6)
AX11(N,L)=Cmplx(0., 0.)
AX11(L,N)=Cmplx(0., 0.)
AX33(N,L)=Cmplx(0., 0.)
AX33(L,N)=Cmplx(0., 0.)
INDX(2)=L
3 CONTINUE

C
DO 103 J=1, 3, 2
JP6=J+6

C
DO 104 LL=1, 2
I=3-LL
I1=INDX(LL)
I2=INDX(I)
TI2=RHO(I2, JP6)
CALL SETUP(I1, I2, JP6, AXI11, AXI33)
AX11(I1,I2)=AX11(I1,I2)-AXI11*TI2
AX33(I1,I2)=AX33(I1,I2)-AXI33*TI2
104 CONTINUE
CONTINUE

CALL SETUP(N,L,B,AXI11,AXI33)
AXI11(N,L)=AL*(W1*AXI11(N,L)-AXI11*W0*TL)
AXI33(N,L)=AL*(W1*AXI33(N,L)-AXI33*W0*TL)
CALL SETUP(L,N,B,AXI11,AXI33)
AXI11(L,N)=AN*(W1*AXI11(L,N)-AXI11*W0*TN)
AXI33(L,N)=AN*(W1*AXI33(L,N)-AXI33*W0*TN)
GO TO 3

IF (FR .EQ. 1.0) GO TO 3

DO 86 I=1,6
     CALL SETUP(N,N,I,ST1(I),ST3(I))

U=.5*(1.0-FR)*AN
POM=PI*(CMPLX(1.,0.)*TAU)/(CMPLX(1.,0.)*TAU)
AXI11(N,N)=POM-U*(W0*(RHO(N,2)*ST1(2)+RHO(N,5)*ST1(5))
& +W1*(RHO(N,1)*ST1(1)+RHO(N,3)*ST1(3)+RHO(N,4)*ST1(4)+
& RHO(N,6)*ST1(6))
AXI33(N,N)=POM-U*(W0*(RHO(N,2)*ST3(2)+RHO(N,5)*ST3(5))
& +W1*(RHO(N,1)*ST3(1)+RHO(N,3)*ST3(3)+RHO(N,4)*ST3(4)+
& RHO(N,6)*ST3(6))

CONTINUE

SOLVE MATRIX EQUATIONS AND INTEGRALS TO GET X11 AND X33

COMPUTE X11

DO 20 I=1,M
   B(I)=-TWOPR*RHO(I,8)

CALL DECOMP(AX11,M)
CALL SOLVE(AX11,X,B,M)

IF (PRINTW,NE.1) GOTO 30
WRITE(6,44) (I,Z(I,8),RHO(I,8),X(I), I=1,M)

CONTINUE

DO 26 I=1,M
   X(I)=RHO(I,8)*C(I,8)*X(I)

CALL INTEGRAL (X,NSEG,NUMPTS,X11)
X11=PI/V0*X11

COMPUTE X33

DO 24 I=1,M
   B(I)=-TWOPR*Z(I,8)

CALL DECOMP(AX33,M)
CALL SOLVE(AX33,X,B,M)

IF (PRINTW,NE.1) GOTO 35
WRITE(6,46) (I,Z(I,8),RHO(I,8),X(I), I=1,M)

CONTINUE
DO 22 I=1,M
  22   X(I)=-2.*RHO(I,8)*X(I)*S(I,8)
C
CALL INTEG(X,NSEGS,NUMPTS,X33)
  X33=PI/V0*X33
C
CCC  WRITE OUTPUT
C
WRITE(6,52)V0
WRITE(6,200) X11
WRITE(6,210) X33
   GO TO 37
990 WRITE(6,991)
999 STOP
C
CCC  FORMATS
C
  34 FORMAT(I5,3E12.6,I5)
  4 FORMAT('1***** PROGRAM DIELCOM *****',
    & '/0',5X,'SEGMENTS',4X,'=',I2)
  1 FORMAT(' ',5X,'BODY #2',5X,'=',I2)
  5 FORMAT(' ',5X,'EXCLUSION   =',F7.4)
150 FORMAT(' ',5X,'TAU     = ',E12.6,' +J',E12.6)
12 FORMAT(2I5,2X,A1,5E12.6)
13 FORMAT('0SEGMENT #',I2,'/ ',5X,'CELLS',9X,'=',',',I2,' ',5X,
   & 'TYPE KEY',6X,'=',',',I2,' ',5X,'VOLUME SENSE = ',A4,' ',5X,
   & '1ST END POINT = (',F10.5,',' ,',F10.5,',')/ ',5X,
   & '2ND END POINT = (',F10.5,',' ,',F10.5,','))
14 FORMAT(' ',5X,'THETA (DEG)  = ',F10.5)
52 FORMAT('0COMPUTED RESULTS; '/ ',5X,'VOLUME = ',F10.5)
200 FORMAT(' ',5X,'X11/V = ',F10.5,' +J',F10.5)
210 FORMAT(' ',5X,'X33/V = ',F10.5,' +J',F10.5)
44 FORMAT('0',6X,'I',11X,'Z',15X,'RHO',21X,'W1(I)'/
   & (' ',5X,I3,5X,2(F12.6,5X),E12.6,' +J',E12.6))
46 FORMAT('0',6X,'I',11X,'Z',15X,'RHO',21X,'W3(I)'/
   & (' ',5X,I3,5X,2(F12.6,5X),E12.6,' +J',E12.6))
991 FORMAT('0*** ERROR IN DATA')
END
SUBROUTINE DATA(IN,MX,MY,ZEP,RHOEP,THETA,B,VOL)

SUBROUTINE DATA is called by the MAIN PROGRAM for each segment profile. It computes the z and rho co-ordinates, \( \sin(\alpha) \), \( \cos(\alpha) \), and the arc length for each cell. Arguments:

- **IN**: Type key for segment
- **MX**: Total number of cells in segments to left
- **MY**: \( MX + ( \text{number of cells in this segment} ) \)
- **ZEP**: \( z \)-coordinate end points of segment:
  - \( ZEP(1)=z1 \), \( ZEP(2)=z2 \)
- **RHOEP**: Rho coordinate end points of segment:
  - \( RHOEP(1)=\rho_01 \), \( RHOEP(2)=\rho_02 \)
- **THETA**: Angle (in radians) subtended by circular arc at its center
- **B**: Fractional exclusion (beta)
- **VOL**: Incremental volume of segment

DIMENSION ZEP(2),RHOEP(2)
COMMON RHO(102,9),Z(102,9),ARC(102),C(102,9),S(102,9)
DATA STEP/.3872988/

MXP1=MX+1
EN=FLOAT(MY-MX)
IF(B .NE. 1) SUBSTEP=.5*(1.0-B)*STEP
IF(IN-2)1,2,3

TYPE KEY 'IN' = 1 (CONCAVE DOWN SEGMENT)

1 CC=-1.0
GO TO 10

TYPE KEY 'IN' = 2 (CONCAVE UP SEGMENT)

2 CC=1.0

COMPUTE THE RADIUS OF CURVATURE

10 ST2=SIN(THETA/2.0)
A=ZEP(2)-ZEP(1)
RAD=0.5*SQRT((RHOEP(1)-RHOEP(2))**2+A*A)/ST2

COMPUTE THE CENTER OF CURVATURE CO-ORDINATES

DD=A/ABS(A)
T=CC*DD*COS(THETA/2.)/ST2
ZCNT=0.5*(ZEP(1)+ZEP(2)+T*(RHOEP(1)-RHOEP(2)))
RHOCNT=0.5*(RHOEP(1)+RHOEP(2)+T*A)

COMPUTE THE INCREMENTAL VOLUME

U2=ZEP(2)-ZCNT
U1=ZEP(1)-ZCNT
VOL=3.141593*ABS(A*(RHOCNT**2+RAD*RAD-(U2**2+U1*U2+U1**2))/3.0)
& -CC*RHOCNT*(U2*(RHOEP(2)-RHOCNT)-U1*(RHOEP(1)-RHOCNT) +RAD*RAD
& *DD*THETA))

C
CCC  COMPUTE COS(ALPHA), SIN(ALPHA), Z, AND RHO AT THE BOUNDARIES
CCC  OF EACH CELL, AND THE ARC LENGTH OF EACH CELL IN THE SEGMENT
C
BETA=CC*DD*THETA/EN
THET1=ATAN2(RHOEP(1)-RHOCNT,ZEP(1)-ZCNT)
U=ABS(BETA*RAD)
B3=STEP*BETA

C
DO 902 I=MXP1,MY
PHI=THET1+(I-MX-.5)*BETA
IF(B .EQ. 1.0) GO TO 1905

C
DO 1902 J=1,2
ANG=PHI+.5*(J-1.5)*BETA*(1.0+B)

C
DO 1903 L=1,3
PSI=ANG+(L-2)*SUBSTP*BETA
M=L+3*(J-1)
C(I,M)=-CC*SIN(PSI)
S(I,M)=CC*COS(PSI)
Z(I,M)=ZCNT+RAD*CC*S(I,M)
1903
RHO(I,M)=RHOCNT-CC*RAD*C(I,M)

C
1902 CONTINUE

C
1905 DO 903 J=7,9
ANG=PHI+(J-B)*B3
C(I,J)=-CC*SIN(ANG)
S(I,J)=CC*COS(ANG)
Z(I,J)=ZCNT+RAD*CC*S(I,J)
903
RHO(I,J)=RHOCNT-CC*RAD*C(I,J)

C
902 ARC(I)=U
C
RETURN

C
CCC  TYPE KEY 'IN' = 3 (LINEAR SEGMENT)
C
CCC  COMPUTE COS(ALPHA), SIN(ALPHA), Z, AND RHO AT THE BOUNDARIES
CCC  OF EACH CELL, AND THE ARC LENGTH OF EACH CELL IN THE SEGMENT
C
3 DX=(ZEP(2)-ZEP(1))/EN
DY=(RHOEP(2)-RHOEP(1))/EN
U =SQRT(DX*DX+DY*DY)
SI=DY/U
CI=DX/U

C
DO 917 I=MXP1,MY
PHI=FLOAT(I-MX)-.5
IF(B .EQ. 1.0) GO TO 1800

C
DO 1802 J=1,2
ANG=PHI+.5*(J-1.5)*(1.0+B)

C
DO 1803 L=1,3
M=L+3*(J-1)
PSI=ANG+(L-2)*SUBSTP
Z(I,M)=ZEP(1)+PSI*DX
RHO(I,M)=RHOEP(1)+PSI*DY
S(I,M)=SI
C(I,M)=CI

CONTINUE

DO 913 J=7,9
       ANG=PHI+(J-8)*STEP
       Z(I,J)=ZEP(1)+ANG*DX
       RHO(I,J)=RHOEP(1)+ANG*DY
       C(I,J)=CI
       S(I,J)=SI

ARC(I)=U

COMPUTE THE INCREMENTAL VOLUME

VOL=1.047198*(ZEP(2)-ZEP(1))*(RHOEP(1)**2+RHOEP(1)*RHOEP(2)+
&       RHOEP(2)**2)

RETURN
END
SUBROUTINE INTEG(V,NSEG,NUMPTS,SUM)

SUBROUTINE INTEG is called by the MAIN PROGRAM to numerically integrate quadratic interpolating polynomials approximating the data on each segment of the profile.

Arguments:

V  Vector of function values, ordered as cells
NSEG  Total number of segments in the profile
NUMPTS  Vector containing in NUMPTS(I) the number of cells on the Ith segment: I = 1, NSEG
SUM  Integral of V across the profile

COMMON RHO(102,9),Z(102,9),ARC(102),C(102,9),S(102,9)
COMPLEX V(102),SUM
DIMENSION NUMPTS(15)
SUM=0.0
JACC=1

DO 3000 I=1,NSEG
   T=ARC(JACC)
   L=NUMPTS(I)
   N=L+JACC-1
   SUM=SUM+T*(0.625*(V(JACC)+V(N))-0.125*(V(JACC+1)+V(N-1)))
   IF(L/2 .NE. (L+1)/2) GO TO 3001
   SUM=SUM+T*(0.66666667*V(N-1)-0.083333333*V(N-2)+0.41666667*V(N))
3001 LM1=N-1
   JLO=JACC+1

   DO 3002 J=JLO,LM1,2
      SUM=SUM+0.333333333*T*(V(J-1)+4.0*V(J)+V(J+1))
3002
   JACC=JACC+1

RETURN
END
SUBROUTINE DECOMP(UL,N)

SUBROUTINE DECOMP is called by the MAIN PROGRAM to perform a L - U decomposition on the N x N matrix UL.

COMPLEX UL(102,102), PIVOT, EM, CDUM
COMMON /SOL/ IPS(102)
CABS1(CDUM)=REAL(CDUM)+AIMAG(CDUM)

DO 5 I=1,N
   IPS(I)=I
5

NM1=N-1

DO 16 K=1,NM1
   BIF=0.0
16

DO 11 I=K,N
   IF(CABS1(UL(IP,K)) .LE. BIF) GO TO 11
   BIF=CABS1(UL(IP,K))
   IDXPIV=I
   CONTINUE
11

IF(IDXPIV .EQ. K) GO TO 15
   J=IPS(K)
   IPS(K)=IPS(IDXPIV)
   IPS(IDXPIV)=J
15
   KP=IPS(K)
   PIVOT=UL(KP,K)
   KP1=K+1

DO 16 I=KP1,N
   IP=IPS(I)
   EM=-UL(IP,K)/PIVOT
   UL(IP,K)=-EM
16

DO 16 J=KP1,N
   UL(IP,J)=UL(IP,J)+EM*UL(KP,J)
   CONTINUE

RETURN
END
SUBROUTINE SOLVE(UL,X,B,N)

SUBROUTINE SOLVE is called by the MAIN PROGRAM to solve the linear system UL * X = B by back substitution after the UL matrix has been decomposed by the SUBROUTINE DECOMP.

COMPLEX UL(102,102),X(102),SUM
DIMENSION B(102)
COMMON /SOL/IPS(102)
NP1=N+1
IP=IPS(1)
X(1)=B(IP)

DO 2 I=2,N
   IP=IPS(I)
   IM1=I-1
   SUM=CMPLX(0.,0.)

      DO 1 J=1,IM1
         SUM=SUM+UL(IP,J)*X(J)
   1

2   X(I)=B(IP)-SUM

   IP=IPS(N)
   X(N)=X(N)/UL(IP,N)

   DO 4 IBACK=2,N
      I=NP1-IBACK
      IP=IPS(I)
      IP1=I+1
      SUM=CMPLX(0.,0.)

      DO 3 J=IP1,N
         SUM=SUM+UL(IP,J)*X(J)
   3

4   X(I)=(X(I)-SUM)/UL(IP,I)

RETURN
END
SUBROUTINE ELLI (M1,K,E,KPR)

SUBROUTINE ELLI is called by the SUBROUTINE SETUP to compute the elliptic integrals $K(m)$, $E(m)$, and derivative $K'(m)$ from their power series approximations. (See sec. 4.2 of "The Numerical Solution of Low Frequency Scattering Problems").

REAL M1,K,KPR
T=-ALOG(M1)
& +4.41787E-3*T)))
E=1.0+M1*(.4432514+.2499837*T+M1*(6.260601E-2+9.20018E-2*T+M1*(
KPR=.5/M1 + 2.83225E-2 -.1249859*T + M1*(-2.999362E-3-.137605*T & +M1*(-7.899336E-2 - 9.985066E-2*T + M1*(-5.362998E-2 -
& 1.767148E-2 * T )))
RETURN
END
SUBROUTINE SETUP(I,J,L,AXI11,AXI33)

SUBROUTINE SETUP is called by the MAIN PROGRAM to compute the quantities AXI11 (f(i,j) of eq. 227 in "The Numerical Solution of Low Frequency Scattering Problems") and AXI33 (a similar quantity for x33).

Arguments:

I Subscript of observer (unprimed) cell
J Subscript of remote (primed) cell
L Index of the point within remote cell for which the kernels are to be computed (See SUBROUTINE DATA)

AXI11 See above
AXI33 See above

COMMON RHO(102,9),Z(102,9),ARC(102),C(102,9),S(102,9)
REAL M,M1,K,KPR
REAL*8 DA1,DA2,DM1,DZD,DRP,DR,DRRP
DZD=DBLE(Z(J,L))-DBLE(Z(I,8))
ZD=SNGL(DZD)
R=RHO(I,8)
DR=DBLE(R)
RP=RHO(J,L)
DRP=DBLE(RP)
DRRP=DR*DRP
RRP=SNGL(DRRP)
DA1=DRRP+DRRP
DA2=DR*DR+DRP*DRP+DZD*DZD
DM1=(DA2-DA1)/(DA2+DA1)
M1=SNGL(DM1)

IF (ABS(M1).LT.1.E-50) WRITE(6,100) M1,Z(J,L),Z(I,8),R,RP,DA1,DA2
100 FORMAT(7E10.3)
M=1.-M1
CALL ELLI(M1,K,E,KPR)
A0=M/RRP
A1=SQRT(A0)
A2=M+M
A3=2.-M
A1=A1*A0
A3=.5*A3
A0=C(J,L)
OM0=.25*A1*(K+2.*M*KPR)
OM1=A1*(.25*K-A3*KPR)
AXI11=R*A0*OM2+(ZD*S(J,L)-RP*A0)*OM1
AXI33=R*A0*OM1+(ZD*S(J,L)-RP*A0)*OM0
RETURN
END

MODIFICATIONS OF MINK TO DIELCOM BY TOM WILLIS
4.2 Main Program

The main program reads, prints and supervises all computations.

A rough flow chart showing the interaction of the subroutines is given below.
References


