

# **Energy Driven Polygonal Approximation for Shape Classification**

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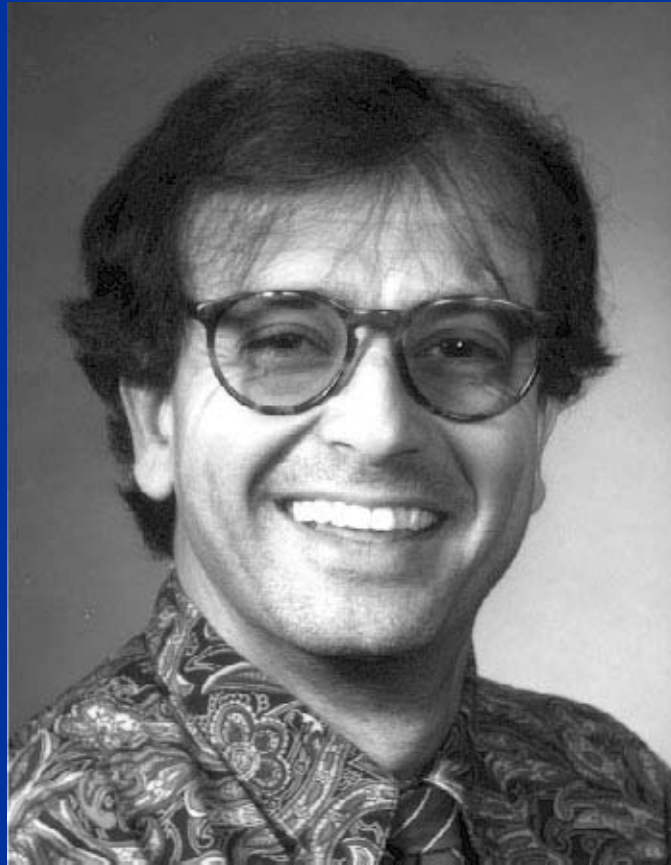
**ECE Dept., NCSU**

**Raleigh, NC**

**A beautiful picture...**



**A not so beautiful...more  
complex picture**



Another...

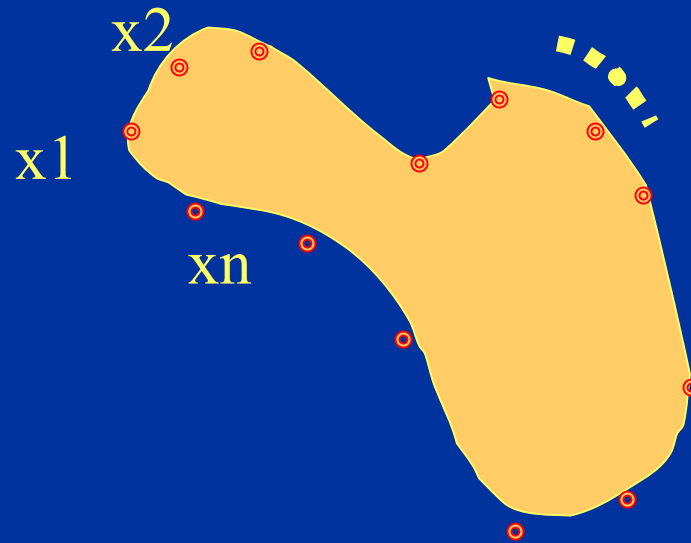


# Ideally...

- **Unique representation of an object contour by a finite number of landmarks**
- **Quotient out *scale, location and rotation***
- **Construct a corresponding probabilistic model to capture variability**
- **Analyze observed shapes in a Bayesian framework**

# Statistical Shape Theory and Analysis

- Deformable templates of Grenander *et al.*
- Statistical Shape theory of Kendall *et al.*
- Given an object contour



# Statistical Shape Theory and Analysis

- The set of landmarks  $\{\mathbf{x}_1, \dots, \mathbf{x}_n\}$  with location and scale information quotient out defines a pre-shape space  $S^{2n-3}$

- Applying rotation transf. to  $\tau$



Shape space

$$\Sigma_2^n$$

$$\Sigma_2^n = \{o(\tau) : \tau \in S^{2n-3}\}$$

# Statistical Shape Theory and Analysis

- The orbit  $\mathcal{O}$  of a shape is the set

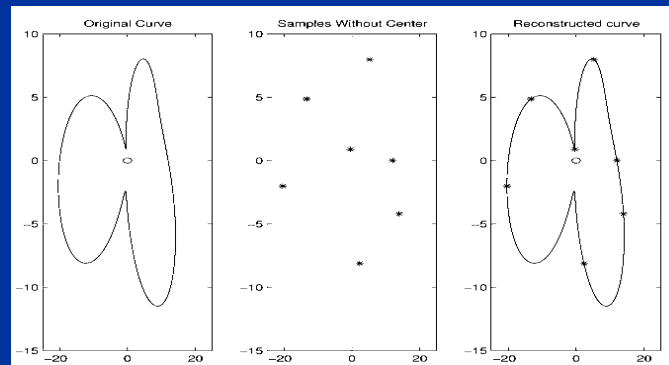
$$\mathcal{O}(\tau) = \{\theta(\tau) : 0 \leq \theta \leq 2\pi\} \subset \mathbb{S}^{2n-3}$$

- For analysis and comparison of shapes, a metric on  $\Sigma_2^n$  is defined as

$$d(\mathcal{O}(\tau_1), \mathcal{O}(\tau_2)) = \inf \{d(\theta_1(\tau_1), \theta_2(\tau_2))\}$$

# Sampling planar curves

- For a Jordan curve parameterized by bandlimited polar radius  $r(\theta)$ , and  $r_0$  as a polar center;
- Subject to some technical conditions, perfect sampling and reconstruction may be obtained [Poliannikov *et. al.* ]



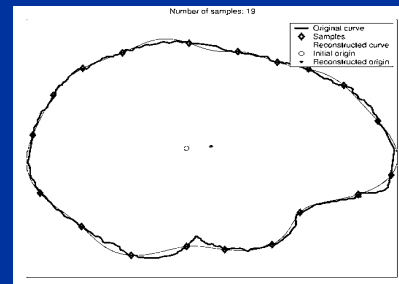
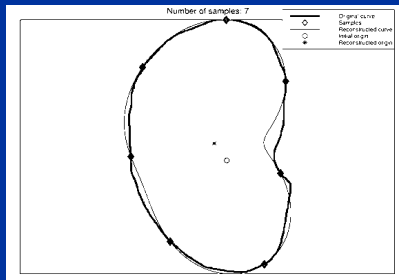
# Curve Sampling

- If  $r(\theta)$  not bandlimited



Optimized approximation!

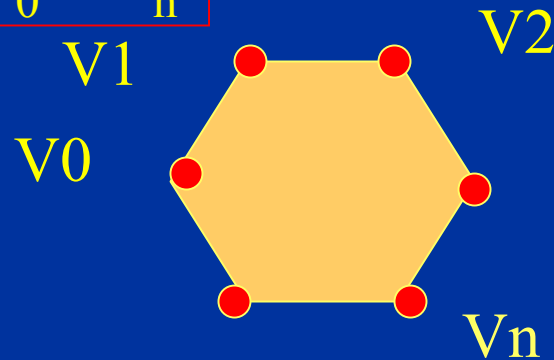
- Minimize reconstruction error of polar center  $r_0$ , and curve: Two-criteria optimization [Poliannikov *et al*]



# Active Landmarks

- For a polygon  $\Gamma$ , as a set of edges and vertices

$$\{V_0, \dots, V_n\}$$



- Define a parametrized curve

$$C(p, V) = L(p - \lfloor p \rfloor, V_{\lfloor p \rfloor}, V_{\lfloor p \rfloor + 1})$$

# Active Landmarks

- Define an information based energy

functional  $JS_{\alpha} = H(\sum a_i p_i(\xi)) - \sum a_i H(p_i(\xi))$

$$E(C) = \iint f(x, y) dx dy$$

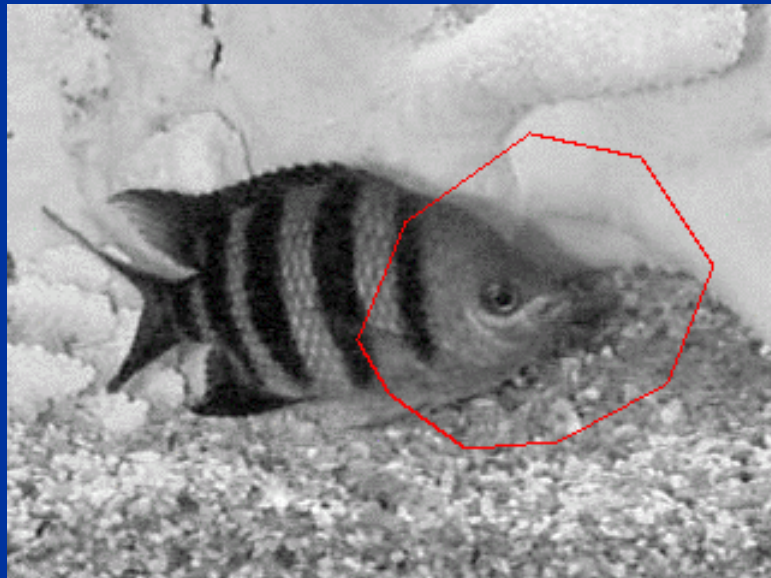


$$\frac{\partial V_k}{\partial t} = \int p f(L(p, V_{k-1}, V_k)) dp N_{k,k-1} +$$

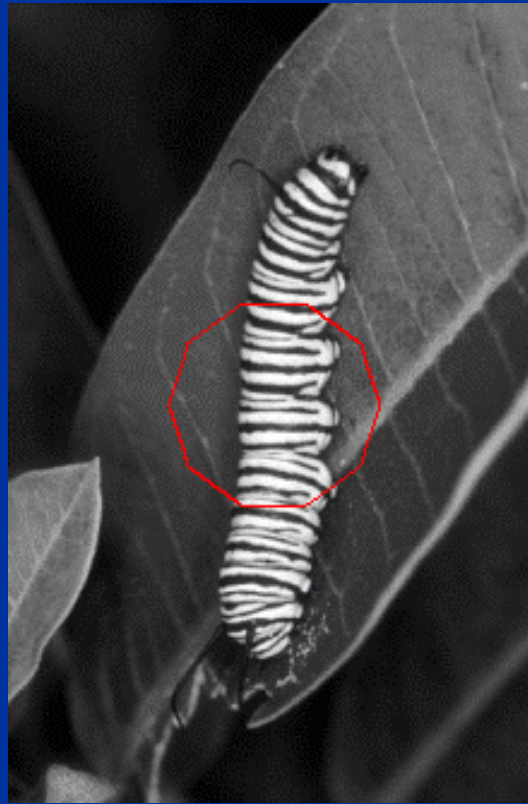
$$\int (1-p) f(L(p, V_k, V_{k+1})) dp N_{k+1,k}$$

# Feature-Based Shape Extraction

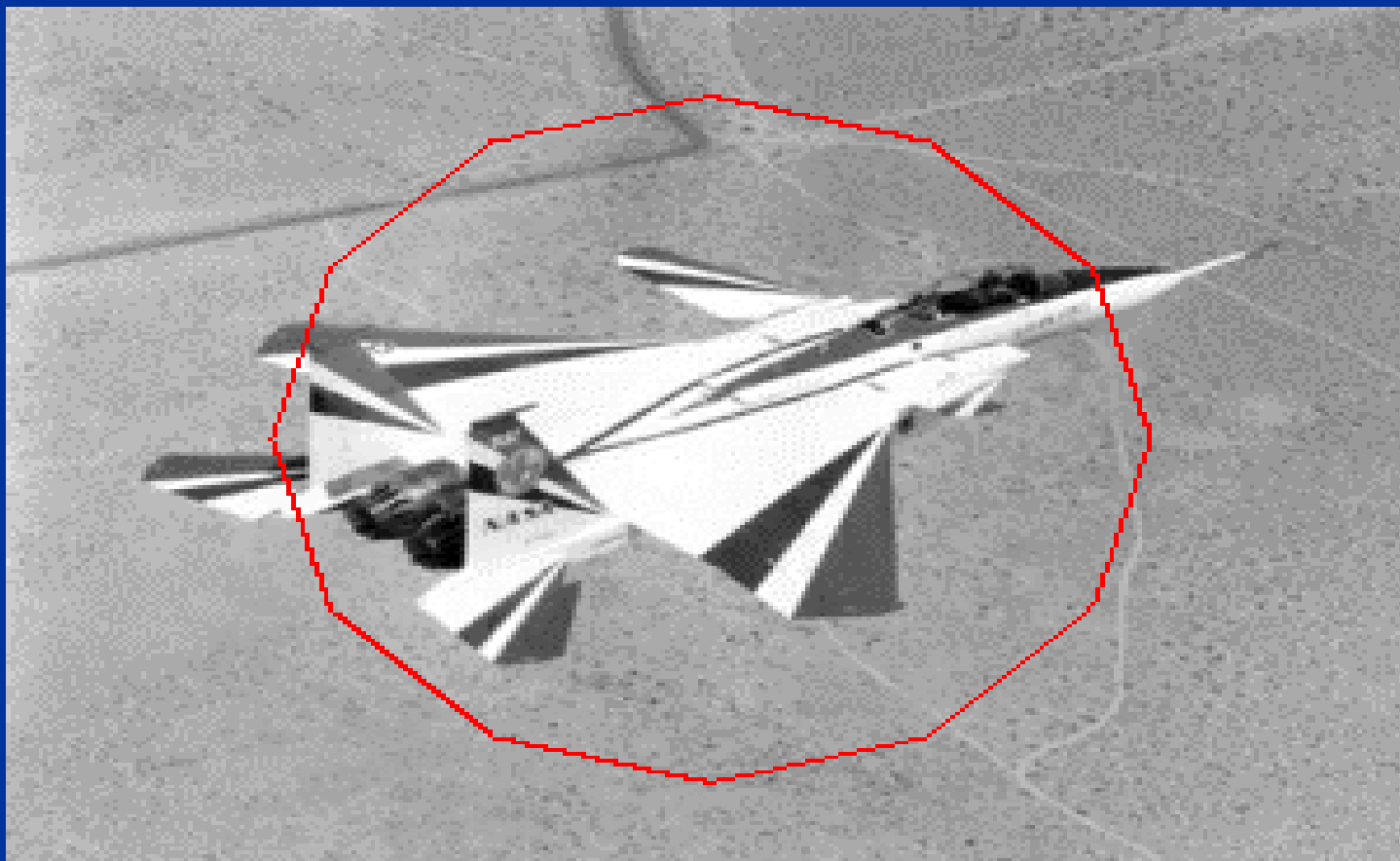
- Pick a generalized Jensen-Shanon IT as a functional to be optimized



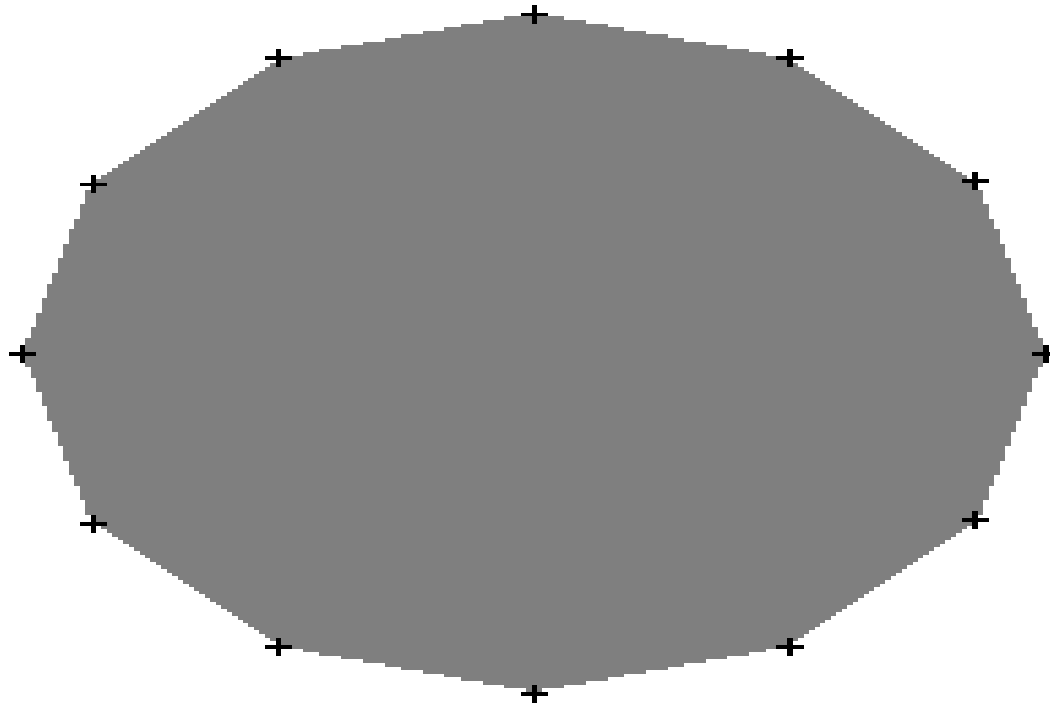
# Feature-Based Shape Extraction



# Feature-Based Shape Extraction



# Feature-Based Shape Extraction



# Rationale

- **The polygonal flow yields :**
  - Usable meaningful feature points from target
  - Applicable to objects within an image (texture etc.)
  - Robust to noise
- **Invariance of feature measures in moderate transformations**

# **A Case Study in classification**

**Problem: Objects in a database, (airplanes) from**

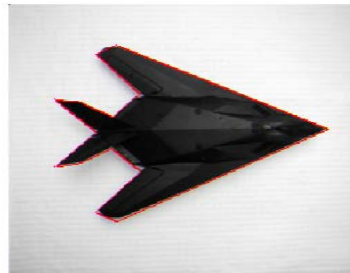
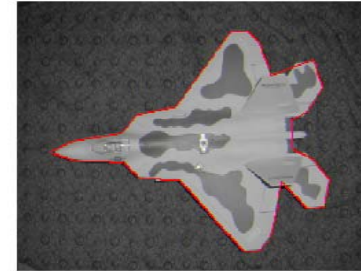
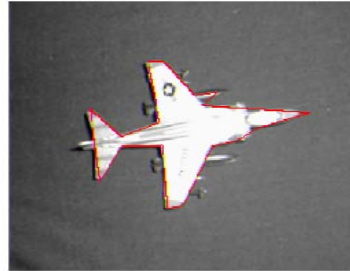
- different viewing angles**
- different lighting and background conditions**

# Retrieval from Model Database

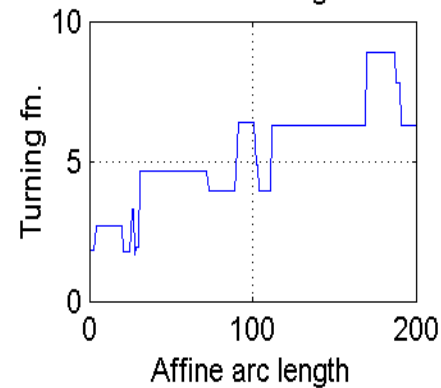
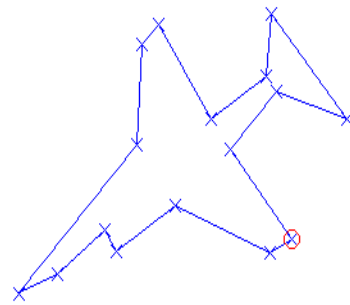
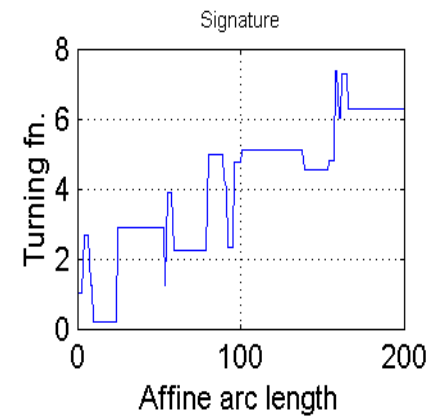
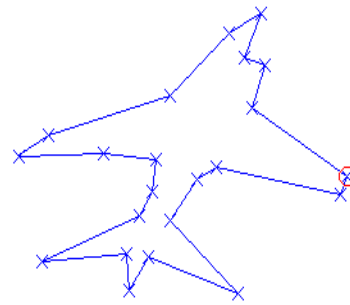
Query  
Image



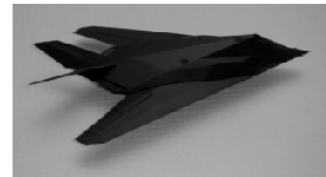
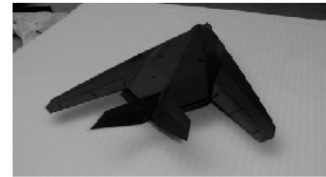
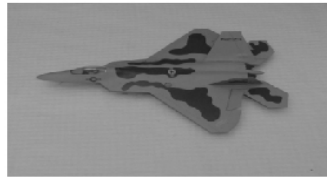
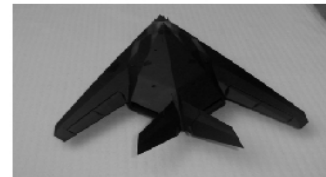
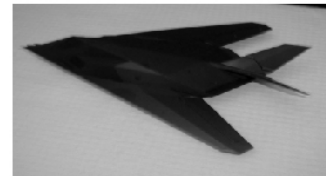
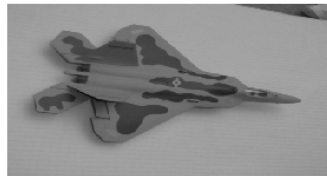
from,



# A Classification signature

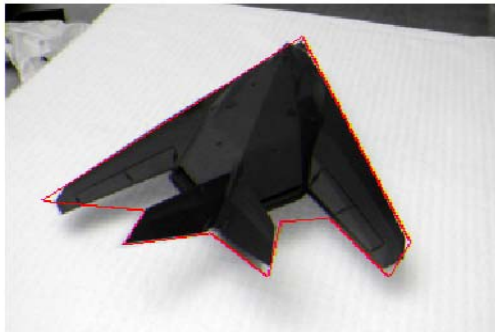


# Different Views

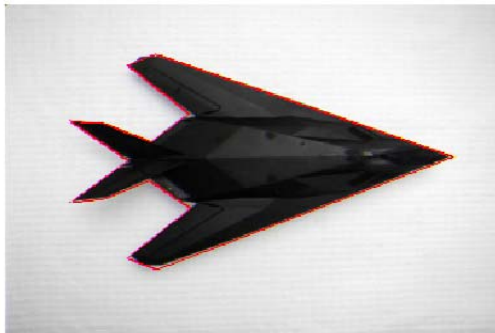


# A Query

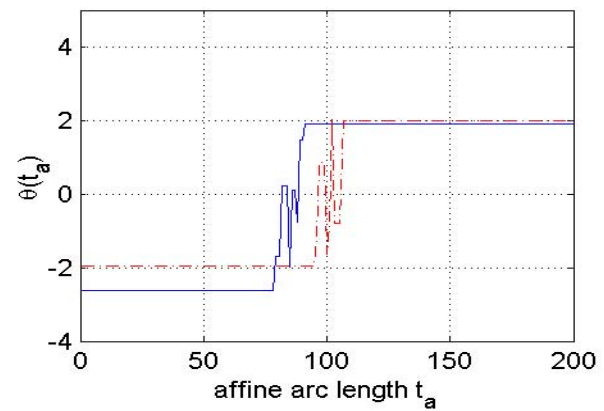
Test image



Match 1



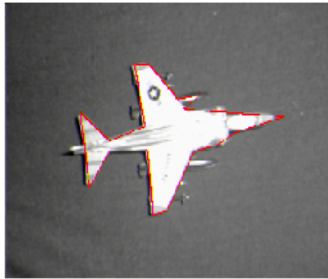
Test : solid, Match : dashed



# A Query to the Image Set

## Consecutive matches

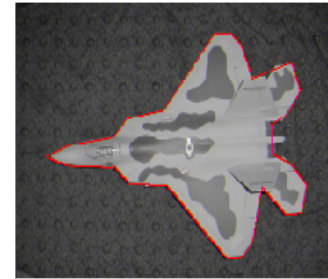
Match 2



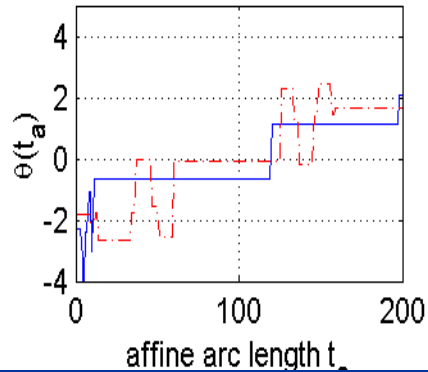
Match 3



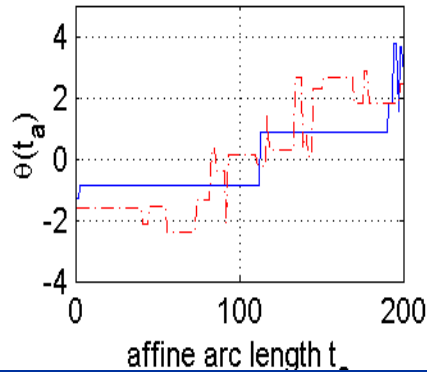
Match 4



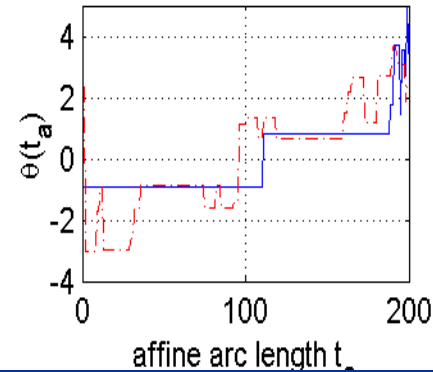
Test: solid, Match: dashed



Test: solid, Match: dashed

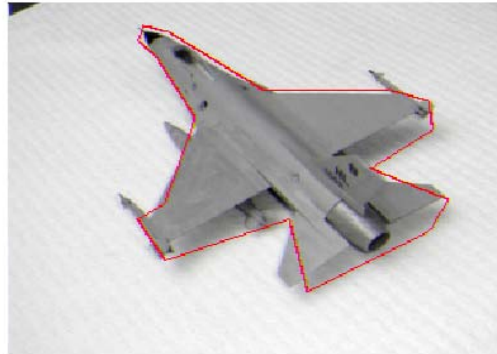


Test: solid, Match: dashed



# A Query to Model Image Set

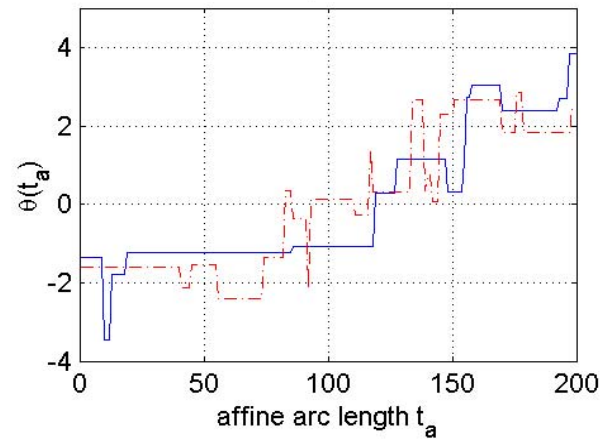
Test image



Match 1

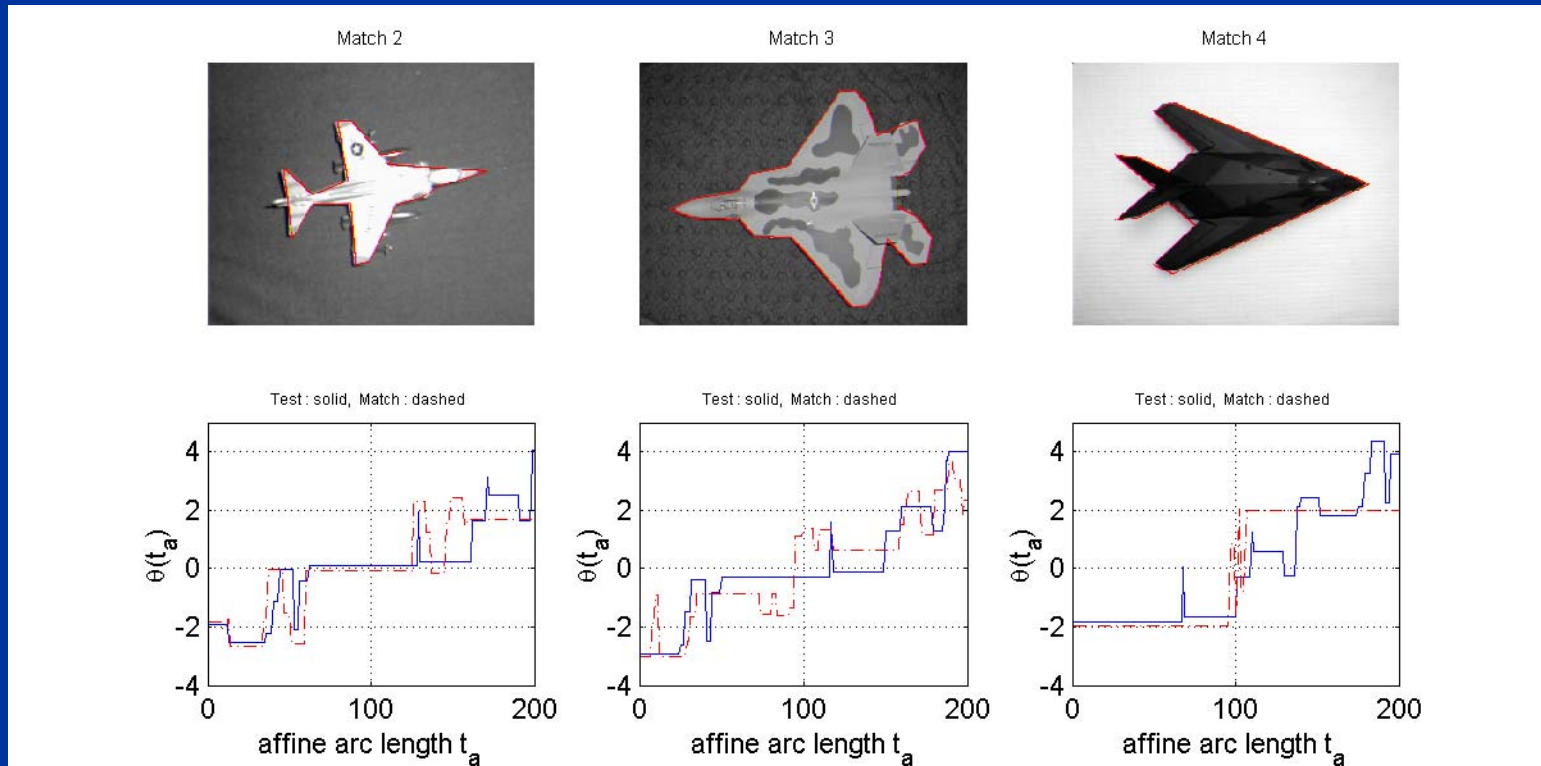


Test: solid, Match: dashed



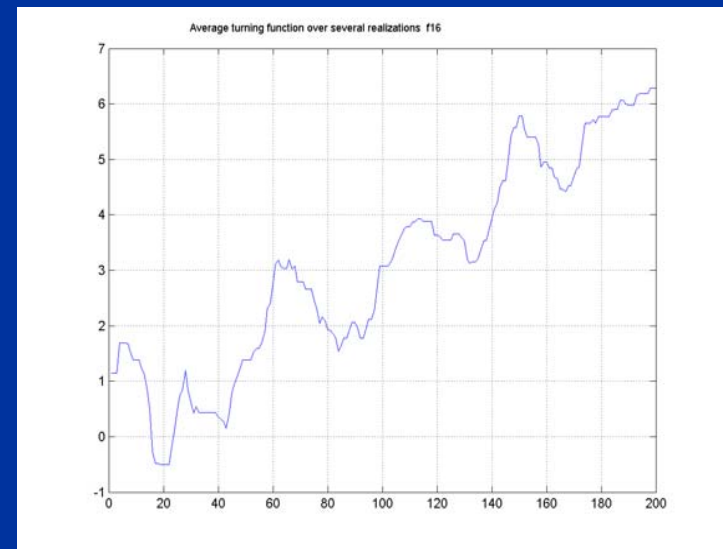
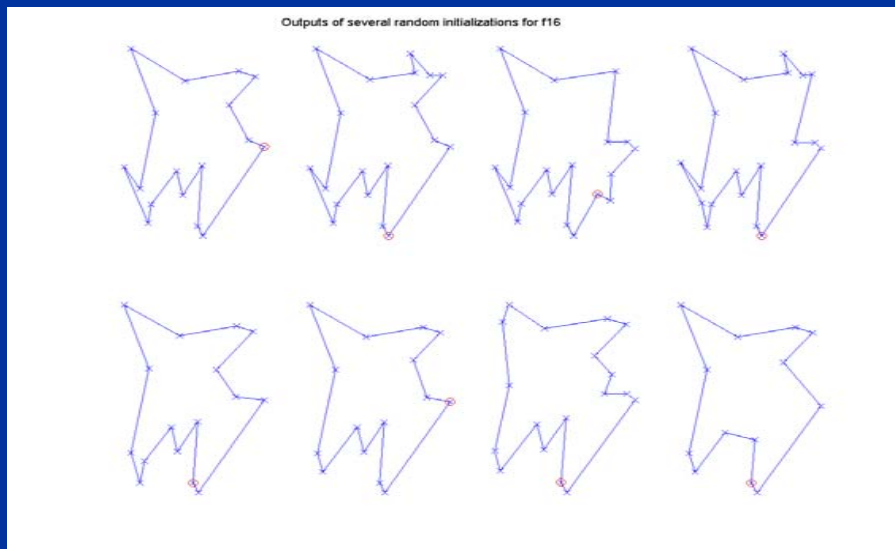
# A Query to Model Image Set

## Consecutive matches



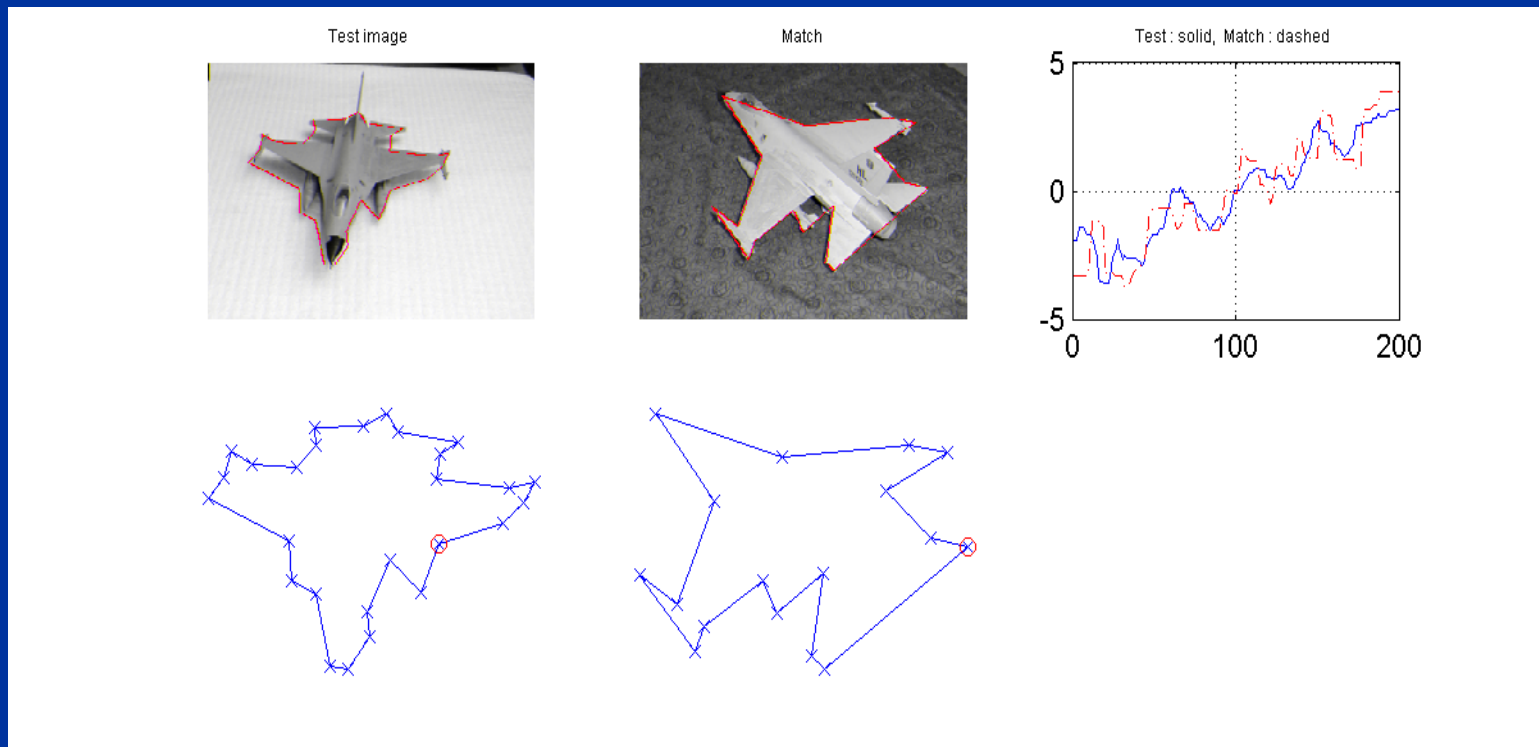
# Statistical Approach

- **Generate random initializations and extract feature points from an image**
- **Obtain several realizations of the target shape: match and average over turning fn vs. arc length**



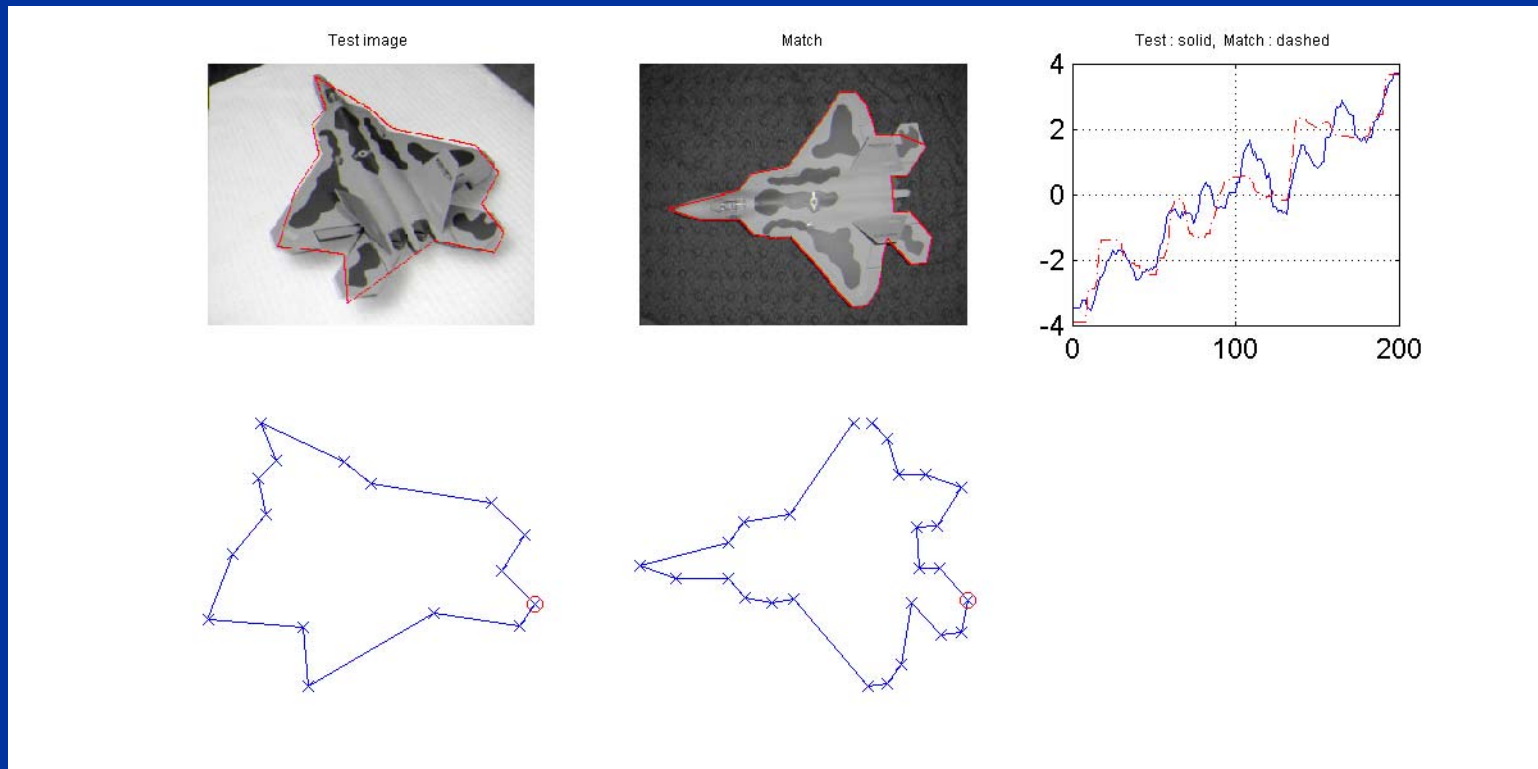
# A Query to Model Image Set

Match using averaged turning functions



# A Query to Model Image Set

Match using averaged turning functions



# Open Problems

- **Development of probabilistic models of arbitrary shapes on the basis of perspective view**
- **Texture models in tandem with shape models for refined classification**
- **Development of occlusion models on the basis of perspective view**