ABSTRACT

PERFORMANCE ANALYSIS OF FREQUENCY-HOPPED SPREAD-SPECTRUM COMMMUNICATION SYSTEMS WITH WORST-CASE JAMMING INTERFERENCE

by

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This thesis focuses on the analysis of frequency-hopped spread-spectrum communication systems in the presence of worst-case jamming interference. Multi-level quantization is employed to improve the performance at the receiver in both coherent and noncoherent systems over hard decision systems. The channel capacity is the performance measure.

The worst-case jamming strategies (from the communicator's perspective) are discussed with various types of noise in coherent/noncoherent systems. For coherent systems, the types of jamming noise considered are tone jamming, phase mismatched tone jamming and partial-band Gaussian jamming. In noncoherent systems, one-dimensional tone jamming and partial-band Gaussian jamming are considered. For both coherent and noncoherent systems two, three or four level quantization detectors are used. The jamming strategies we consider allow the jammer to pulse between several power values subject to an average power constraint.

It is found that four level tone jamming makes the performance significantly worse than two level tone jamming, whereas with Gaussian jamming, four level

jamming does not significantly degrade the channel capacity compared with two level jamming.

To My Parents and My Wife

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LIST OF SYMBOLS

A	Channel input alphabet
B	Channel output alphabet
C	Channel capacity
E_b	Information bit energy (i.e. $E_b = E_c / r$)
E_c	Channel symbol energy
I	Jamming noise random variable (i.e. $I = ZN$)
N	Jamming noise type
N_J	Jammer power density
P	Distribution of jamming noise level Z
P_s	Signal power
q	Number of frequency slots
Q_L	L level quantization rule
r	Channel code rate
R	Output of correlation receiver
T	Signal duration
W	Channel bandwidth
X	Channel input symbol
Y	Channel output symbol
Z	Jamming noise level
λ	Channel symbol energy to jammer power density ratio
θ	Quantization level

CHAPTER I

INTRODUCTION

1.1. Spread-Spectrum Communication Systems

In many communication systems it is desired to transmit and receive signals over channels with interference from other sources. There are several types of interference in communication channels. One of them is unavoidable noise, which is called *background noise.* Other examples of interference include *jamming* and *multi-user interference.* A commonly used way to mitigate interference is through the use of spread-spectrum which produces a signal with a bandwidth much wider than the message bandwidth [DIX76], [COO83], [SCH82], [UTL78], [VIT79b]. The most widely used spread-spectrum methods are *direct sequence* and *frequency hopping* [PIC82]. In direct sequence systems each user has his own code sequence and occupies the full channel bandwidth at all times during the transmission [PUR77]. In frequency hopped systems the carrier frequency of the signal hops among a set of frequencies during transmission according to a specified hopping pattern. Errors may occur when a signal is hopped to a certain frequency that is occupied by another signal (i.e. interference source) [COO83],[GER81b],[PIC82].

Spread spectrum systems have been used to (a) provide low probability of intercept, (b) combat multipath problems, (c) provide anti-jam capability and (d)

provide multiple access capability. These systems have been mainly used in military communications for several decades because of the extreme difficulty in detecting the presence of the signal and to jam the signal. In commercial communications spread-spectrum allows multiple access capability. In a spread-spectrum multiple access system, each user is assigned a particular quasi-orthogonal code sequence which is modulated on the carrier along with the digital data. Unlike traditional frequency and time division multiple access for multi-user communication, spread spectrum multiple access technique (SSMA) does not require precise time or frequency coordination between the transmitters in the system.

In certain communication systems (especially military communication systems), the channel contains hostile interference, that is, a source of interference whose goal is to make the performance as bad as possible. We call this kind of interference intentional jamming. Under this situation it is very difficult to maintain reliable communications. Spread-spectrum systems have been used for many years when there is intentional jamming.

1.2. Background

Over the last ten years there has been significant research into the design of frequency-hopped systems that achieve reliable communication in the presence of jamming. It was recognized early that spread-spectrum alone is not sufficient to overcome a jamming threat. While a communication system employing frequency-hopped spread-spectrum, for example, forces a jammer to use large bandwidths, a simple two-level partial-band jammer (see Fig. 1.2.1) which places all his power in a fraction of the overall bandwidth instead of spreading the same power over a larger bandwidth, can cause a degradation (compared to additive white Gaussian noise) on

the order of 30-40 *dB* [VIT75]. This led to the use of error-correcting codes [OMU82], [PUR82a], [STA82a]. Initially decoding algorithms for additive white Gaussian channels where considered. However, it was recognized that the performance of these decoding algorithms in the presence of two-level (on-off) partial-band jamming was extremely poor. Then many realized that for a two-level partial-band jammer it would be reasonably easy to detect, on each hop, if there was interference or not. This led to modified decoding algorithms which incorporated this side information. Later algorithms for channels subject to jamming were considered which did not require the use of side information [LEE84]. However many of these algorithms were only analyzed for the case of two-level partial-band jamming, without paying attention to what the worst-case jammer would be.

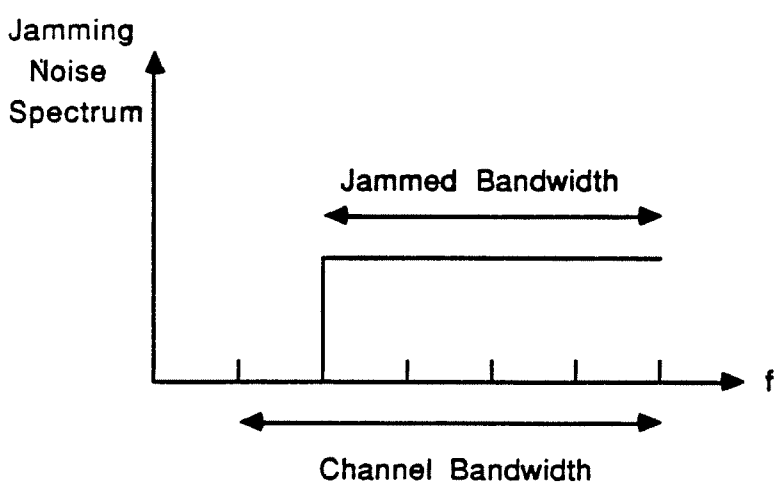

Figure 1.2.1. Two-level partial-band jamming.

In this thesis we examine the performance of a frequency-hopped spread-spectrum system subject to intentional jamming. The performance measure considered is the channel capacity. We do not assume any side informance is available. We allow the jammer to pulse between two, three and four levels with a fixed average power. The perspective taken in this thesis will be that of the communicator. The communicator is interested in providing reliable communication at the largest possible rate. A worst case strategy is a jamming strategy that minimizes the rate below which reliable communication is possible. We find that for a partial-band Gaussian jammer (to be described) two-level jamming, three- and four-level jamming have nearly the same performance. For a tone jammer with random phase (also described later) we find that the jammer can cause a nonnegligible degradation if he uses more than two levels. Similar results hold for a tone jammer with perfect phase information.

The remainder of this chapter is organized as follows. Section 1.3 describes the frequency-hopped spread-spectrum communication systems to be considered. Two-types of modulation/demodulation are considered, namely, coherent PSK and non-coherent FSK. Each of these is described in the next section. Section 1.4 describes the types of jamming signals we will consider. In section 1.5 we will introduce the performance measure to be used. We will also describe some of the difficulties in evaluating the performance. Finally in section 1.6 we will outline the remainder of the thesis.

1.3. Frequency-Hopped Spread-Spectrum

In this section we describe the communications system to be analyzed in this thesis. This will include the modulation, demodulation and frequency-hopped

spread-spectrum. The system block diagram is shown in Fig. 1.3.1. We first assume that there are one of two symbols to be transmitted. Each symbol is equally likely. While this last assumption may be restrictive (i.e. channel capacity could be achieved by a non-uniform probability distribution), in practice it is very difficult to take advantage of any possible gain by using symbols that are not equally likely. Two different modulation schemes are considered, Phase Shift-Keying (PSK) and Frequency Shift-Keying (FSK).

Binary Phase Shift-Keying

Let $b(t)$ be a random process representing the input to the modulator with

$$b(t) = \sum_{n=-\infty}^{\infty} X_n \, P_T(t-nT) \tag{1.3.1}$$

where X_n is the data symbol input to the modulator during $[nT, (n+1)T)$, $P_T(t)$ is a unit pulse function (i.e. $P_T(t)=1$ for $0 \leq t < T$ and zero elsewhere) and T is the

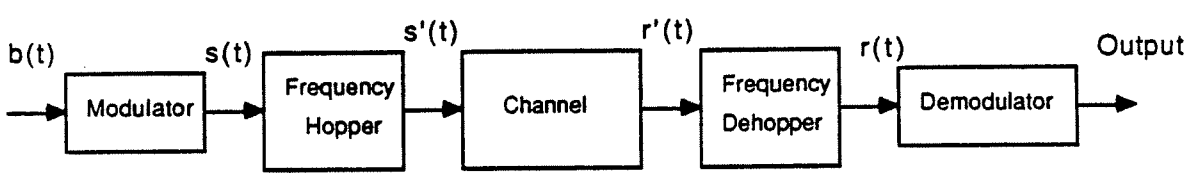

Figure 1.3.1. Frequency-hopped spread-spectrum system block diagram.

duration of each data bit. As mentioned above we will assume that $\{X_n\}_{n=-\infty}^{\infty}$ is a sequence of independent and identically distributed (i.i.d.) random variables equally likely to be +1 or -1. The PSK signal then is

$$s(t) = \sqrt{2P_s}\, b(t) \cos 2\pi f_c t \tag{1.3.2}$$

where P_s is the power of the signal and f_c is the carrier frequency. The energy of each channel symbol transmitted will be denoted by E_c and is given by $E_c = P_s T$.

Binary Frequency Shift-Keying

In frequency shift-keying we start with the same data signal as input to the modulator. The output of the modulator however is the signal

$$s(t) = \sqrt{2P_s} \cos\left[2\pi(f_c + b(t)\Delta f)t + \psi(t)\right] \tag{1.3.3}$$

where Δf is one half the spacing between the FSK tones and $\psi(t)$ is a phase signal introduced by the modulator. We will assume that Δf is chosen so that tones of duration T at the frequencies $f_c \pm \Delta f$ are orthogonal and that the phase signal $\psi(t)$ is constant on every interval of the form $lT \leq t < (l+1)T$ for some integer l.

Frequency-Hopping

As mentioned before we will consider frequency-hopped spread-spectrum signals. We will assume there are q frequency slots. We will now describe the frequency hopped signal for PSK and FSK.

For PSK the input to the frequency-hopper is the modulated output signal given by (1.3.2). The output of the frequency-hopper is the signal

$$s'(t) = \sqrt{2P_s}\, b(t) \cos\left[2\pi(f_c + f_h(t))t\right] \tag{1.3.4}$$

The frequency hopping waveform, $f_h(t)$, is given by

$$f_h(t) = \sum_{n=-\infty}^{\infty} f_n P_T(t-nT_h) \tag{1.3.5}$$

where $\{f_n\}_{n=-\infty}^{\infty}$ is the frequency hopping pattern, and T_h is the hop duration. In this thesis we will only consider one symbol per hop so that $T_h = T$. For the analysis in this thesis we will assume that f_n is equally likely to be any one of q frequencies $\{f_1, f_2, \cdots, f_q\}$. We model the sequence $\{f_n; -\infty < n < \infty\}$ as a sequence of i.i.d. random variables uniformly distributed over the q frequencies. Usually q will be very large (on the order of 100 or 1000 or more).

For FSK the output of the frequency-hopper is a signal of the form

$$s'(t) = \sqrt{2P_s} \cos\left[2\pi(f_c + b(t)\Delta f + f_h(t))t + \phi(t)\right] \tag{1.3.6}$$

where $\phi(t)$ is a random phase term introduced by the frequency-hopper. As with the random phase term introduced by the FSK modulator $\phi(t)$ is assumed to be constant on intervals of the form $[nT_h, (n+1)T_h)$.

The received signal $r'(t)$ for PSK is given by

$$r'(t) = s'(t) + j'(t) \tag{1.3.7}$$

where $j'(t)$ is the jamming signal. We will assume throughout that the receiver is able to coherently demodulate the signal. We also assume that perfect symbol synchronization is achieved. Finally we assume there is no background noise. In chapter four we will describe how background noise can be incorporated. We will consider an ideal frequency-dehopper which is synchronized to the hopping signal. The output of the frequency-dehopper is a signal of the form

$$r(t) = s(t)+j(t) \tag{1.3.8}$$

where $j(t)$ is the output of the frequency-dehopper due to the jamming signal. We will describe the signal $j(t)$ and $j'(t)$ in the next section. Thus the frequency-hopper and frequency-dehopper combination appears transparent to the modulated signal. However the jamming signal is affected by the frequency-dehopper.

For FSK we will assume the received signal is of the form

$$r'(t) = \sqrt{2P_s}\cos\left[2\pi(f_c+b(t)\Delta f+f_h(t))t+\Phi(t)\right] + j'(t) \tag{1.3.9}$$

where $\Phi(t)$ is a random phase introduced by the channel. We will assume that $\Phi(t)$ is constant on intervals of the form $[nT, (n+1)T)$ and is uniformly distributed on the interval $[0, 2\pi)$. The effect of the dehopper then is to produce a signal of the form

$$r(t) = \sqrt{2P_s}\cos\left[2\pi(f_c+b(t)\Delta f)t+\Psi(t)\right) + j(t) \tag{1.3.10}$$

where again $\Psi(t)$ is a random phase term having the same statistics as $\Phi(t)$. Again the frequency-hopper and frequency-dehopper combination appears transparent to the modulated signal (except for an additional phase term). The next section describes the models we use for the jamming signal.

1.4. Jamming Signal Model

The jammer is assumed to know the location of the q frequency slots being used by the transmitter but not the particular hopping pattern. The jamming signal $j'(t)$ at the receiver will be modeled as a weighted sum of noise signals :

$$j'(t) = \sum_{i=1}^{q} Z_i(t)\,\hat{j}(t)\,\sqrt{2}\cos 2\pi f_i t \tag{1.4.1}$$

where $\hat{j}(t)\sqrt{2}\cos 2\pi f_i t$ is the noise signal in the i-th frequency slot and $Z_i(t)$ determines the amplitude of jamming signal in the i-th frequency slot. We assume $Z_i(t)$ is a pulse train with each pulse of duration T:

$$Z_i(t) = \sum_{l=-\infty}^{\infty} Z_{i,l} P_T(t-lT) \tag{1.4.2}$$

where $Z_{i,l}$ represents the amplitude of the jamming signal in frequency slot i in the interval $[lT \leq t \leq (l+1)T)$. For a Gaussian type noise, $\hat{j}(t)$ is a Gaussian process with two-sided power spectral density $N_J/2$ over a bandwidth W/q Hz centered at f_c. Here W is the total spread bandwidth of the transmitted signal so that W/q is the bandwidth of each frequency slot. Another type of noise of interest is tone jamming. For coherent PSK we assume $\hat{j}(t)$ is of the form

$$\hat{j}(t) = \sqrt{2N_J/T}\ b_j(t)\cos(2\pi f_c t + \hat{\phi}(t)) \tag{1.4.3}$$

where $\hat{\phi}(t)$ is a random phase introduced by the jammer, N_J/T is the noise power, T is signal duration. Also in (1.4.3) $b_j(t)$ is a random signal given by

$$b_j(t) = \sum_{n=-\infty}^{\infty} V_n\ P_T(t-nT) \tag{1.4.4}$$

where V_n is equally likely to be +1 or -1. In (1.4.3) we assume that $\hat{\phi}(t)$ is constant on intervals of the form $[nT, (n+1)T)$ and is uniformly distributed on the interval $[0, 2\pi)$. If the phase in (1.4.3) is mismatched to the phase of the demodulating signal at the receiver, then the output of the demodulator has a random phased signal. If there is no phase difference between jammer signal and demodulating signal then the output of the demodulator due to the jammer will be a constant. For non-coherent FSK we consider a one-dimensional form and a two-dimensional form of $\hat{j}(t)$, that is, for one-dimensional jamming $\hat{j}(t)$ is given by

$$\hat{j}(t) = \sqrt{2N_J/T}\ \cos\left[2\pi(f_c+b_j(t)\Delta f)t+\hat{\psi}(t)\right] \tag{1.4.5}$$

where Δf is the FSK frequency spacing in (1.3.3) and $\hat{\psi}(t)$ is a uniformly distributed random phase introduced by the jammer. For two-dimensional jamming $\hat{j}(t)$ is given by

$$\hat{j}(t)=\sqrt{N_J/T}\left\{\cos\left[2\pi(f_c+\Delta f)t+\hat{\psi}(t)\right]+\cos\left[2\pi(f_c-\Delta f)t+\overline{\psi}(t)\right]\right\} \tag{1.4.6}$$

where $\overline{\psi}(t)$ is another random phase. The output of the frequency-dehopper due to the jamming signal is of the form

$$j(t) = Z_{i,l}\ \hat{j}(t)\ , \qquad lT \le t < (l+1)T,\ -\infty < l < \infty \tag{1.4.7}$$

where i is such that $f_h(t)=f_i$, $lT \le t < (l+1)T$. The model we use for the jammer allows $Z_{i,l}$ to be a sequence of i.i.d. random variables with average power

$$E\left[Z_{i,l}^2\right] = 1 \tag{1.4.8}$$

where $E(X)$ is the expected value of X.

For a two level jammer the distribution for $Z_{i,l}$ is given by [HOU75], [MCE81]

$$\begin{aligned} &Pr\{Z_{i,l}=0\}=1-\rho \\ &Pr\{Z_{i,l}=\sqrt{1/\rho}\}=\rho\ , \qquad 0<\rho\le 1 \end{aligned} \tag{1.4.9}$$

where ρ is a constant. For a three level jammer the distribution for $Z_{i,l}$ is given by

$$\begin{aligned} &Pr\{Z_{i,l}=0\}=1-\rho_1-\rho_2 \\ &Pr\{Z_{i,l}=\sqrt{J_1}\}=\rho_1\ , \qquad 0<\rho_1\le 1 \\ &Pr\{Z_{i,l}=\sqrt{J_2}\}=\rho_2\ , \qquad 0<\rho_2\le 1 \end{aligned} \tag{1.4.10}$$

where ρ_1 and ρ_2 are constants, and $\sum_{k=1}^{2} J_k\rho_k=1$. For a four level jammer the distribution for $Z_{i,l}$ is given by

$$
\begin{aligned}
&Pr\{Z_{i,l}=0\}=1-\rho_1-\rho_2-\rho_3 \\
&Pr\{Z_{i,l}=\sqrt{J_1}\}=\rho_1\,, \qquad 0<\rho_1\leq 1 \\
&Pr\{Z_{i,l}=\sqrt{J_2}\}=\rho_2\,, \qquad 0<\rho_2\leq 1 \\
&Pr\{Z_{i,l}=\sqrt{J_3}\}=\rho_3\,, \qquad 0<\rho_3\leq 1
\end{aligned}
\tag{1.4.11}
$$

where ρ_1, ρ_2 and ρ_3 are constants, and $\sum_{k=1}^{3} J_k\rho_k=1$.

1.5. Performance Measure

The demodulators we consider will map the received waveform in the interval $[lT, (l+1)T)$ into a finite number of outputs. The output of the demodulator is the input to the decoder. Because of our assumption regarding $Z_i(t)$ the resulting channel will be memoryless with a finite number of inputs and outputs. Let us denote the channel input alphabet by $A=\{+1,-1\}$ and the channel output alphabet by $B=\{\beta_1,\beta_2, \cdots ,\beta_K\}$. In our model K is 2, 3 or 4. Let Z be a generic random variable with distribution identical to $Z_{i,l}$. Let the transition probability of the channel for $Z=z$ be denoted by $p(y \mid x,z)$ for $y \in B$, $x \in A$. Let the distribution of z be denoted by $P(z)$. The jammer chooses the distribution for Z subject to $E[Z^2]=J$. With the jammer distributing the power level the channel transition probabilities are

$$
p(y \mid x) = \int_{z=0}^{\infty} p(y \mid x,z)\, dP(z). \tag{1.5.1}
$$

One measure of performance that we are interested is the capacity of the channel when the decoder knows only the conditional transition probabilities $p(y \mid x,z)$, but not $P(z)$. This is known as a compound channel. Since the input and output alphabets are finite the capacity of the compound channel is given by [CSI81]

$$
C = \max_{X} \min_{P} I(X;Y) \tag{1.5.2}
$$

where $I(X;Y)$ is the mutual information between input X and output Y, the maximization is over all possible distributions on X and the minimization is over all possible distributions on the random variable Z. All channels considered are such that for every distribution P on the power of the jammer, the optimum input distribution on X is the uniform distribution. We will denote the mutual information with a uniform input distribution by $C(P,\lambda)$ where P is the jamming strategy and λ is the signal-to-noise ratio. With this definition (1.5.2) becomes

$$C = \min_{P} C(P,\lambda). \tag{1.5.3}$$

A code for a compound channel is a pair of mappings: $f: \mathfrak{M} \longrightarrow A^n$ and $g: B^n \longrightarrow \mathfrak{M}$ where $\mathfrak{M}=\{0, 1, \cdots, M-1\}$ is the message set. The rate of a code is $r=\frac{\log_2 M}{n}$ information bits/channel symbol.

The significance of the channel capacity C is given by a coding theorem for a compound channel [CSI81]. That theorem is the following : there exist codes of rate r with arbitrarily small error probability provided the code rate r is less than the channel capacity C, no matter which strategy $P(z)$ the jammer chooses.

In this thesis we are interested in determining the minimum energy needed for each information bit in order for reliable (arbitrarily small error probability) communication to be possible. Naturally this is a function of the rate of the code used. If we use codes of rate r then the relation between the symbol energy and the information bit energy, E_b, is given by

$$E_b = E_c/r \tag{1.5.4}$$

The capacity is a function of the symbol signal-to-noise ratio, $\lambda \triangleq E_c/N_J$. Reliable

communication is possible provided the code rate satisfies

$$r \leq C(\lambda) \tag{1.5.5}$$

or equivalently

$$\frac{E_b}{N_J} \geq \frac{C^{-1}(r)}{r} \tag{1.5.6}$$

where $C^{-1}(r)$ is the inverse function of $C(\lambda)$. The right hand side of (1.5.6) is the minimum bit signal-to-noise ratio necessary for reliable communication. Throughout this thesis we display figures showing the minimum bit signal-to-noise ratio needed to obtain the optimum code rate.

Another measure of performance of interest is the channel cutoff rate. This is defined as

$$R_0 = \max_X \min_P \left(-\log_2 \sum_{y \in B} \left[\sum_{x \in A} p(x) \sqrt{p(y \mid x)} \right]^2 \right). \tag{1.5.7}$$

The cutoff rate R_0 is considered as a practical limit to the set of rates for which reliable communication is possible [WOZ65]. As with channel capacity the maximum cutoff rate is obtained by a uniform distribution on the input X for most systems. We do not compute the cutoff rate of the system in this thesis. Many of the conclusions drawn based on the channel capacity will also hold with the cutoff rate. Our main focus is to determine the structure of the distribution of jamming levels Z which is optimal from the jammer's point of view (worst-case from the communicator's perspective). While it would be of interest to consider all distribution on Z with $E[Z^2] \doteq J$, we will only consider distribution concentrated on two, three and four levels.

For the numerical calculation of the channel capacity we have used VMCON[1] which is a nonlinear optimization package, developed at the Argonne National Laboratory, that can be used to solve constrained or unconstrained nonlinear programming problems. When we compared the numerical results using VMCON with that using brute force method, we found VMCON worked fine when the number of variables to be optimized is small. However, as the number of variables increases (i.e. four jammer levels) we could not compare the accuracy of VMCON with brute force methods because for brute force method the computing time increases exponentially as the number of variables of the jammer increases. However the numerical results obtained using VMCON for higher level jamming seemed to be reasonable. The results obtained using VMCON is an upper bound on the channel capacity or the signal-to-noise ratio computed using VMCON is a lower bound on signal-to-noise ratio. This is true since VMCON produces a feasible jamming strategy but there is no guarantee this is the optimal strategy.

1.6. Outline of the Thesis

Chapter 2 discusses worst-case jamming strategies when the communicator employs the coherent BPSK modulation scheme. The types of jamming noise are tone jamming, phase mismatched tone jamming and partial-band Gaussian jamming. The channel capacity under phase mismatched tone jamming is shown to be much better than that under tone jamming, and compare the above three jamming noise types.

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[1]VMCON is authored by R. L. Crane, K. E. Hillstrom, and M. Minkoff

Chapter 3 considers noncoherent BFSK. The jamming noise type considered is one-dimensional tone jamming and partial-band Gaussian jamming. The number of jammer levels are two, three and four. We show that higher level tone jamming degrades the performance significantly compared to lower level tone jamming. However for Gaussian jamming the performance of two, three and four level jamming is nearly the same.

Chapter 4 lists conclusions and suggests some future research topics.

CHAPTER II

CHANNELS WITH WORST-CASE JAMMING AND COHERENT DEMODULATION

2.1. Introduction

In this chapter we analyze a frequency-hopped spread-spectrum system with jamming. We will assume a binary signaling scheme, for which the modulation process corresponds to changing the phase of the carrier between one of two possible values corresponding to binary symbols +1 and -1. We assume the receiver is perfectly synchronized to the transmitter.

We investigate the performance of several different channels with different jamming strategies, namely, two level jamming, three level jamming and four level jamming. The limitation to the jammer is the average power. For a hard decision channel the worst-case (from the communicator's perspective) average-power-limited Gaussian jammer is known to pulse between two values, one of them being zero. Thus an on-off jammer is the worst possible jammer. The proof of this statement is in Appendix A. In [VIT82] Viterbi introduced a ratio threshold technique for mitigating interference in a spread-spectrum communication system with noncoherent demodulation. This technique used an additional channel output, called a quality bit, to improve the performance over a hard decision receiver. The quality bit is

obtained by quantizing the output of the matched filter into four levels. In this thesis we analyze systems with two, three or four level quantization detectors with two, three or four levels of jamming.

In section 2.2 the channel models and assumptions that we will make for this chapter are described. Section 2.3 introduces the channels with hard and soft decision detectors under two, three and four level tone jamming interference. The channel models we define allow for the jammer's signal to match one of two communicator's signals, i.e. there is no phase difference between jammer's signal and communicator's signal. In section 2.4, we show the performance of the system with random phase tone jamming. Unlike section 2.2, with random phase tone jamming there is uniformly distributed random phase term in the received signal. Both hard and soft decision detectors are considered. In section 2.5, we consider Gaussian jamming interference with mean 0 and variance depending upon the jammer's power distribution. Finally in section 2.6 we compare the performances of the three different worst-case jamming strategies introduced in sections 2.3, 2.4, and 2.5.

2.2. Channel Models

In this section we describe the channel models and assumptions that we will make in our analysis. The key assumption that we make is that there is a finite number of inputs (two) and a finite number of outputs of the channel (two, three or four). We model the jamming strategies as distributing the power level of a certain type of noise. The noise types discussed in this chapter are tone jamming, random phase tone jamming and Gaussian jamming.

Tone Jamming

We first discuss tone and random phase tone jamming signal. In this type of jamming the output of the correlation receiver in Fig. 2.2.1 due to the jamming signal given by (1.4.6) is given by

$$I = \int_0^T Z_{i,0}\, \hat{j}(t)\, \sqrt{2/E_c T}\, \cos\,(2\pi f_c t + \psi)\, dt \tag{2.2.1}$$

$$= \int_0^T Z_{i,0}\, V_0\, \sqrt{2N_J/T}\, \sqrt{2/E_c T}\, \cos\,(2\pi f_c t + \psi)\, \cos\,(2\pi f_c t + \hat{\phi})\, dt$$

$$= Z_{i,0}\, \sqrt{N_J/E_c}\, V_0 \cos\phi$$

where $\phi = \psi - \hat{\phi}$ is a random phase and $Z_{i,0}$ is a random variable with average power 1 given by (1.4.7). In deriving (2.2.1) we have assumed the frequency-hopped signal is in the i-th slot during $0 \leq t < T$.

For tone jamming with perfect phase information ($\psi = \hat{\phi}$) we let $N = V_0$ in (2.2.1) and let $Z = Z_{i,0}\, \sqrt{N_J/E_c}$. The additive noise random variable I given by (2.2.1) then is of the form

$$I = Z\, N \tag{2.2.2}$$

with

$$E[N^2] = 1 \tag{2.2.3}$$

and

$$\begin{aligned} E[Z^2] &= E[Z_{i,0}^2]\, N_J/E_c \\ &= 1/\lambda \end{aligned} \tag{2.2.4}$$

where $\lambda = E_c/N_J$.

For random phase tone jamming in (2.2.1) we let $N=\cos\phi$ and let $Z=Z_{i,0}\sqrt{N_J/E_c}$. The random variable I is then of the form

$$I = Z\ N \tag{2.2.5}$$

with

$$E[N^2]=1/2 \tag{2.2.6}$$

and

$$E[Z^2] = 1/\lambda. \tag{2.2.7}$$

Gaussian Jamming

For Gaussian type of jamming, $\hat{j}(t)$ in (1.4.1) is a Gaussian process with power spectral density $N_J/2$ centered at f_c over bandwidth $\frac{W}{q}$. The noise random variable I is then given by

$$\begin{aligned} I &= \int_0^T Z_{i,0}\sqrt{2/E_c T}\ \cos 2\pi f_c t\ \hat{j}(t)\ dt \\ &= Z_{i,0}\sqrt{2/E_c T}\int_0^T \hat{j}(t)\cos 2\pi f_c t\ dt \\ &= Z_{i,0}\sqrt{2/E_c T}\ \sqrt{N_J T/4}\left(\sqrt{4/N_J T}\int_0^T \hat{j}(t)\cos 2\pi f_c t\ dt\right). \end{aligned} \tag{2.2.8}$$

In (2.2.8) we let $N=\sqrt{4/N_J T}\int_0^T \hat{j}(t)\cos 2\pi f_c t\ dt$ and let $Z=Z_{i,0}\sqrt{2/E_c T}\ \sqrt{N_J T/4}$. The noise random variable I is then of the form

$$I = Z\ N \tag{2.2.9}$$

where N is Gaussian random variable with mean 0 and variance 1, and Z is a random variable with average power $E[Z^2]=1/2\lambda$. For calculating I discussed above

we have assumed that $f_c T$ is integer.

The jamming strategies consist of all distributions on Z with an average power $E[Z^2]$. In a frequency-hopped spread-spectrum system the distribution on Z corresponds to the distribution of the power levels in different frequency slots.

As mentioned in chapter 1 we are considering a channel with a finite number of inputs and a finite number of outputs. Denote the channel input alphabet by $A=\{+1,-1\}$ and the channel output alphabet by $B=\{\beta_1,\beta_2,\cdots,\beta_K\}$, where $K=2$, 3, or 4. For two level quantization $\beta_1=+1$ and $\beta_2=-1$. For three level quantization $\beta_1=+1$, $\beta_2=?$ and $\beta_3=-1$. For four level quantization $\beta_1=+1$,good, $\beta_2=+1$,bad, $\beta_3=-1$,bad and $\beta_4=-1$,good. Let the transition probability of the channel for $Z=z$ be denoted by $p(y \mid x,z)$ for $y \in B$, $x \in A$. Let the distribution of Z be denoted by $P(z)$. With the jammer distributing the power, the channel transition probabilities are

$$p(y \mid x) = \int_{z=0}^{\infty} p(y \mid x,z)\, dP(z) \,. \tag{2.2.10}$$

The performance measure that we are interested in finding is the capacity of the channel, as described in chapter 1, when the decoder knows only the conditional transition probabilities $p(y \mid x,z)$, but not $P(z)$. Our main focus is to determine the structure of the distribution Z which is optimal from the jammer's point of view (worst-case from the communicator's perspective).

Now we discuss three different channel models mentioned earlier. The input to the channel will be binary and the output will be binary (hard decisions), ternary or quarternary. The three types of interference considered in section 2.3, 2.4 and 2.5 are: (1) Tone jamming (no phase error), (2) Tone jamming with uniformly

distributed phase error, and (3) Gaussian jamming. In each case we consider jamming strategies which concentrate the total jamming power on one of two, three or four levels to degrade the performance.

The channel model is shown in Fig. 2.2.1. Since we are considering a memoryless compound channel with finite input and output, we need only calculate the transition probabilities for the channel. To this end consider input signals and output signals in the interval $[0, T)$. The input to the channel is the random variable X_0 and the output of the correlation receiver is the random variable

$$R = X_0 + I \tag{2.2.11}$$

where $I = ZN$ is noise random variable discussed in chapter 1.

The quantized output, Y, is

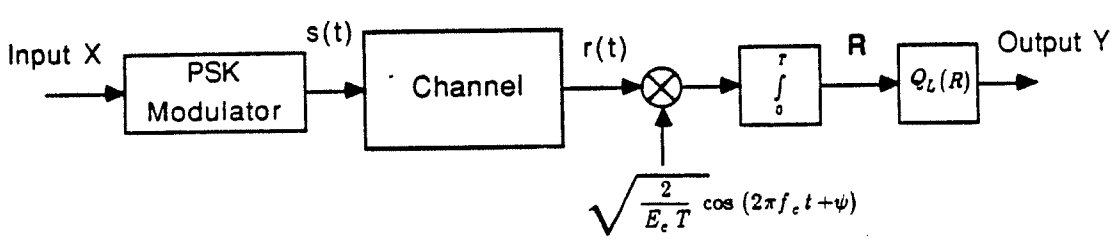


Figure 2.2.1. Channel model for coherent system.

$$\begin{aligned} Y &= Q_L(X_0+I) \\ &= Q_L(X_0+N\cdot Z) \end{aligned} \tag{2.2.12}$$

where Q_L is a L level quantizer,

$$Pr\ (\ X_0=1\ ) = Pr\ (\ X_0=-1\ ) = 1/2. \tag{2.2.13}$$

In (2.2.12) N is a symmetric ($Pr\{N<n\} = Pr\{N>-n\}$) random variable depending upon the type of jammer. N is a constant, the cosine of a uniformly distributed phase, or a Gaussian random variable. The two level quantizer is given by

$$Q_2(R) = \begin{cases} 1 & R>0 \\ -1 & R\leq 0. \end{cases} \tag{2.2.14}$$

With this quantization the channel is a binary symmetric channel with crossover probability $p = Pr\{R>0 \mid X_0=-1\}$. The capacity of the channel is then

$$C = 1-H_2(p) \tag{2.2.15}$$

where $H_2(x)=-x\log_2 x-(1-x)\log_2(1-x)$ is the binary entropy function. The three level quantizer is given by

$$Q_3(R) = \begin{cases} 1 & R>\theta \\ ? & -\theta\leq R<\theta \\ -1 & R\leq -\theta \end{cases} \tag{2.2.16}$$

where $0\leq\theta\leq 1$. The transition probabilities for the resulting channel will be denoted as follows:

$$p(y \mid x) = \begin{cases} p_a & x=1,\ y=1 \text{ or } x=-1,\ y=-1 \\ p_b & x=1,\ y=? \text{ or } x=-1,\ y=? \\ p_c=1-p_a-p_b & x=1,\ y=-1 \text{ or } x=-1,\ y=1. \end{cases} \tag{2.2.17}$$

The resulting capacity is given by

$$C = p_a \log_2 p_a + p_c \log_2 p_c - (p_a + p_c)\log_2(p_a + p_c) + p_a + p_c . \tag{2.2.18}$$

The four level quantizer is given by

$$Q_4(R) = \begin{cases} 1, \text{ good} & R > \theta \\ 1, \text{ bad} & 0 < R \leq \theta \\ -1, \text{ bad} & -\theta < R \leq 0 \\ -1, \text{ good} & R \leq -\theta \end{cases} \tag{2.2.19}$$

where $0 \leq \theta \leq 1$. The transition probabilities induced from the four level quantizer will be denoted as follows:

$$p(y \mid x) = \begin{cases} p_a & x=1,\ y=1, \text{ good} \\ p_b & x=1,\ y=1, \text{ bad} \\ p_c & x=1,\ y=-1, \text{ bad} \\ p_d & x=1,\ y=-1, \text{ good.} \end{cases} \tag{2.2.20}$$

The transition probabilities for $x=-1$ can be obtained by noting that $Pr(Y=y \mid X_0=-1) = Pr(Y=-y \mid X_0=1)$. The channel capacity for four level quantization is given by

$$\begin{aligned} C = {} & p_a \log_2 p_a + p_b \log_2 p_b + p_c \log_2 p_c + p_d \log_2 p_d + 1 \\ & -(p_a + p_d)\log_2(p_a + p_d) - (p_b + p_c)\log_2(p_b + p_c). \end{aligned} \tag{2.2.21}$$

The goals of this thesis then are to minimize the capacity over all distributions on Z concentrated on two, three and four values. The capacity depends on the distribution of Z through the transition probabilities. Because of the nonlinear nature of C our minimization is done mainly using a nonlinear programming package.

2.3. Tone Jamming

In tone jamming, since we assume that there is no phase term in (2.2.1), random variable I is given by $I = Z\,N$ where $N = V_0$ (i.e. +1 or -1) and $Z = Z_{i,0}\sqrt{N_J/E_c}$. N has power $E[N^2]=1$ and Z has the distribution $P(z)$ with the average power $1/\lambda$. The jamming strategies are then distributions on Z such that $E[Z^2]=1/\lambda$.

2.3.1. Two Level Jamming

Three different quantizers under two level jamming environment are considered. For a hard decision quantizer the worst-case jamming strategy is to concentrate Z on two or fewer power levels; one just large enough to overcome the signal and the other zero. It is easy to see that a jammer using any two power levels, one of which is greater than 1, can be replaced with a jammer with power levels 0 and 1 without sacrificing performance or average power. Thus the optimal jamming strategy for hard decision receiver is to concentrate his power on at most two levels.

Two level jamming, two level quantization

With two level quantization the conditional transitional probabilities are given by

$$\begin{aligned} Pr(Y=1 \mid X=-1,Z) &= Pr(Y=-1 \mid X=1,Z) \\ &= Pr\{1+NZ \leq 0\} \\ &= 1/2\; u(Z-1) \end{aligned} \tag{2.3.1.1}$$

where $u(x)$ is one if $x \geq 0$ and is zero otherwise. The worst-case distribution the jammer employs is then

$$Pr\{Z=z\} = \begin{cases} 1 & z=\sqrt{1/\lambda} \\ 0 & \text{otherwise} \end{cases} \tag{2.3.1.2}$$

for $\lambda \leq 1$ and

$$Pr\{Z=z\} = \begin{cases} 1/\lambda & z=1 \\ 1-1/\lambda & z=0 \end{cases} \tag{2.3.1.3}$$

for $\lambda > 1$. The error probability p is given by

$$p = \begin{cases} 0.5 & \lambda \leq 1 \\ 0.5/\lambda & \lambda > 1. \end{cases} \tag{2.3.1.4}$$

The channel capacity is then given by

$$C = 1 - H_2(p) = \begin{cases} 0 & \lambda \leq 1 \\ 1-H_2(1/2\lambda) & \lambda > 1 \end{cases} \tag{2.3.1.5}$$

where $H_2(x) = -x\log_2 x - (1-x)\log_2(1-x)$. Fig. 2.3.1.1 shows the channel capacity as a function of λ. The bit signal-to-noise ratio needed to obtain the code rate r for a hard decision receiver is shown in Fig. 2.3.1.2. There are several remarks to make concerning the curve in Fig. 2.3.1.2. These remarks also apply to most of the latter figures as well. First the interpretation is that for information bit signal-to-noise ratio E_b/N_J above the curve reliable communication is possible while below the curve reliable communication is impossible. The fact that there is an optimal code rate is a result of the following: First, as the code rate decreases the error correcting capability of the code is increasing so the E_c/N_J necessary for reliable communication decreases. However, this offset by the fact that there are more redundant symbol using energy E_c. For this reason the curve in Fig. 2.3.1.2 has a minimum. It should be noted that not all such curves will exhibit a minimum as seen in Fig. 2.3.1.2. The existence of an optimal code rate (>0) is determined by the behavior of

the capacity as a function of the channel signal-to-noise ratio.

Two level jamming, three level quantization

For two level jamming and three level quantization the optimum jamming strategies can also be easily determined. To proceed we consider several ranges of the signal-to-noise ratio. For each range we determine the optimum jamming strategy and the resulting channel capacity. We start by considering small values of λ, where λ is the channel symbol signal-to-noise ratio.

For $\lambda \leq 1/(1+\theta)^2$ the jammer has enough power so that the strategy $Z=1+\theta$ (with probability one) is a feasible strategy ($E[Z^2] \leq 1/\lambda$). For this strategy, the resulting channel is a binary symmetric channel with error probability 1/2 (N is equally likely to be +1 or -1). The resulting capacity is zero.

For $1/(1+\theta)^2 < \lambda < 1/(1-\theta)^2$ the jammer no longer has enough power to cause an error with probability one half. The possible jamming strategies are

$$Z = \begin{cases} 1+\theta & w.p. \quad 1/\lambda(1+\theta)^2 \\ 0 & w.p. \quad 1-1/\lambda(1+\theta)^2 \end{cases} \tag{2.3.1.6}$$

or

$$Z = \begin{cases} 1-\theta & w.p. \quad 1 \\ 0 & w.p. \quad 0. \end{cases} \tag{2.3.1.7}$$

The first strategy results in a binary symmetric channel with crossover probability $1/(2\lambda(1+\theta)^2)$ and thus the capacity is $1-H_2(1/(2\lambda(1+\theta)^2))$. The second strategy results in a binary symmetric erasure channel with transition probability one half. The capacity for the second strategy is one half. The capacity in this region is then the minimum of the two capacities

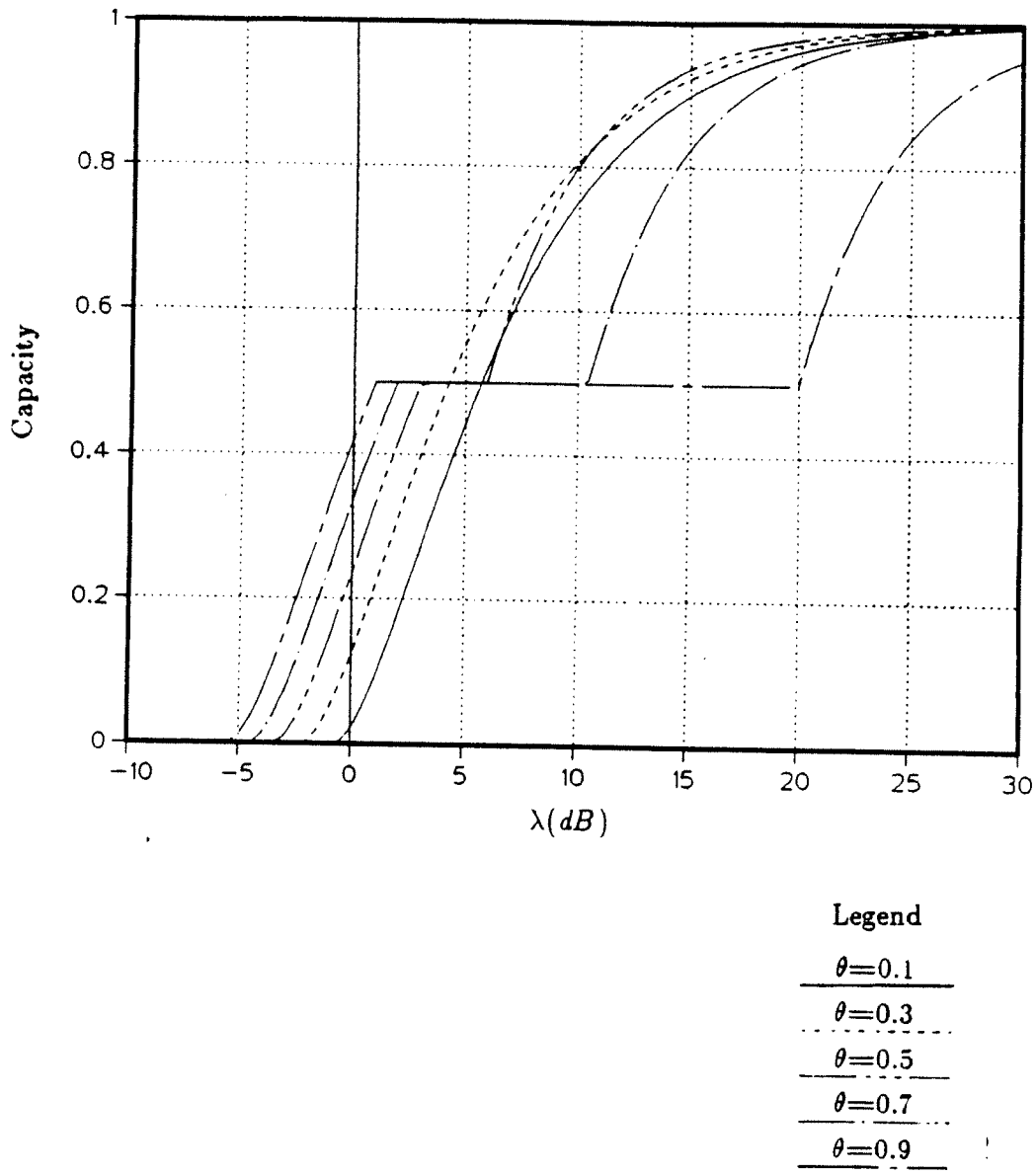


Figure 2.3.1.1. Channel capacity vs. channel symbol signal-to-noise ratio in two level tone jamming and three level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

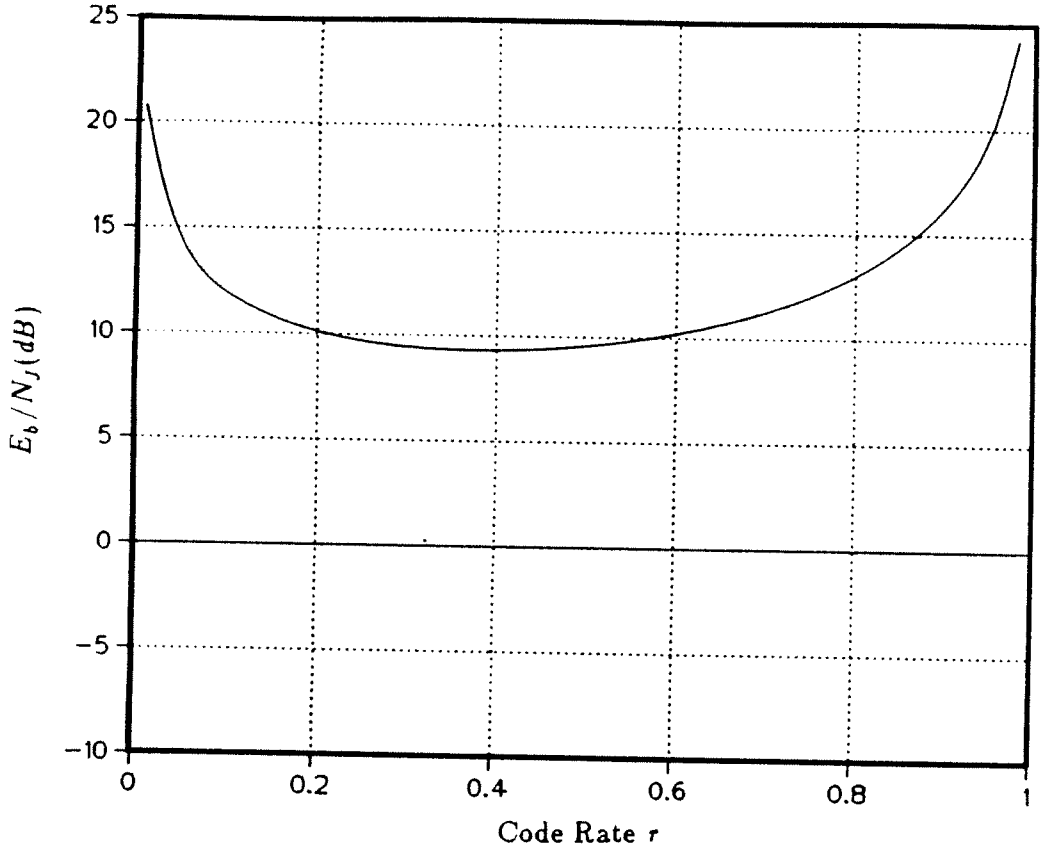


Figure 2.3.1.2. E_b/N_J needed for reliable communication at code rate r in two level tone jamming and hard decision system.

$$C = \min(1/2,\ 1-H_2(1/2\lambda(1+\theta)^2)). \tag{2.3.1.8}$$

It is easy to see that for $p > 0.11$, $1-H_2(p) < 1/2$ so that if $1/2\lambda(1+\theta)^2 < 0.11$ the capacity will be 1/2. Thus for $1/(1+\theta)^2 < \lambda \leq 1/(1-\theta)^2$ the capacity is given by

$$C = \begin{cases} 1-H_2(1/2\lambda(1+\theta)^2) & 1/(1+\theta)^2 < \lambda \leq 4.545/(1+\theta)^2 \\ 1/2 & 4.545/(1+\theta)^2 < \lambda \leq 1/(1-\theta)^2. \end{cases} \tag{2.3.1.9}$$

For $\lambda > 1/(1-\theta)^2$ the two feasible jamming strategies are

$$Z = \begin{cases} 1+\theta & w.p.\ \ 1/\lambda(1+\theta)^2 \\ 0 & w.p.\ \ 1-1/\lambda(1+\theta)^2 \end{cases} \tag{2.3.1.10}$$

or

$$Z = \begin{cases} 1-\theta & w.p.\ \ 1/\lambda(1-\theta)^2 \\ 0 & w.p.\ \ 1-1/\lambda(1-\theta)^2. \end{cases} \tag{2.3.1.11}$$

The resulting two capacities are $C = 1-H_2(1/2\lambda(1+\theta)^2)$ and $C = 1-1/2\lambda(1-\theta)^2$ respectively. Thus in general the overall channel capacity is given by

$$C = \begin{cases} 0 & 0 \leq \lambda \leq 1/(1+\theta)^2 \\ 1-H_2(1/2\lambda(1+\theta)^2) & 1/(1+\theta)^2 < \lambda \leq 4.545/(1+\theta)^2 \\ 1/2 & 4.545/(1+\theta)^2 < \lambda \leq 1/(1-\theta)^2 \\ \min(1-1/2\lambda(1-\theta)^2,\ 1-H_2(1/2\lambda(1+\theta)^2)) & \lambda > 1/(1-\theta)^2. \end{cases} \tag{2.3.1.12}$$

Notice for $\theta < 0.36$, $4.545/(1+\theta)^2 > 1/(1-\theta)^2$ so that the third region in (2.3.1.12) is empty. For $\theta > 0.36$ this region is nonempty and the capacity is a constant independent of λ in this region. Notice also that as a function of λ the capacity is not a concave function. This is due to our restriction of not letting the transmitter use a distribution on the power subject to an average power constraint other than constant power for all channel symbols. If the transmitter could pulse (as the jammer does) between various power levels, the capacity as a function of λ would be a con-

cave function. Figure 2.3.1.3 shows the minimum bit signal-to-noise ratio necessary for reliable communication as a function of code rate given $\theta = 0.1$, 0.3, 0.5, 0.7 and 0.9. Fig. 2.3.1.4 shows the minimum bit signal-to-noise ratio needed to obtain code rate when the optimal θ is chosen. In Fig. 2.3.1.3 we see that for large θ, a very small jammer power can cause an erasure.

Two level jamming, four level quantization

For four level quantization the channel capacity can be calculated in a similar fashion. First if the jammer has enough power so that $Z=1+\theta$ with probability one is a feasible strategy then this is the optimal strategy and the capacity is zero. This will be the case when $\lambda \leq 1/(1+\theta)^2$. To determine the optimal strategies in other cases we note that for the channel considered the channel capacity is one if $p_b=p_d=0$ or $p_c=p_d=0$ (see (2.2.21)). Thus since the jammer can only choose one of p_b, p_c, p_d to be nonzero the capacity will be less than one only if the jammer can make $p_d>0$. This is possible by using the following strategy:

$$Z = \begin{cases} 1+\theta & w.p. \quad 1/\lambda(1+\theta)^2 \\ 0 & w.p. \quad 1-1/\lambda(1+\theta)^2. \end{cases} \tag{2.3.1.13}$$

The resulting capacity is that of a binary symmetric channel with crossover probability $1/2\lambda(1+\theta)^2$:

$$C = 1-H_2(1/2\lambda(1+\theta)^2). \tag{2.3.1.14}$$

Thus the capacity is given by

$$C = \begin{cases} 0 & \lambda \leq 1/(1+\theta)^2 \\ 1-H_2(1/2\lambda(1+\theta)^2) & \lambda > 1/(1+\theta)^2. \end{cases} \tag{2.3.1.15}$$

Notice that the capacity is at least as large as the capacity of three level

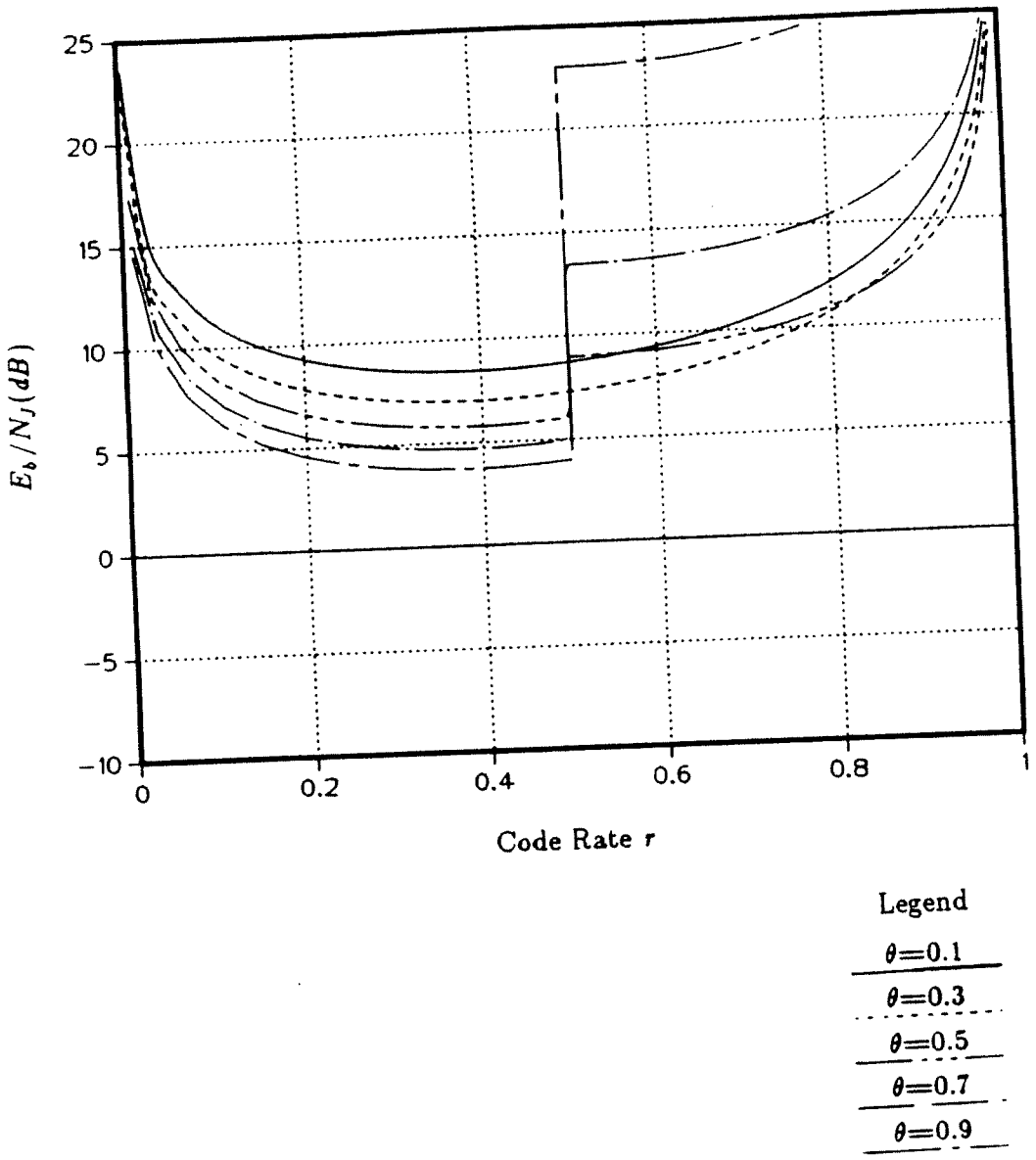


Figure 2.3.1.3. E_b/N_J needed for reliable communication at code rate r in two level tone jamming and three level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

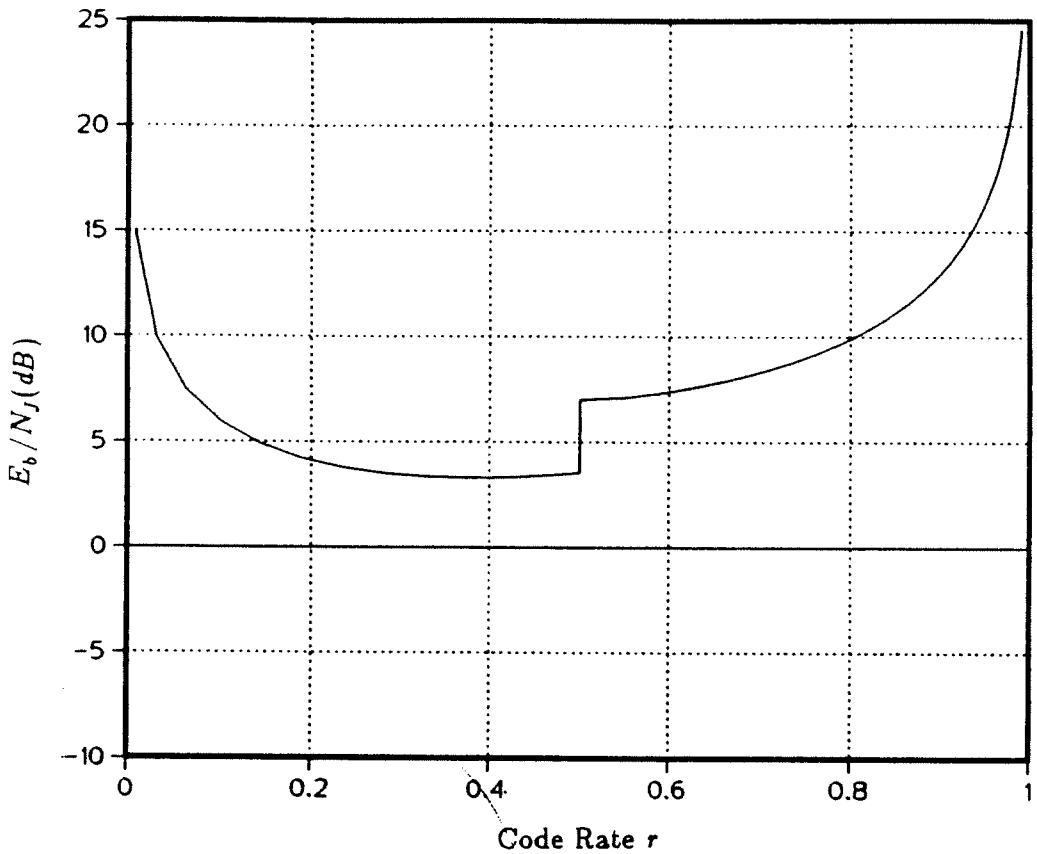


Figure 2.3.1.4. E_b/N_J needed for reliable communication at code rate r in two level tone jamming and three level quantization system with optimal θ.

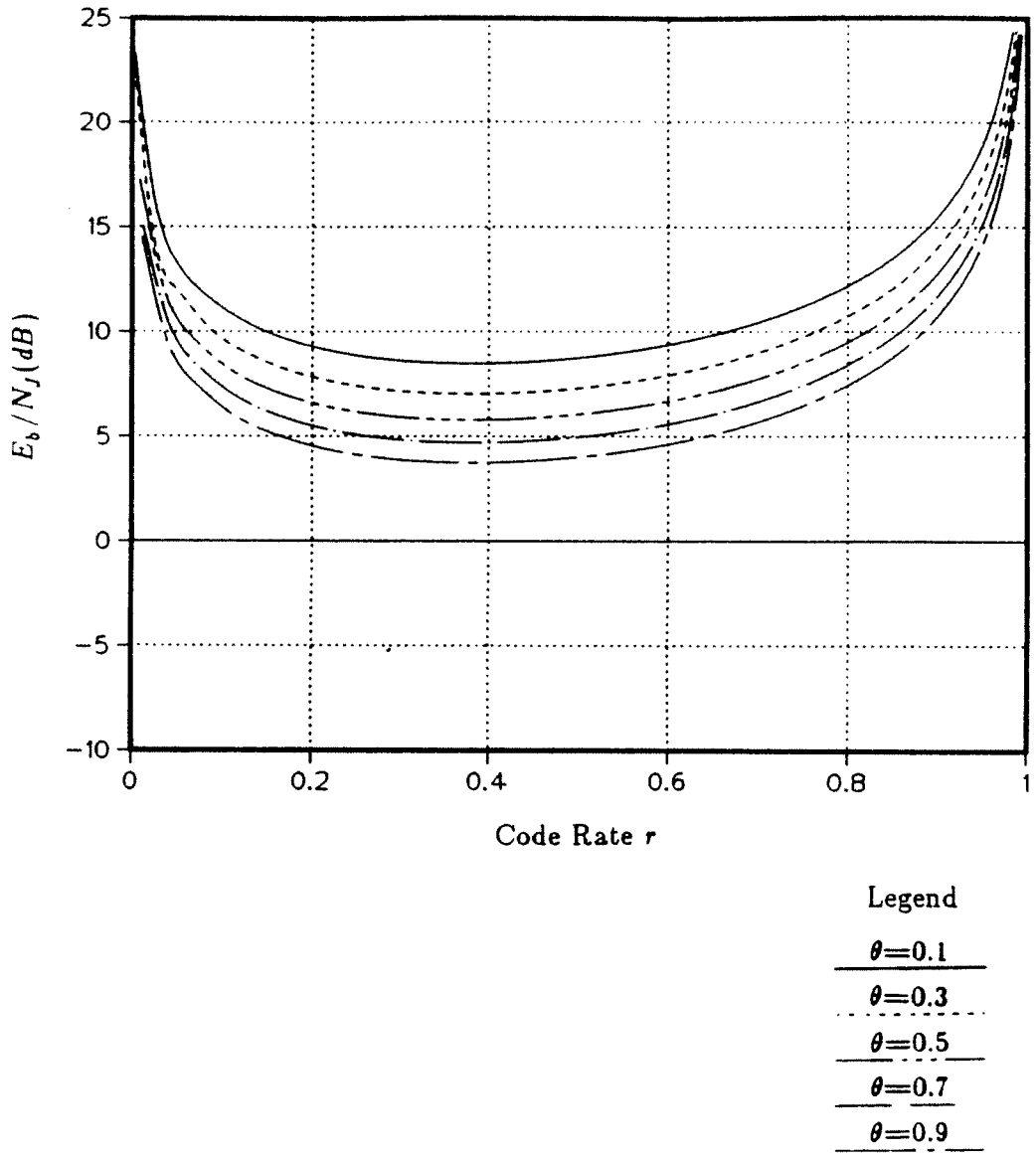


Figure 2.3.1.5. E_b/N_J needed for reliable communication at code rate r in two level tone jamming and four level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

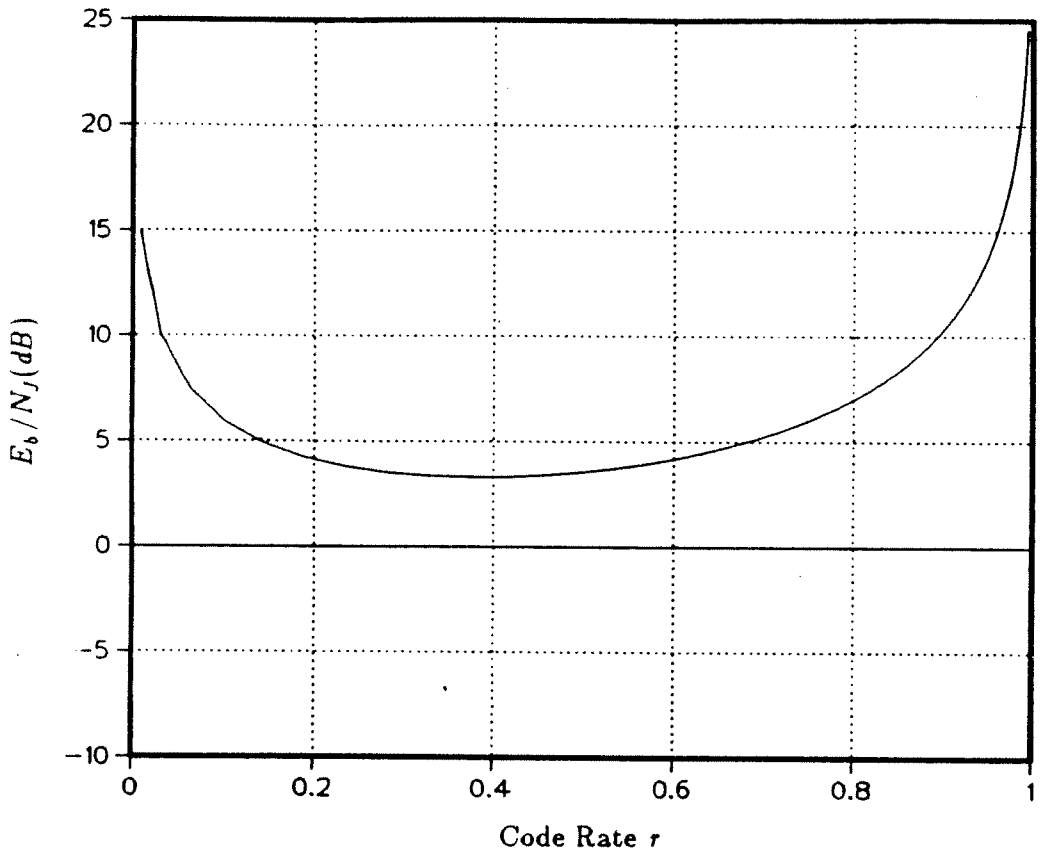


Figure 2.3.1.6. E_b/N_J needed for reliable communication at code rate r in two level tone jamming and four level quantization system with optimal θ.

quantization. Fig. 2.3.1.5 shows the minimum bit signal-to-noise ratio needed to obtain code rate for $\theta = 0.1$, 0.3, 0.5, 0.7 and 0.9. From the communicator's view point, he must have θ as large as possible in order to obtain optimum performance. For example, in Fig. 2.3.1.5, at rate $1/2$, the system with θ=0.9 is about $4.5dB$ better than that with θ=0.1. Fig. 2.3.1.6 shows the performance of the system with optimal θ.

2.3.2. Three Level Jamming

In this section we will discuss the three level jamming strategy. Three and four level quantization detectors are considered for analysis. The optimal strategy for the jammer is to concentrate Z on three or fewer power levels: one zero and the others (one or two) just large enough to overcome the signals.

Three level jamming, three level quantization

With three level quantization the optimal distribution of Z is concentrated at three points, namely zero, $1-\theta$ and $1+\theta$.

For $\lambda \leq 1/(1+\theta)^2$ the jammer has enough power so that $Z=1+\theta$ with probability one. This strategy makes the channel capacity zero. For $\lambda > 1/(1+\theta)^2$ the distribution of Z is given by

$$Pr\{Z=z\} = \begin{cases} \rho_1 & z=1-\theta \\ \rho_2 & z=1+\theta \\ 1-\rho_1-\rho_2 & z=0 \end{cases} \tag{2.3.2.1}$$

where ρ_1 and ρ_2 are probabilities being jammed corresponding to $1-\theta$ and $1+\theta$ respectively. The transition probabilities p_a, p_b and p_c are as follows :

$$\begin{aligned} p_a &= 1-0.5(\rho_1+\rho_2) \\ p_b &= 0.5\rho_1 \\ p_c &= 0.5\rho_2. \end{aligned} \qquad \textbf{(2.3.2.2)}$$

The channel capacity is given by

$$C = p_a \log_2 p_a + p_c \log_2 p_c - (p_a + p_c)\log_2(p_a + p_c) + p_a + p_c. \qquad (2.3.2.3)$$

Optimum ρ_1 and ρ_2 are computed numerically. In Fig. 2.3.2.1 the numerical results are given. For small θ (e.g. θ=0.1) the worst-case ρ_1 is zero and ρ_2=1. In Fig. 2.3.2.2 we show the minimum E_b/N_J needed to obtain code rate r given $\theta = 0.1$, 0.3, 0.5, 0.7 and 0.9, and Fig. 2.3.2.3 shows the best performance of the system with three level jamming and three level quantization. For θ=0.5, at rate 1/ 4 three level jamming strategy requires $2.4dB$ larger information bit signal-to-noise ratio than compared to two level jamming and three level quantization. However, at rate 3/ 4, the performance of these two systems is almost same. When the optimal θ is chosen, at rate 1/4, the performance degradation compared to two level jamming strategy is about 3.9 dB. However as the code rate increases, the degradation becomes almost zero.

Three level jamming, four level quantization

With four level quantization and three level jamming the distribution of Z is concentrated on three levels. These values are in the set $\{0, 1-\theta, 1, 1+\theta\}$. We assume z_0=0 is one of the levels. Let $p_a(z)$, $p_b(z)$, $p_c(z)$ and $p_d(z)$ be transition probabilities conditioned on $Z=z$. Let ρ_1 be the probability of $Z=z_1$ and let ρ_2 be the probability of $Z=z_2$. Then the transition probabilities conditioned on $Z=z$ are given by

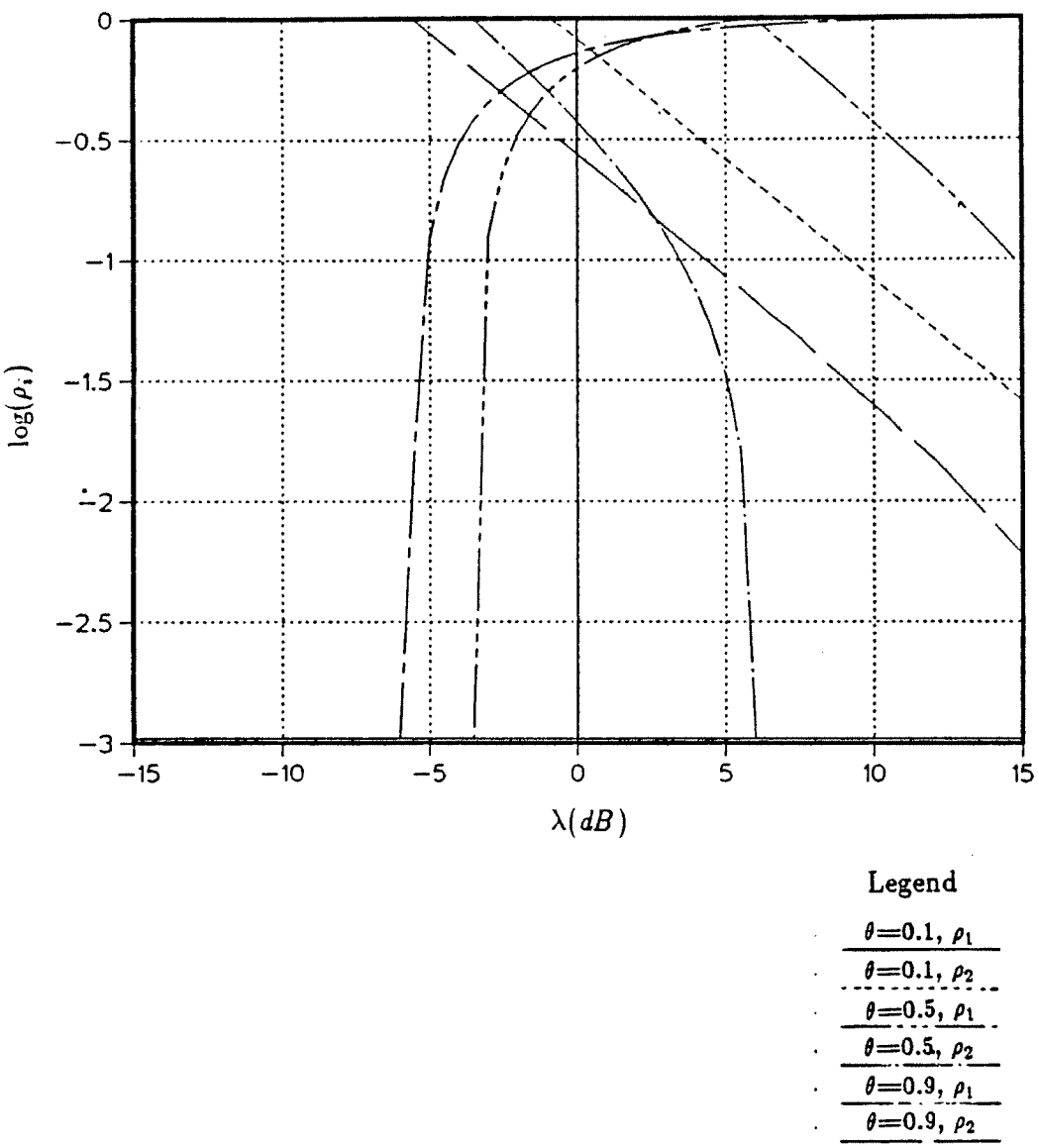


Figure 2.3.2.1. Optimal probabilities ρ_1 and ρ_2 in three level tone jamming and three level quantization system with θ=0.1, 0.5, and 0.9.

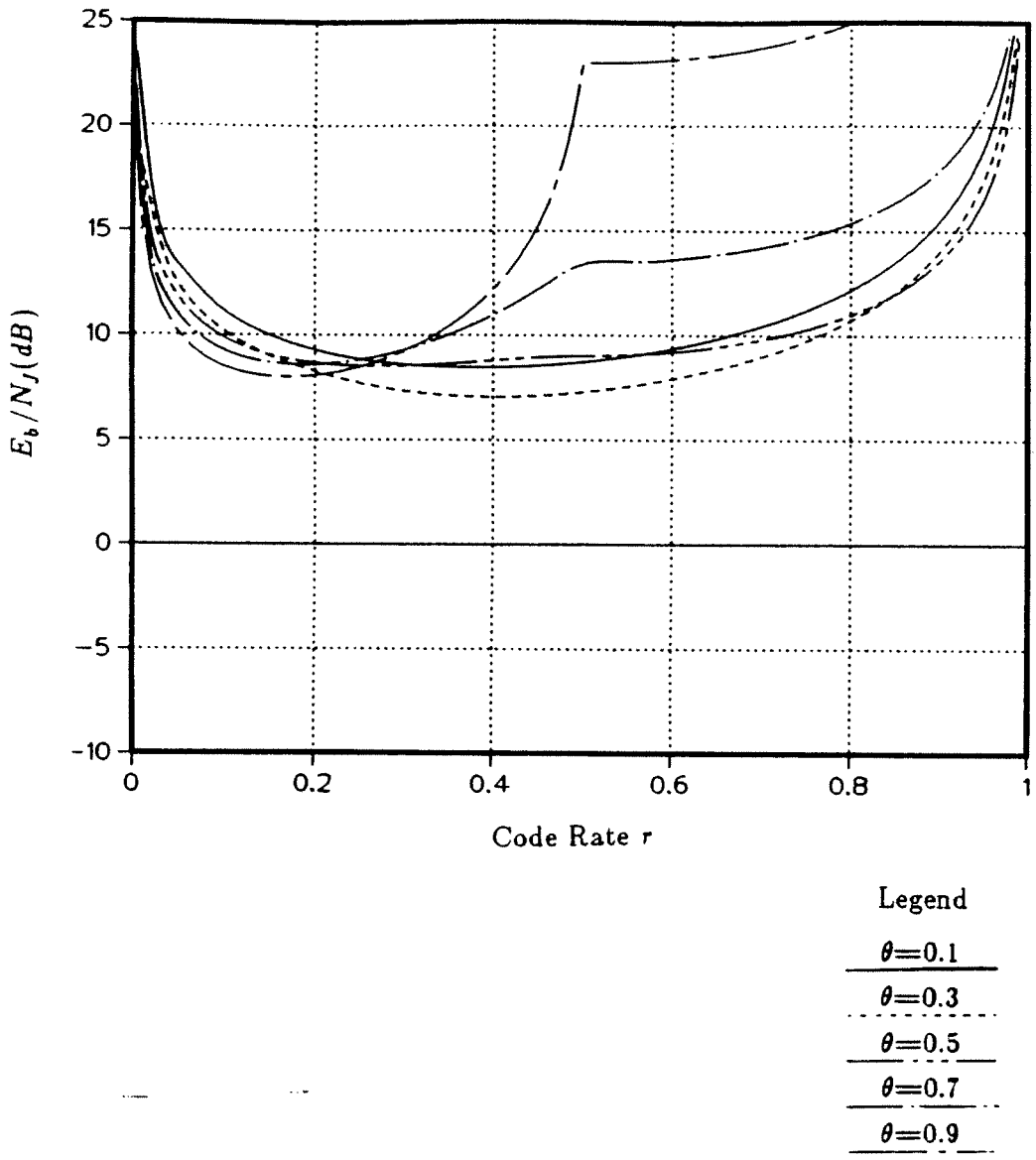


Figure 2.3.2.2. E_b/N_J needed for reliable communication at code rate r in three level tone jamming and three level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

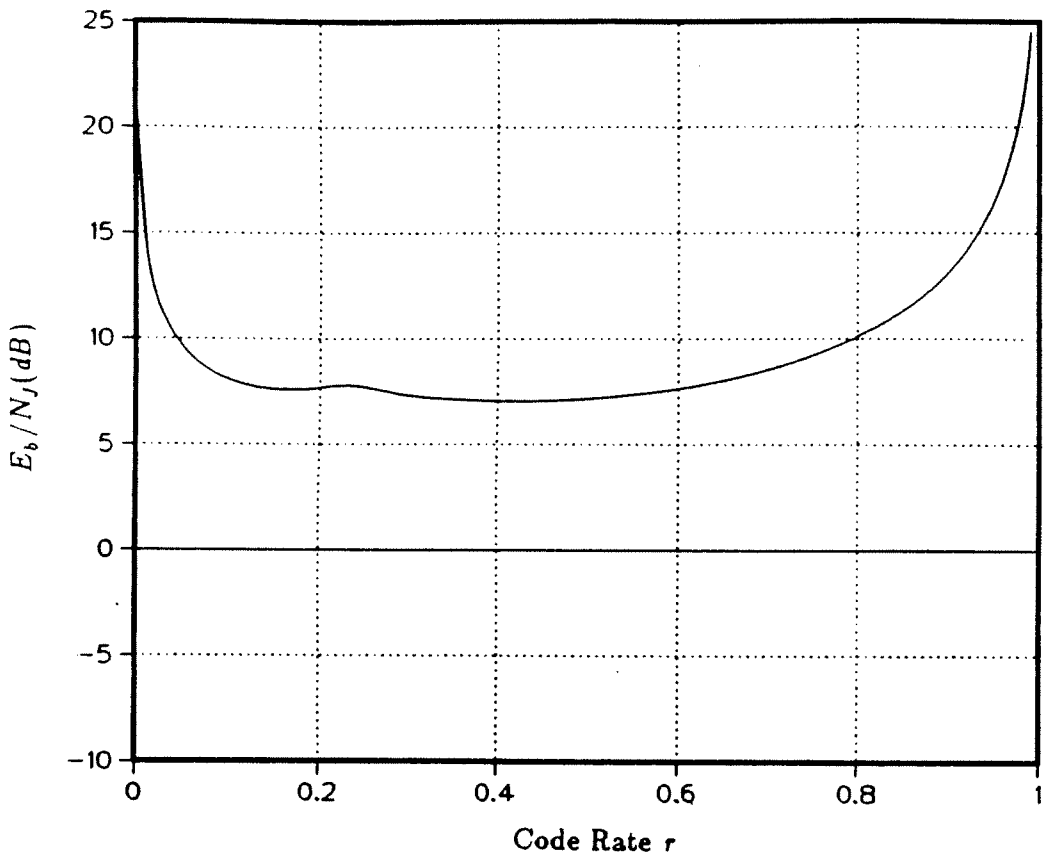


Figure 2.3.2.3. E_b/N_J needed for reliable communication at code rate r in three level tone jamming and three level quantization system with optimal θ.

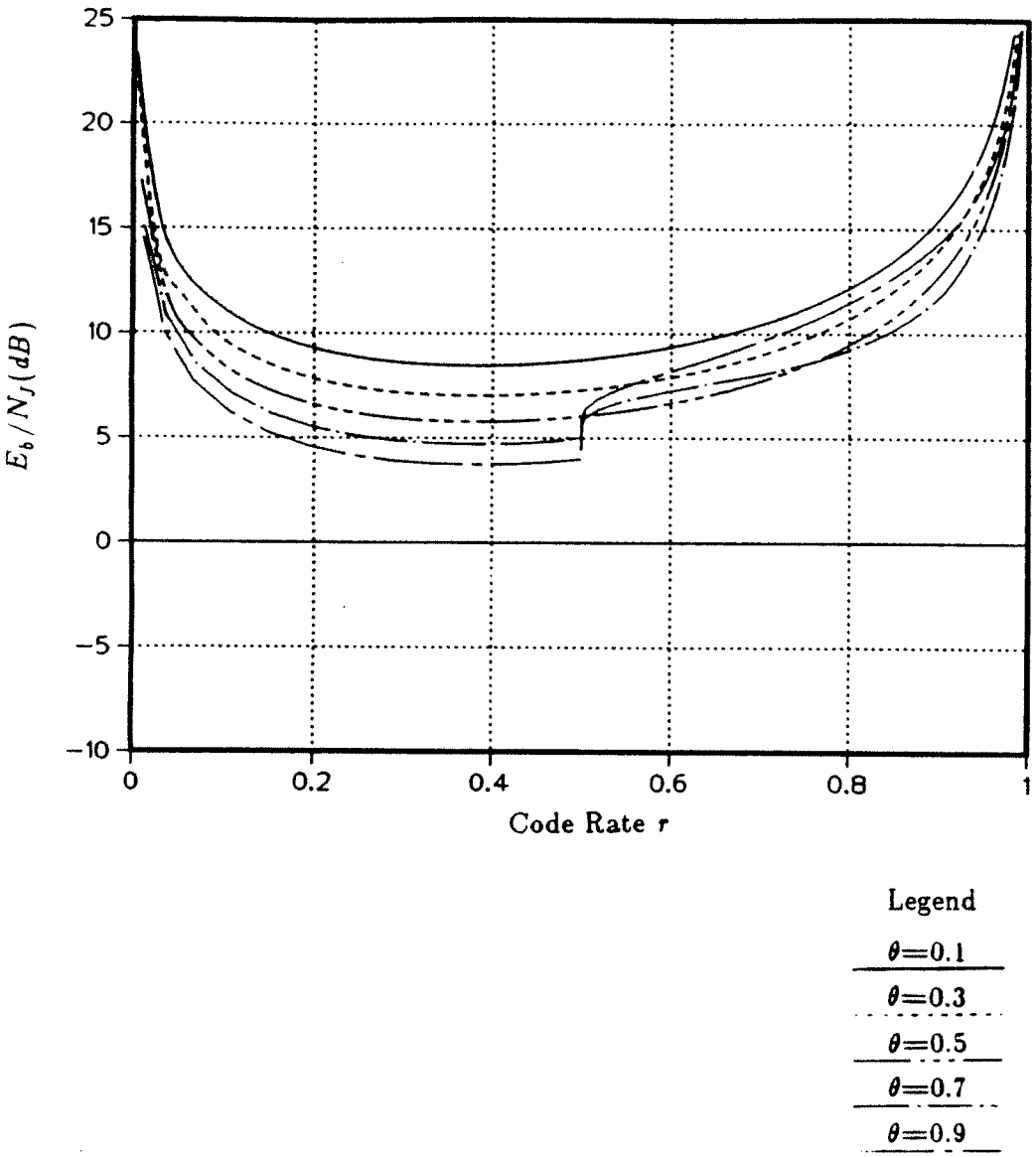


Figure 2.3.2.4. E_b/N_J needed for reliable communication at code rate r in three level tone jamming and four level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

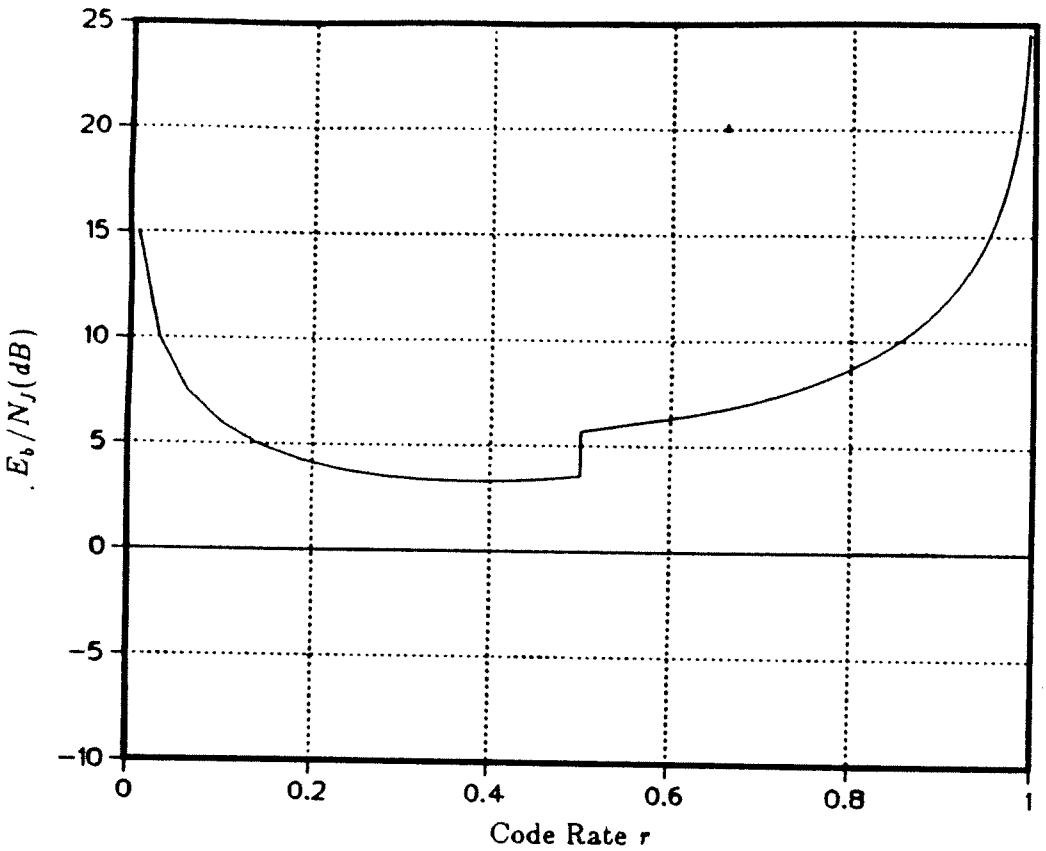


Figure 2.3.2.5. E_b/N_J needed for reliable communication at code rate r in three level tone jamming and four level quantization system with optimal θ.

(1) If $z \in [1-\theta, 1)$

$$\begin{aligned} p_a(z) &= 0.5 \\ p_b(z) &= 0.5 \\ p_c(z) &= 0 \\ p_d(z) &= 0. \end{aligned} \tag{2.3.2.4}$$

(2) If $z \in [1, 1+\theta)$

$$\begin{aligned} p_a(z) &= 0.5 \\ p_b(z) &= 0 \\ p_c(z) &= 0.5 \\ p_d(z) &= 0. \end{aligned} \tag{2.3.2.5}$$

(3) If $z \in [1+\theta, \infty)$

$$\begin{aligned} p_a(z) &= 0.5 \\ p_b(z) &= 0 \\ p_c(z) &= 0 \\ p_d(z) &= 0.5. \end{aligned} \tag{2.3.2.6}$$

The transition probabilities are then given by

$$\begin{aligned} p_a &= \sum_{i=1}^{2} p_a(z_i)\rho_i + \rho_0 \\ p_b &= \sum_{i=1}^{2} p_b(z_i)\rho_i \\ p_c &= \sum_{i=1}^{2} p_c(z_i)\rho_i \\ p_d &= \sum_{i=1}^{2} p_d(z_i)\rho_i . \end{aligned} \tag{2.3.2.7}$$

The channel capacity is given by

$$\begin{aligned} C = {} & p_a \log_2 p_a + p_b \log_2 p_b + p_c \log_2 p_c + p_d \log_2 p_d + 1 \\ & -(p_a + p_d)\log_2(p_a + p_d) - (p_b + p_c)\log_2(p_b + p_c). \end{aligned} \tag{2.3.2.8}$$

The numerical results for this system are shown in Fig. 2.3.2.4 for different values of

θ . For small θ, the worst-case jamming strategy is $z_1=1$ and $z_2=1+\theta$. However, for large λ, as θ increases the optimum jamming strategy is $z_1=1-\theta$ and $z_2=1$.

At rate less than $1/2$, the performance of three level jamming is the same as two level jamming for all θ. At rate $3/4$, three level jamming degrades the performance by about 1 dB and about 4 dB for $\theta=0.7$ and 0.9 compared to two-level jamming. This implies that for large θ, a small amount of jammer power is sufficient for the communicator to choose the bad channel state. We show the minimum E_b/N_J needed to obtain code rate r with several thresholds in Fig. 2.3.2.4. The performance with the optimal threshold under three level jamming is shown in Fig. 2.3.2.5.

2.3.3. Four Level Jamming

In this section we discuss four level jamming and four level quantization. The distribution of Z is concentrated at four points, that is, zero, $1-\theta$, 1 and $1+\theta$.

For $\lambda \leq 1/(1+\theta)^2$, $Pr\{Z=z\}$ is given by

$$Pr\{Z=z\} = \begin{cases} 1 & z=\sqrt{1/\lambda} \\ 0 & \text{otherwise} \end{cases} \tag{2.3.3.1}$$

and the resulting capacity is zero. For $\lambda > 1/(1+\theta)^2$, $Pr\{Z=z\}$ is given by

$$Pr\{Z=z\} = \begin{cases} \rho_1 & z=z_1=1-\theta \\ \rho_2 & z=z_2=1 \\ \rho_3 & z=z_3=1+\theta \\ 1-\rho_1-\rho_2-\rho_3 & z=0. \end{cases} \tag{2.3.3.2}$$

The transition probabilities p_a, p_b, p_c and p_d are then given by

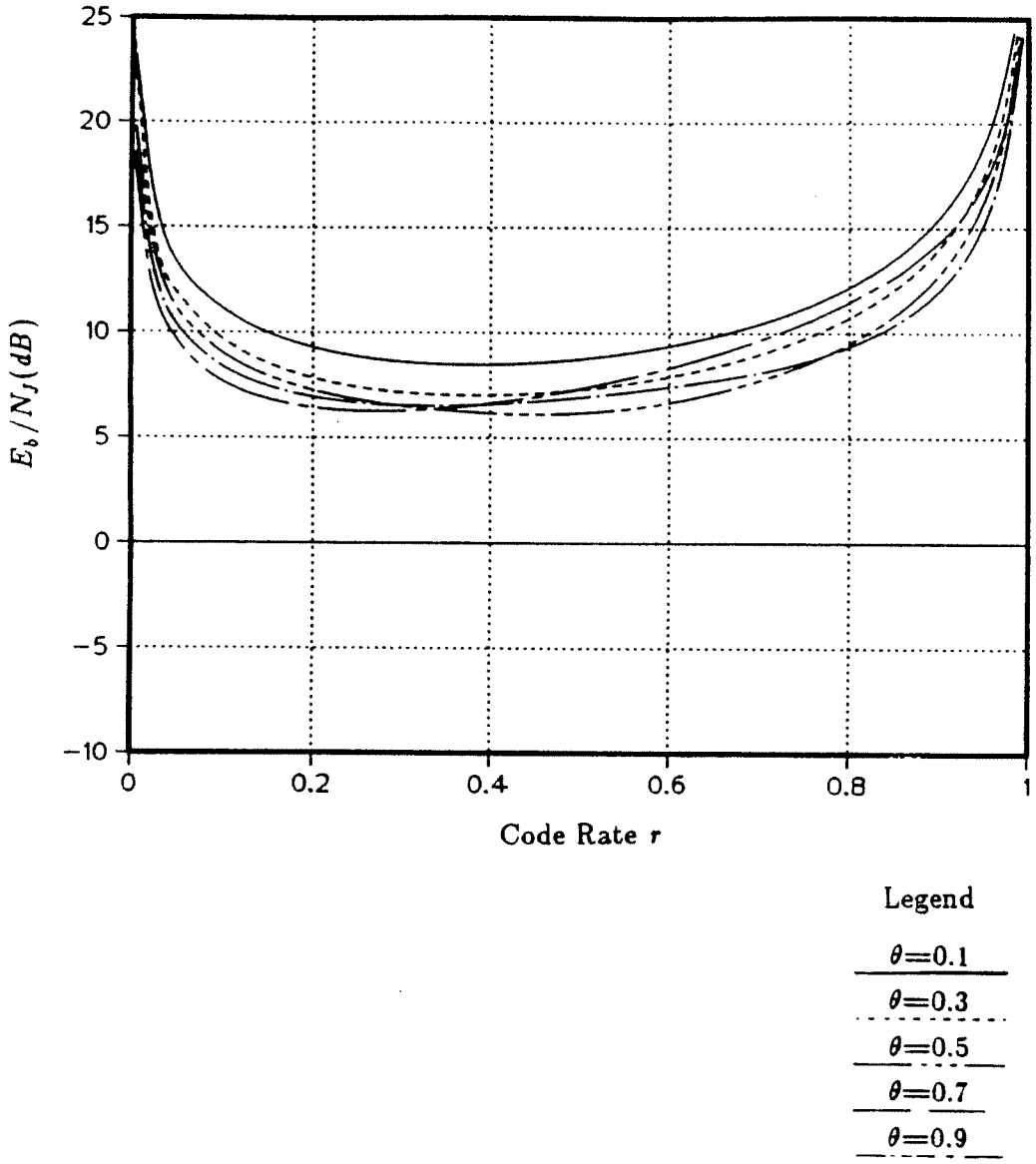


Figure 2.3.3.1. E_b/N_J needed for reliable communication at code rate r in four level tone jamming and four level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

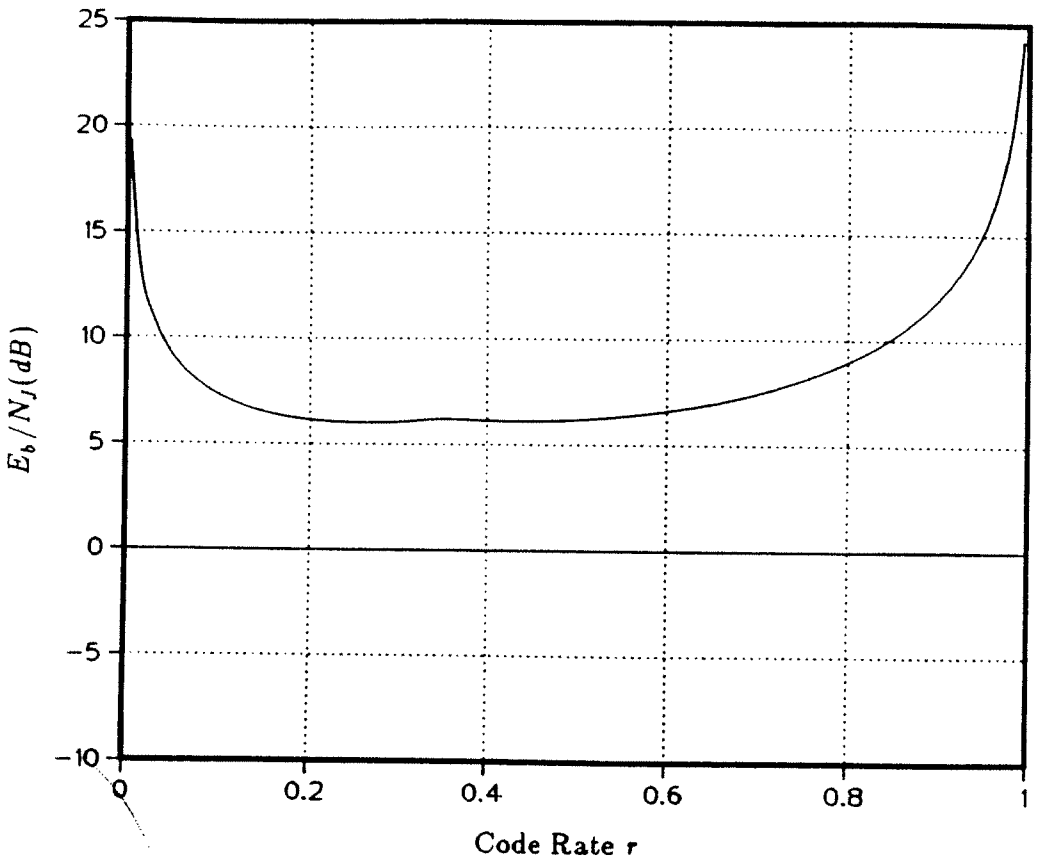


Figure 2.3.3.2. E_b/N_J needed for reliable communication at code rate r in four level tone jamming and four level quantization system with optimal θ.

$$
\begin{aligned}
p_a &= 1\text{–}0.5(\rho_1+\rho_2+\rho_3) \\
p_b &= 0.5\rho_1 \\
p_c &= 0.5\rho_2 \\
p_d &= 0.5\rho_3.
\end{aligned}
\tag{2.3.3.3}
$$

The channel capacity is given by (2.3.2.8). The numerical results are shown in Fig. 2.3.3.1 for several value of θ. For small θ the optimal strategy of the jammer is the same as two or three level jamming strategy. In Fig. 2.3.3.2. we show the performance of this system with the optimal threshold θ.

2.4. Tone Jamming with Random Phase

In this section we consider a uniformly distributed random phase in the jammed slot of a frequency band being used by the communicator. For this case the noise random variable I is defined in (2.2.5). The jamming strategies are then to distribute Z subject to average power $1/\lambda$ given in (2.2.7).

In the following subsections 2.4.1, 2.4.2 and 2.4.3 we consider two, three and four level jamming strategies as discussed in the previous section 2.3.

2.4.1. Two Level Jamming

We first consider two, three and four level quantization at the receiver for two level jamming.

Two level jamming, two level quantization

For a hard decision (two level quantization) the two or fewer values on which the jammer concentrate Z are zero and a value greater than or equal to 1. Before we compute the nonzero value of the jammer power we determine the transition probabilities of the channel.

The crossover transition probability given $Z=z$ is same as (2.3.1.1) replacing Z by $Z \cos \phi$, and is given by

$$\begin{aligned} Pr(Y=1 \mid X=-1,Z) &= Pr(Y=-1 \mid X=1,Z) \\ &= Pr(1+Z \cos \phi \leq 0). \end{aligned} \tag{2.4.1.1}$$

The crossover probability p is given by

$$p = \rho_1 \frac{1}{\pi} \cos^{-1} \frac{1}{z_1} \tag{2.4.1.2}$$

where ρ_1 is the probability being jammed with power z_1 and $\rho_0=1-\rho_1$ is the probability not being jammed.

Since the channel capacity C is a decreasing function of the crossover probability p, maximizing p produces the minimum of C. The average power constraint of the jammer is

$$E\left[Z^2\right] = z_1^2 \rho_1 = 1/\lambda. \tag{2.4.1.3}$$

Thus p can be written as

$$p = \rho_1 \frac{1}{\pi} \cos^{-1} \sqrt{\rho_1 \lambda}. \tag{2.4.1.4}$$

The worst-case distribution is then given by

$$Pr\{Z=z\} = \begin{cases} 1 & z=\sqrt{1/\lambda} \\ 0 & \text{otherwise} \end{cases} \tag{2.4.1.5}$$

for $\lambda \leq \gamma_0$ and

$$Pr\{Z=z\} = \begin{cases} \gamma_0/\lambda & z=\sqrt{1/\gamma_0} \\ 1-\gamma_0/\lambda & z=0 \end{cases} \tag{2.4.1.6}$$

for $\lambda > \gamma_0$, where γ_0 is the solution of

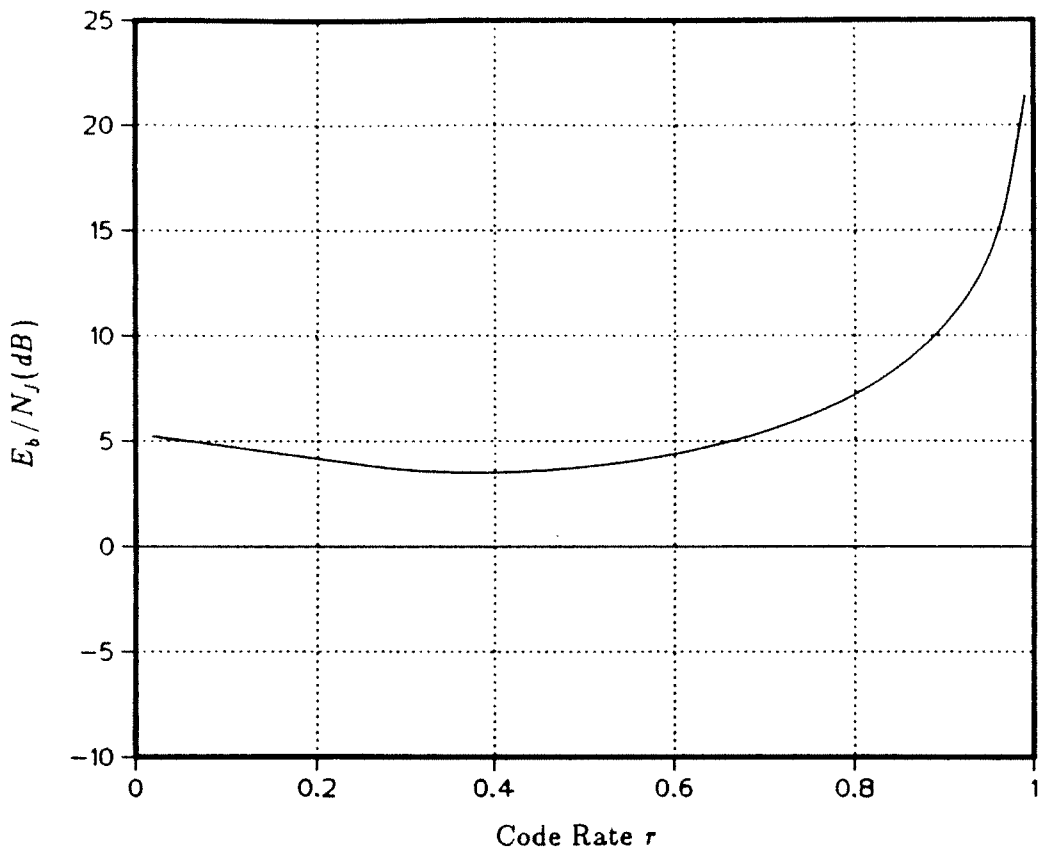


Figure 2.4.1.1. E_b/N_J needed for reliable communication at code rate r in two level random phase tone jamming and two level quantization system.

$$\cos^{-1}\sqrt{\gamma_0} - \frac{1}{2}\sqrt{\frac{\gamma_0}{1-\gamma_0}} = 0. \tag{2.4.1.7}$$

The value of γ_0 is 0.63. The resulting channel capacity is then

$$C = \begin{cases} 0 & \lambda \leq \gamma_0 \\ 1-H_2(\gamma_0/\lambda) & \lambda > \gamma_0. \end{cases} \tag{2.4.1.8}$$

Fig. 2.4.1.1 shows the signal-to-noise ratio needed for reliable communication for two level tone jamming and hard decision system. Comparing with Fig. 2.3.1.2 we see that at rate one half the penalty on the jammer for not knowing the phase is 5.8 dB.

Two level jamming, three level quantization

With three level quantization the distribution of Z is given by

$$Pr\{Z=z\} = \begin{cases} \rho_1 & z=z_1 \\ \rho_0 & z=0 \end{cases} \tag{2.4.1.9}$$

where z_1 is greater than or equal to $1-\theta$, ρ_1 is $1/z_1^2\lambda$ and $\rho_0=1-\rho_1$. Since it is difficult to compute the optimal distribution of the jammer analytically, it has been done numerically. The conditional transition probabilities $p_a(z)$, $p_b(z)$ and $p_c(z)$ are calculated depending upon the value of z.

(1) If z is less than $1-\theta$ then $p_a(z)=1$.

(2) If $z \in [1-\theta, 1+\theta)$ then

$$\begin{aligned} p_a(z) &= \frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z} \\ p_b(z) &= \frac{1}{\pi}\cos^{-1}\frac{1-\theta}{z} \\ p_c(z) &= 0. \end{aligned} \tag{2.4.1.10}$$

(3) If $z \in [1+\theta, \infty)$ then

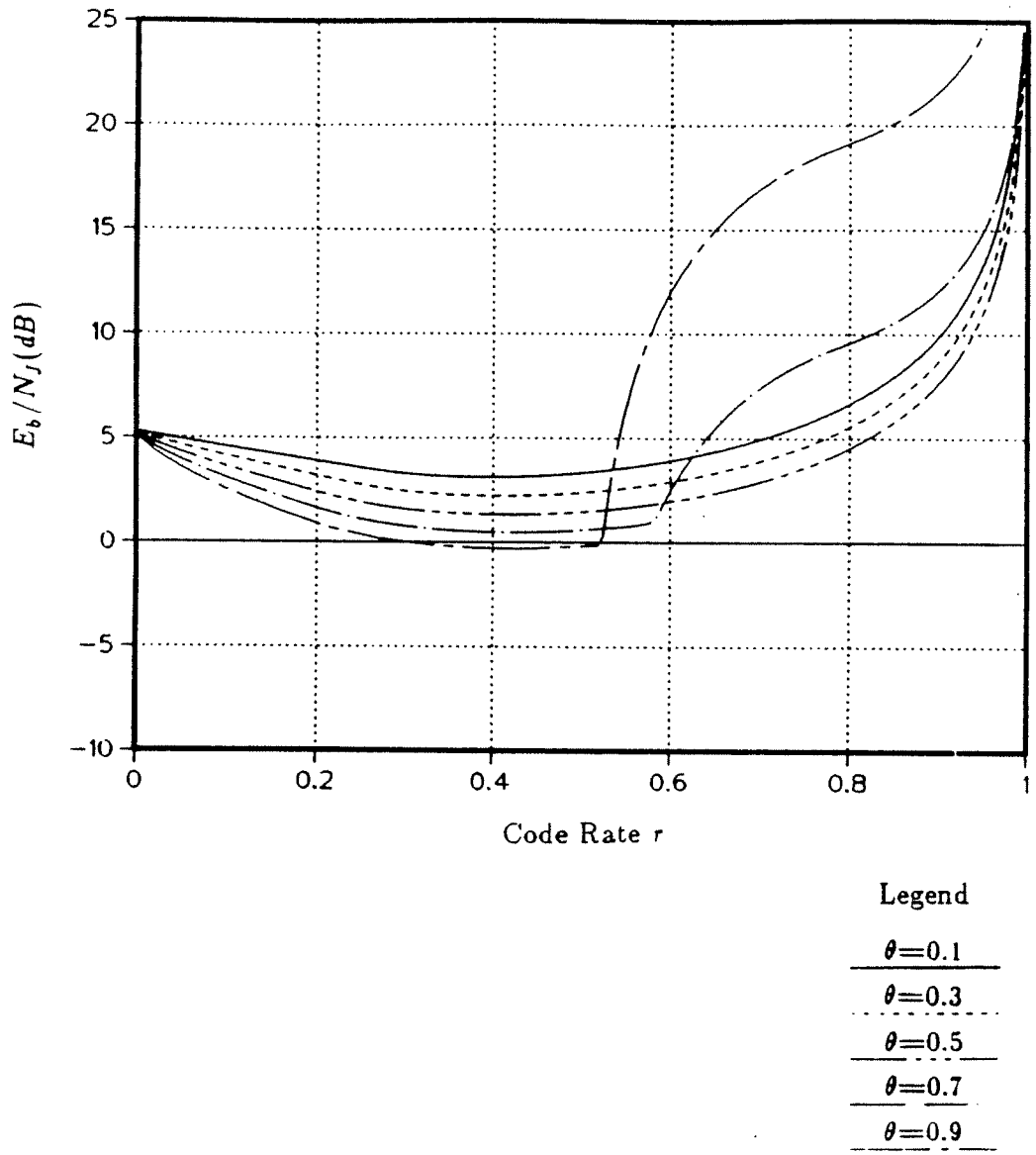


Figure 2.4.1.2. E_b/N_J needed for reliable communication at code rate r in two level random phase tone jamming and three level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

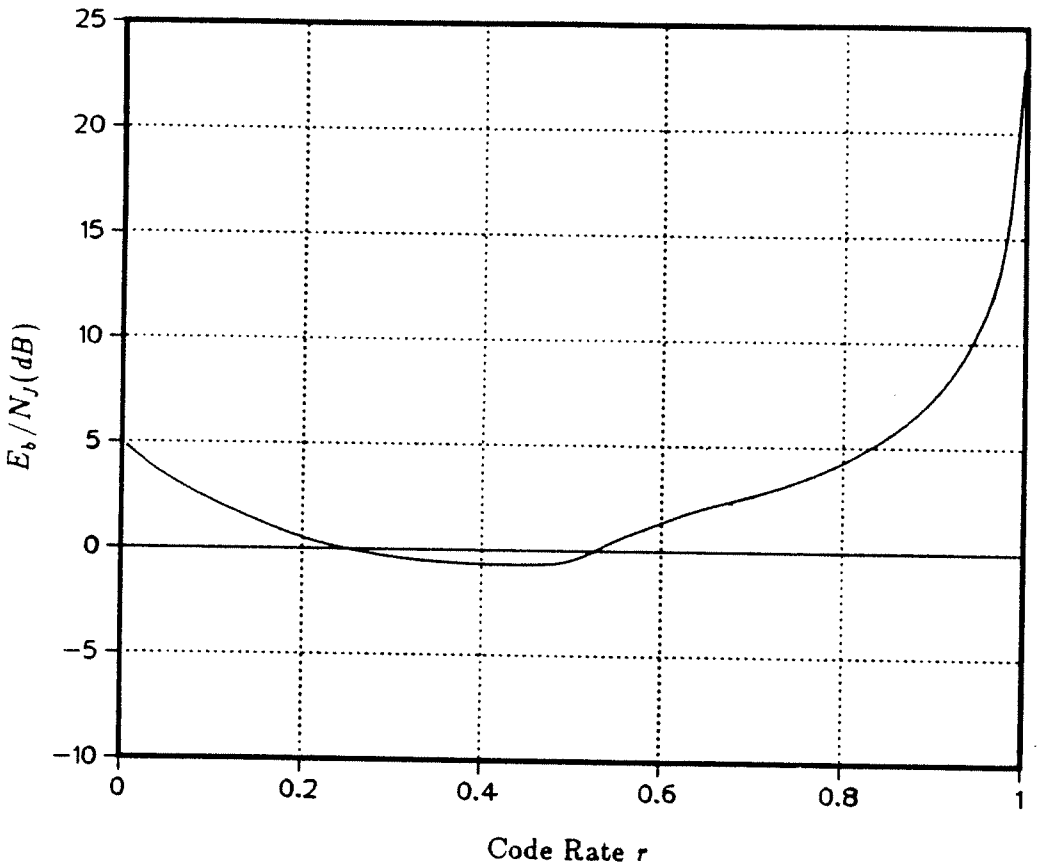


Figure 2.4.1.3. E_b/N_J needed for reliable communication at code rate r in two level random phase tone jamming and three level quantization system with optimal θ.

$$
\begin{aligned}
p_a(z) &= \frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z} \\
p_b(z) &= \frac{1}{\pi}\left(\cos^{-1}\frac{1-\theta}{z} - \cos^{-1}\frac{1+\theta}{z}\right) \\
p_c(z) &= \frac{1}{\pi}\cos^{-1}\frac{1+\theta}{z} \quad .
\end{aligned}
\tag{2.4.1.11}
$$

The transition probabilities are given by

$$
\begin{aligned}
p_a &= p_a(z)\rho_1+\rho_0 \\
p_b &= p_b(z)\rho_1 \\
p_c &= p_c(z)\rho_1 \ .
\end{aligned}
\tag{2.4.1.12}
$$

Fig. 2.4.1.2 shows the necessary signal-to-noise ratio for various θ. Fig. 2.4.1.3 shows the performance with optimal θ. For small λ, the jammer has enough power so that $z \geq 1+\theta$ with proability one. However, as θ increases the performance becomes better for small λ. Hence, from the communicator's point of view when λ is small the optimum θ is close to 1. However, for large θ and large λ, small jamming power is sufficient to to cause an erasure. Fig. 2.4.1.2 shows that for a rate less than half, large θ is optimum while for rates greater than one half, $\theta \approx 0.5$ is optimum.

Two level jamming, four level quantization

With four level quantization the optimal distribution of Z is given by (2.4.1.9). The jamming level z must be greater than $1+\theta$, otherwise the capacity is 1. To evaluate the performance of two level random phased tone jamming and four level quantization system we must consider four cases.

(1) If z is less than $1-\theta$ then $p_a(z)=1$.

(2) If $z \in [1-\theta, 1)$ then

$$
\begin{aligned}
p_a(z) &= \frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z} \\
p_b(z) &= \frac{1}{\pi}\cos^{-1}\frac{1-\theta}{z} \\
p_c(z) &= 0 \\
p_d(z) &= 0.
\end{aligned}
\tag{2.4.1.13}
$$

(3) If $z \in [1, 1+\theta)$ then

$$
\begin{aligned}
p_a(z) &= \frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z} \\
p_b(z) &= \frac{1}{\pi}\left(\cos^{-1}\frac{1-\theta}{z} - \cos^{-1}\frac{1}{z}\right) \\
p_c(z) &= \frac{1}{\pi}\cos^{-1}\frac{1}{z} \\
p_d(z) &= 0.
\end{aligned}
\tag{2.4.1.14}
$$

(4) If $z \in [1+\theta, \infty)$ then

$$
\begin{aligned}
p_a(z) &= \frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z} \\
p_b(z) &= \frac{1}{\pi}\left(\cos^{-1}\frac{1-\theta}{z} - \cos^{-1}\frac{1}{z}\right) \\
p_c(z) &= \frac{1}{\pi}\left(\cos^{-1}\frac{1}{z} - \cos^{-1}\frac{1+\theta}{z}\right) \\
p_d(z) &= \frac{1}{\pi}\cos^{-1}\frac{1+\theta}{z} \quad .
\end{aligned}
\tag{2.4.1.15}
$$

The transition probabilities are given by

$$
\begin{aligned}
p_a &= p_a(z)\rho_1 + \rho_0 \\
p_b &= p_b(z)\rho_1 \\
p_c &= p_c(z)\rho_1 \\
p_d &= p_d(z)\rho_1 \ .
\end{aligned}
\tag{2.4.1.16}
$$

The capacity is given by (2.2.21). In Fig. 2.4.1.4 and 2.4.1.5 we show $C^{-1}(r)/r$ for various θ and the optimal θ, respectively. We can see that the behavior of this system is similar to the system with two level tone jamming and four level

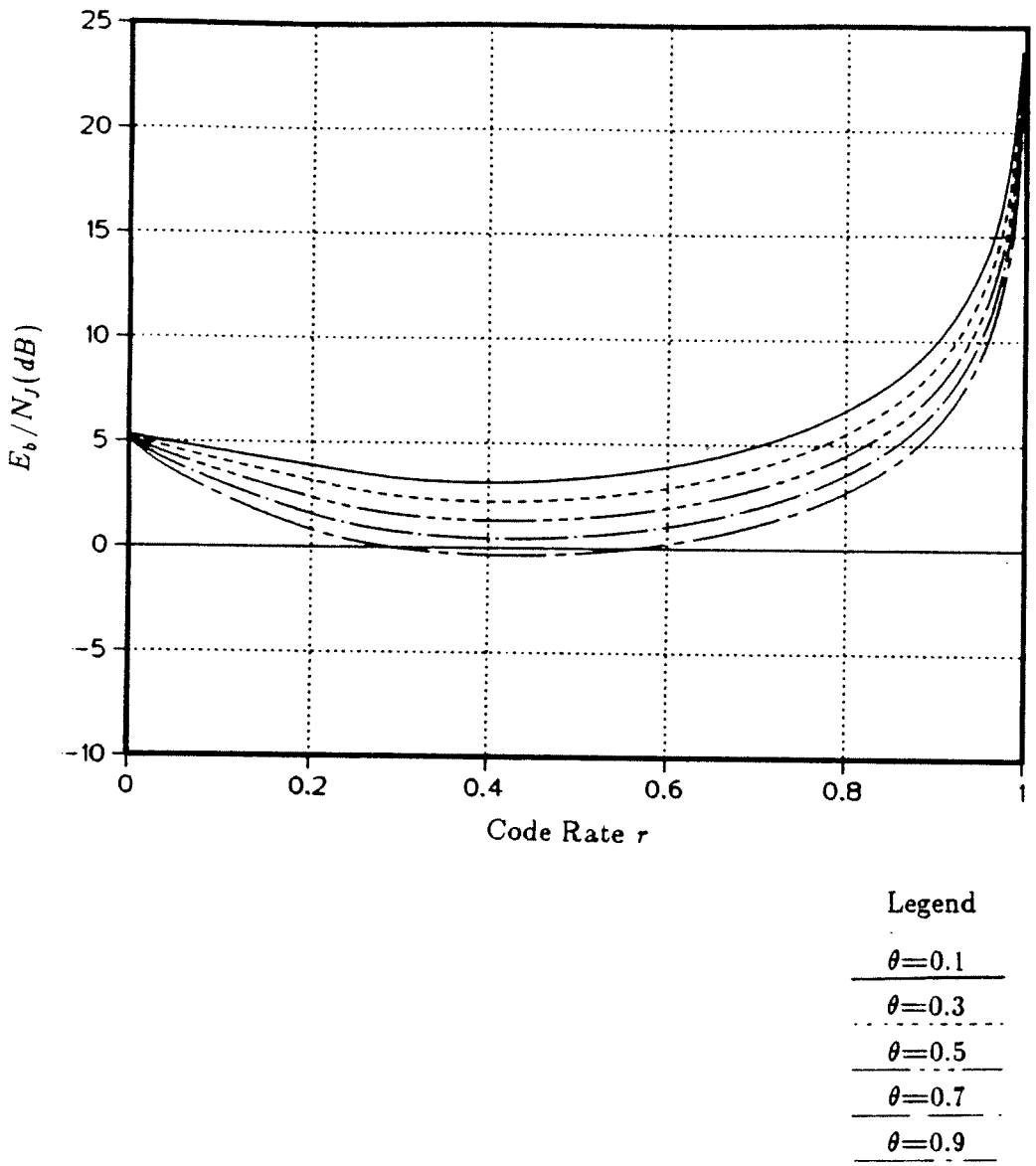


Figure 2.4.1.4. E_b/N_J needed for reliable communication at code rate r in two level random phase tone jamming and four level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

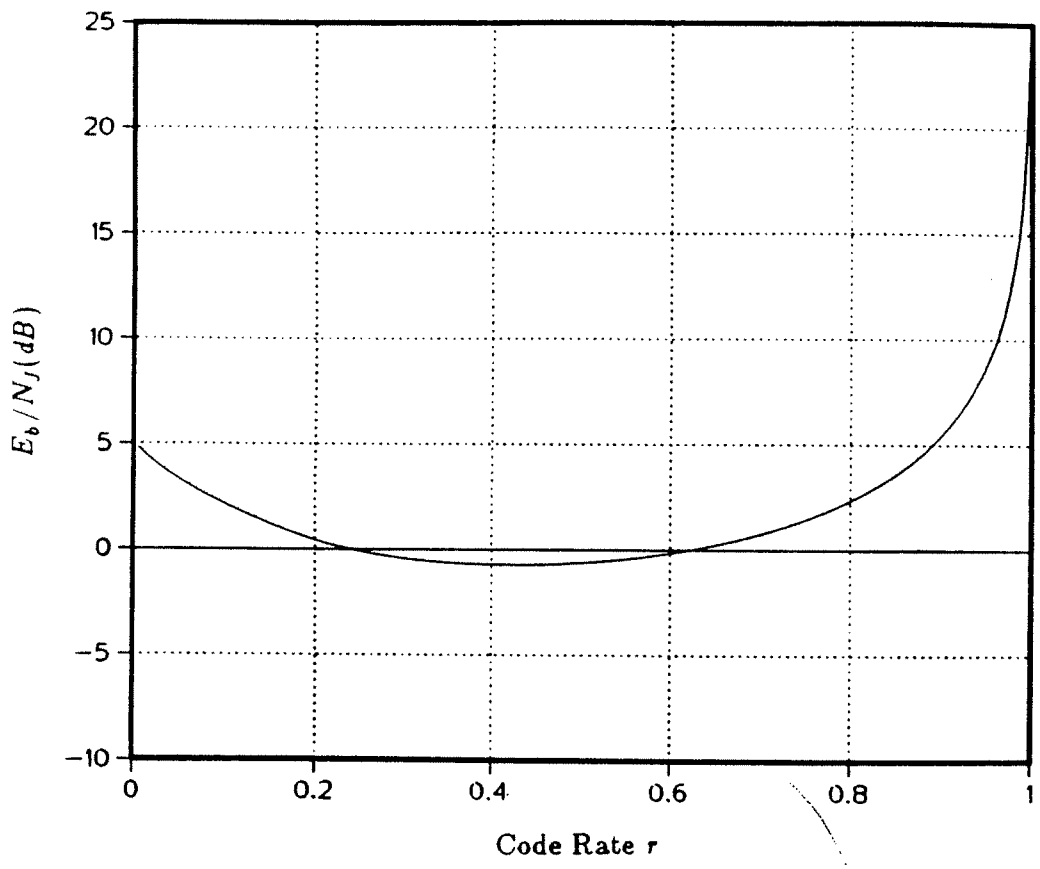


Figure 2.4.1.5. E_b/N_J needed for reliable communication at code rate r in two level random phase tone jamming and four level quantization system with optimal θ.

quantization.

2.4.2. Three Level Jamming

For three level jamming we will consider three and four level quantization at the receiver. The optimal jamming strategies are to distribute the power with at most three different levels, and each level must greater than the communicator's level.

Three level jamming, three level quantization

With three level quantization the jammer level Z has at most three values, that is, zero, z_1, (greater than or equal to $1-\theta$) and z_2, (greater than or equal to $1+\theta$). The distribution of Z is then given by

$$Pr\{Z=z\} = \begin{cases} \rho_1 & z=z_1 \\ \rho_2 & z=z_2 \\ \rho_0 & z=0 \end{cases} \tag{2.4.2.1}$$

where ρ_1 and ρ_2 are probabilities corresponding to z_1 and z_2, respectively, and $\rho_0=1-\rho_1-\rho_2$. The resulting transition probabilities p_a, p_b and p_c are then given by

$$\begin{aligned} p_a &= \rho_0+\rho_1\frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z_1}+\rho_2\frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z_2} \\ p_b &= \rho_1\frac{1}{\pi}\cos^{-1}\frac{1-\theta}{z_1}+\rho_2\frac{1}{\pi}\left(\cos^{-1}\frac{1-\theta}{z_2}-\cos^{-1}\frac{1+\theta}{z_2}\right) \\ p_c &= \rho_2\frac{1}{\pi}\cos^{-1}\frac{1+\theta}{z_2} \quad . \end{aligned} \tag{2.4.2.2}$$

The optimal ρ_1, ρ_2 and the corresponding jammer levels z_1, z_2 are shown in Fig. 2.4.2.1 and 2.4.2.2, respectively. For small θ (e.g. $\theta=0.1$) and small λ, the optimal jamming distribution is concentrated on a single value $z_2\geq 1+\theta$. For $\theta=0.5$, ρ_1

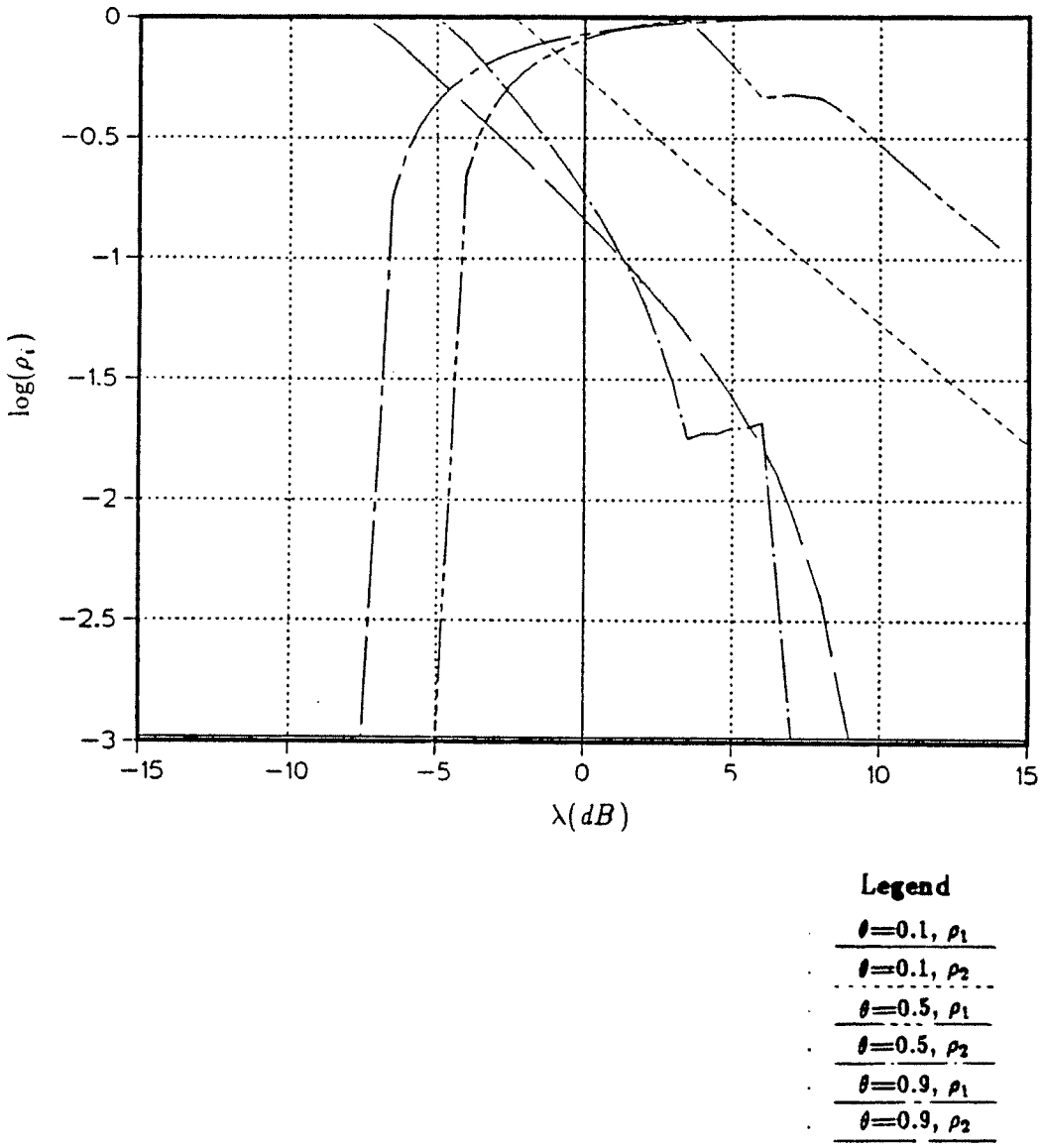


Figure 2.4.2.1. Optimal probabilities ρ_1 and ρ_2 in three level random phase tone jamming and three level quantization system with θ=0.1, 0.5 and 0.9.

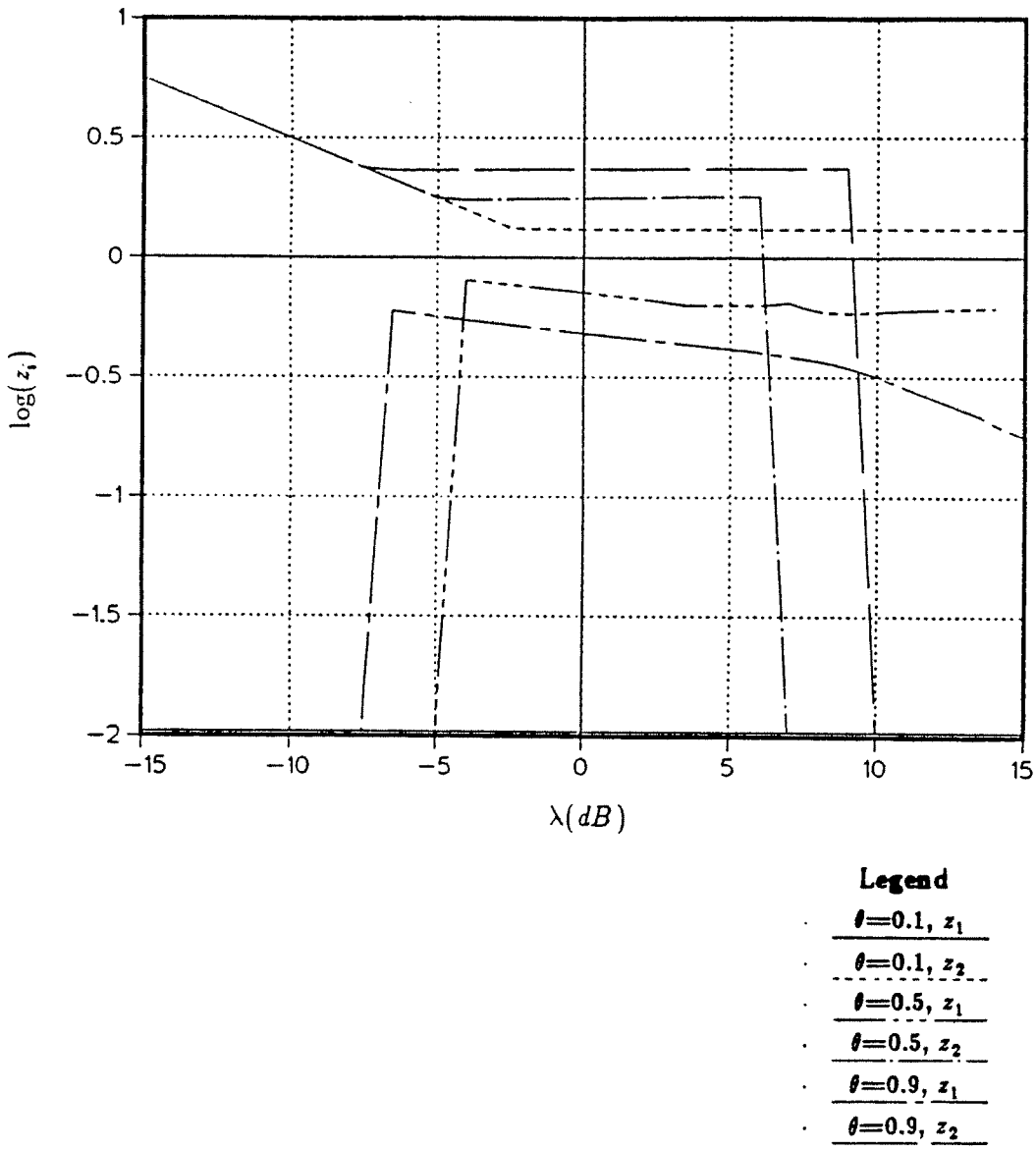


Figure 2.4.2.2. Optimal strength z_1 and z_2 in three level random phase tone jamming and three level quantization system with θ=0.1, 0.5 and 0.9.

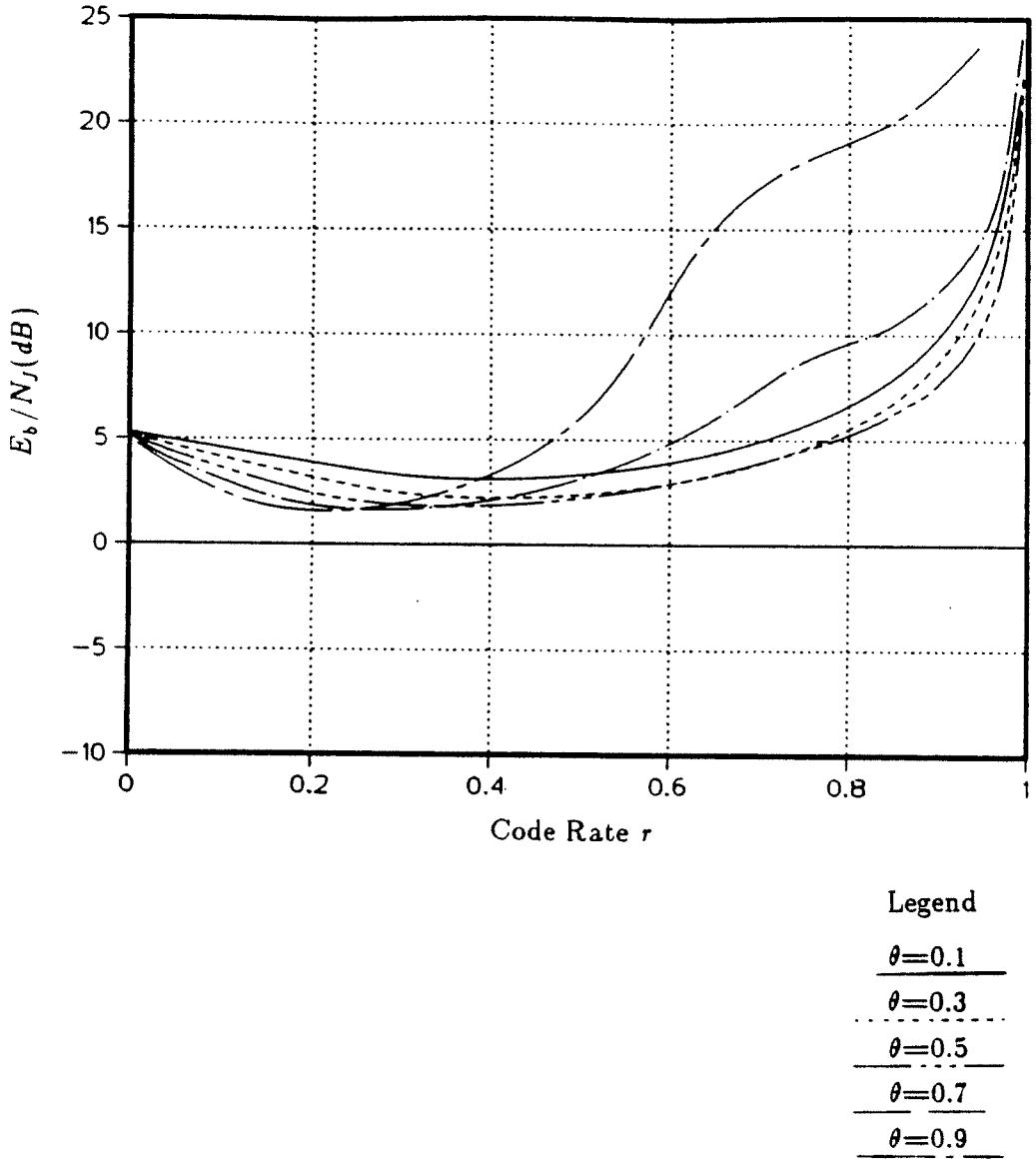


Figure 2.4.2.3. E_b/N_J needed for reliable communication at code rate r in three level random phase tone jamming and three level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

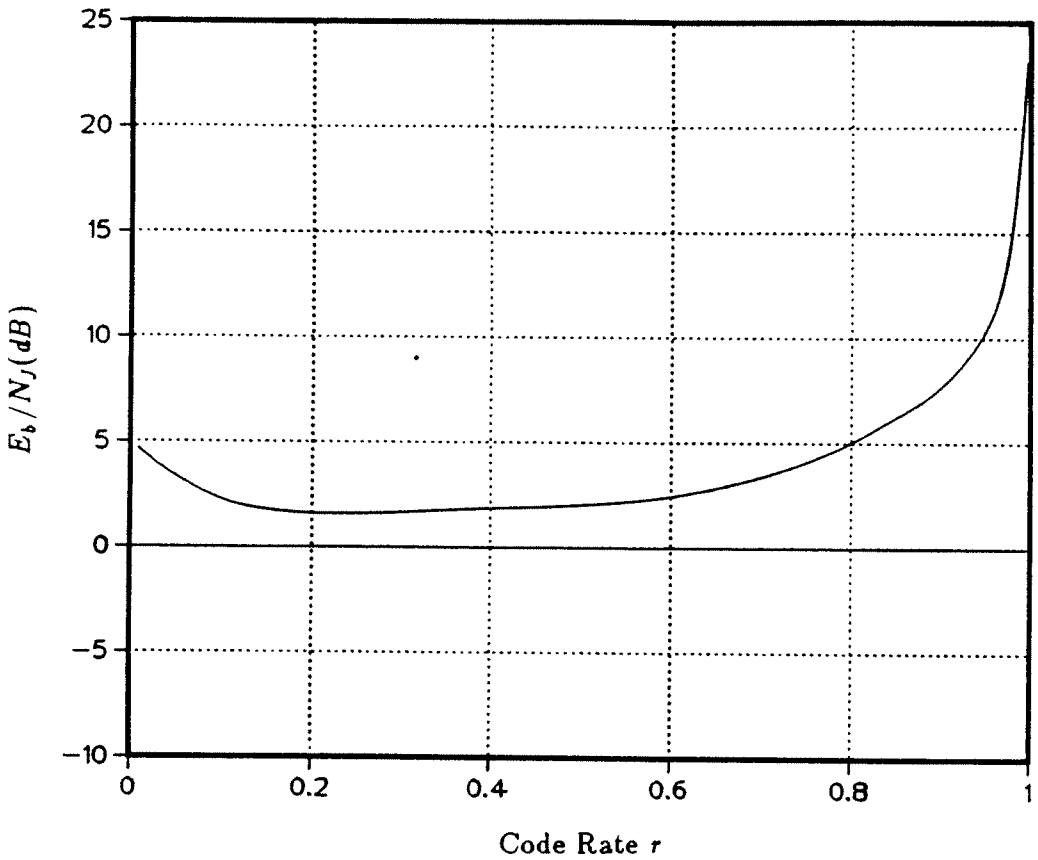


Figure 2.4.2.4. E_b/N_J needed for reliable communication at code rate r in three level random phase tone jamming and three level quantization system with optimal θ.

increases in the range of $-4.5\ dB < \lambda < 3.5\ dB$. The numerical results are shown in Fig. 2.4.2.3 and Fig. 2.4.2.4 for fixed θ and the optimum θ respectively. Compared to the two level jamming, three level jamming degrades performance at rate one half (e.g. at $\theta=0.5$, $r=1/2$, 0.8 dB loss and at $\theta=0.9$, $r=1/2$, 6.3 dB loss). For small θ the optimal distribution of Z for three level jamming is concentrated on two levels. This is because with a small increase in jamming power the jammer can cause an error. In other words, little additional power is needed to cause an error as opposed to an erasure. As discussed in previous section 2.4.1, with three level quantization, for large θ, the jammer can easily force the capacity to be close to one half with small power.

Three level jamming, four level quantization

With four level quantization Z is distributed at three values: one zero and the other two values $z_1 \geq 1-\theta$ and $z_2 \geq 1$. Let ρ_1, ρ_2 be probabilities corresponding to z_1 and z_2, respectively, and $\rho_0 = 1-\rho_1-\rho_2$. The conditional transition probabilities are then given by the following based on the value of Z.

(1) If $z \in [1-\theta, 1)$ then

$$\begin{aligned} p_a(z) &= \frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z} \\ p_b(z) &= \frac{1}{\pi}\cos^{-1}\frac{1-\theta}{z} \\ p_c(z) &= 0 \\ p_d(z) &= 0. \end{aligned} \tag{2.4.2.3}$$

(2) If $z \in [1, 1+\theta)$ then

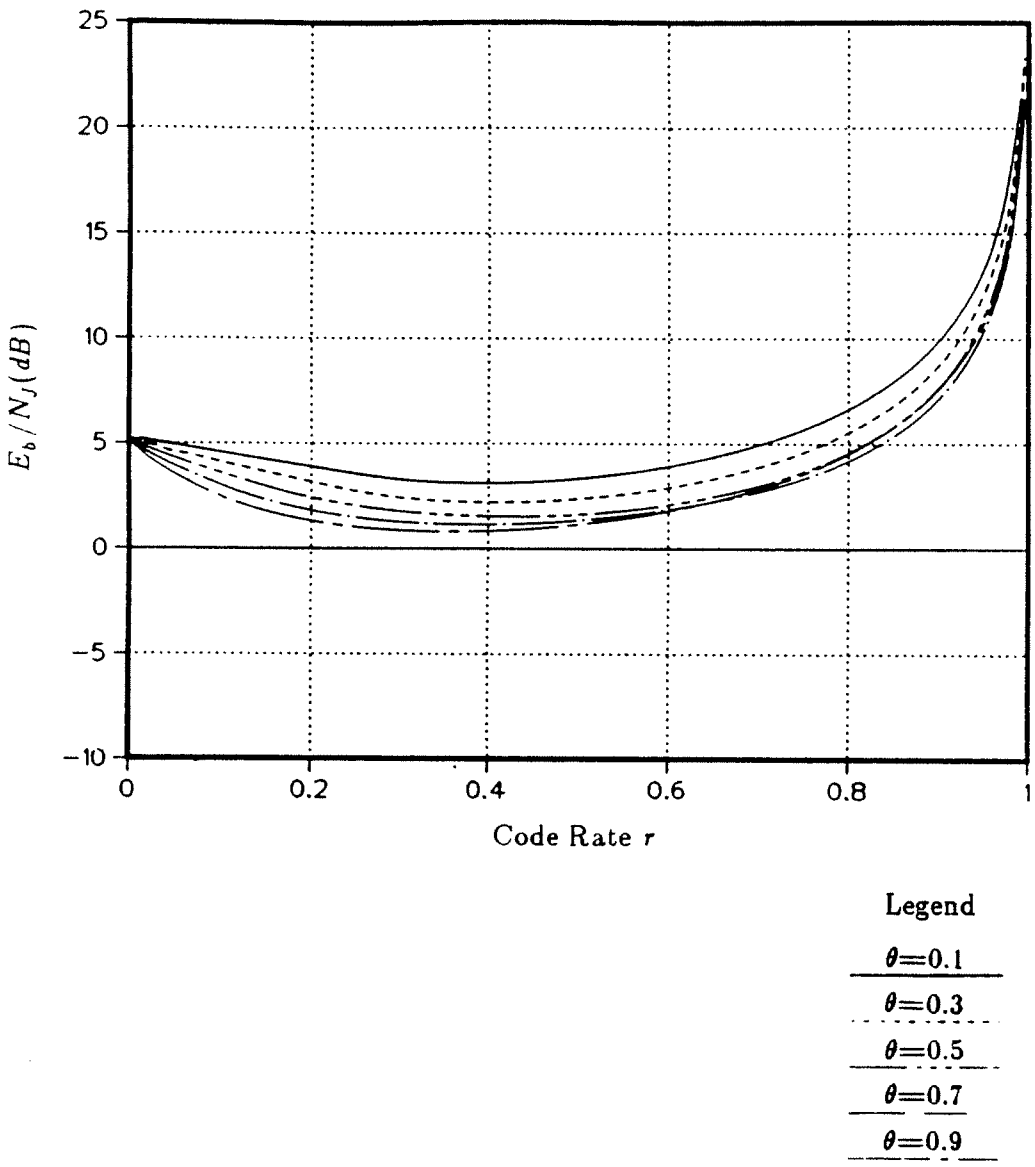


Figure 2.4.2.5. E_b/N_J needed for reliable communication at code rate r in three level random phase tone jamming and four level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

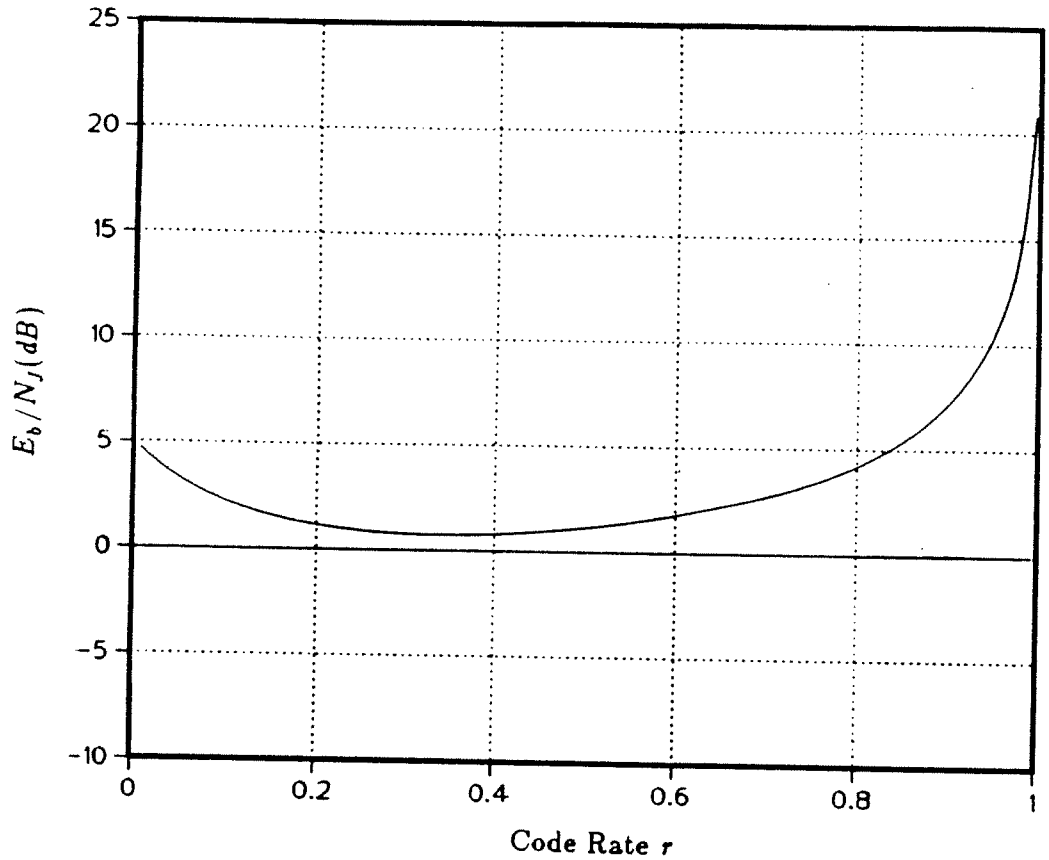


Figure 2.4.2.6. E_b/N_J needed for reliable communication at code rate r in three level random phase tone jamming and four level quantization system with optimal θ.

$$
\begin{aligned}
p_a(z) &= \frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z} \\
p_b(z) &= \frac{1}{\pi}\left(\cos^{-1}\frac{1-\theta}{z} - \cos^{-1}\frac{1}{z}\right) \\
p_c(z) &= \frac{1}{\pi}\cos^{-1}\frac{1}{z} \\
p_d(z) &= 0.
\end{aligned}
\tag{2.4.2.4}
$$

(3) If $z \in [1+\theta, \infty)$ then

$$
\begin{aligned}
p_a(z) &= \frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z} \\
p_b(z) &= \frac{1}{\pi}\left(\cos^{-1}\frac{1-\theta}{z} - \cos^{-1}\frac{1}{z}\right) \\
p_c(z) &= \frac{1}{\pi}\left(\cos^{-1}\frac{1}{z} - \cos^{-1}\frac{1+\theta}{z}\right) \\
p_d(z) &= \frac{1}{\pi}\cos^{-1}\frac{1+\theta}{z} \quad .
\end{aligned}
\tag{2.4.2.5}
$$

The expression (2.3.2.7) is used to compute the channel transition probabilities and the channel capacity is given by (2.3.2.8). In Fig. 2.4.2.5 it is shown that three level jamming degrades the performance for large θ (e.g. θ=0.7, r=0.6, 0.8 dB difference) compared to two level jamming. In this system, for small θ, the jamming strategy is two level jamming strategy (i.e. ρ_1 or ρ_2=0). However, for large λ as θ increases, three level jamming degrades the performance compared to two level jamming. Figure 2.4.2.6 shows the minimum bit signal-to-noise ratio needed for reliable communication at code rate r when the optimal θ is chosen.

2.4.3. Four Level Jamming

The distribution of Z for the system with four level quantization concentrated on at most four points is given by

$$Pr\{Z=z\} = \begin{cases} \rho_1 & z=z_1 \\ \rho_2 & z=z_2 \\ \rho_3 & z=z_3 \\ \rho_0 & z=0 \end{cases} \tag{2.4.3.1}$$

where $z_1 \in [1-\theta, 1)$, $z_2 \in [1, 1+\theta)$ and $z_3 \in [1+\theta, \infty)$, and ρ_1, ρ_2 and ρ_3 are probabilities being jammed corresponding to jammer values z_1, z_2 and z_3, respectively, and $\rho_0=1-\rho_1-\rho_2-\rho_3$. The transition probabilities based on z_1, z_2 and z_3 are given by

$$\begin{aligned} p_a &= \rho_0+\rho_1\frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z_1}+\rho_2\frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z_2}+\rho_3\frac{1}{\pi}\cos^{-1}\frac{\theta-1}{z_3} \\ p_b &= \rho_1\frac{1}{\pi}\cos^{-1}\frac{1-\theta}{z_1}+\rho_2\frac{1}{\pi}\left(\cos^{-1}\frac{1-\theta}{z_2}-\cos^{-1}\frac{1}{z_2}\right)+\rho_3\frac{1}{\pi}\left(\cos^{-1}\frac{1-\theta}{z_3}-\cos^{-1}\frac{1}{z_3}\right) \\ p_c &= \rho_2\frac{1}{\pi}\cos^{-1}\frac{1}{z_2}+\rho_3\frac{1}{\pi}\left(\cos^{-1}\frac{1}{z_3}-\cos^{-1}\frac{1+\theta}{z_3}\right) \\ p_d &= \rho_3\frac{1}{\pi}\cos^{-1}\frac{1+\theta}{z_3} \quad . \end{aligned} \tag{2.4.3.2}$$

It is very hard to obtain the optimal distribution of Z analytically. We have computed the optimal distribution numerically. As done in the previous two sections 2.4.1 and 2.4.2, for small θ the worst-case jamming strategy is concentrated on two levels. For large θ, the jammer's strategy is to use three different power levels in some range of λ. The numerical results of channel capacity are shown in Fig. 2.4.3.1 and Fig. 2.4.3.2. Unlike the tone jamming with perfect knowledge of signal phase, a four level jamming strategy with unknown phase does not degrade the performance significantly compared to a three level jamming strategy with unknown phase, even though θ is large. For example, for $r<1/2$ and $\theta=0.9$ the performance of four level jamming is the same as three level jamming. However, for $r=0.8$ and $\theta=0.9$ the degradation is about 0.5 dB.

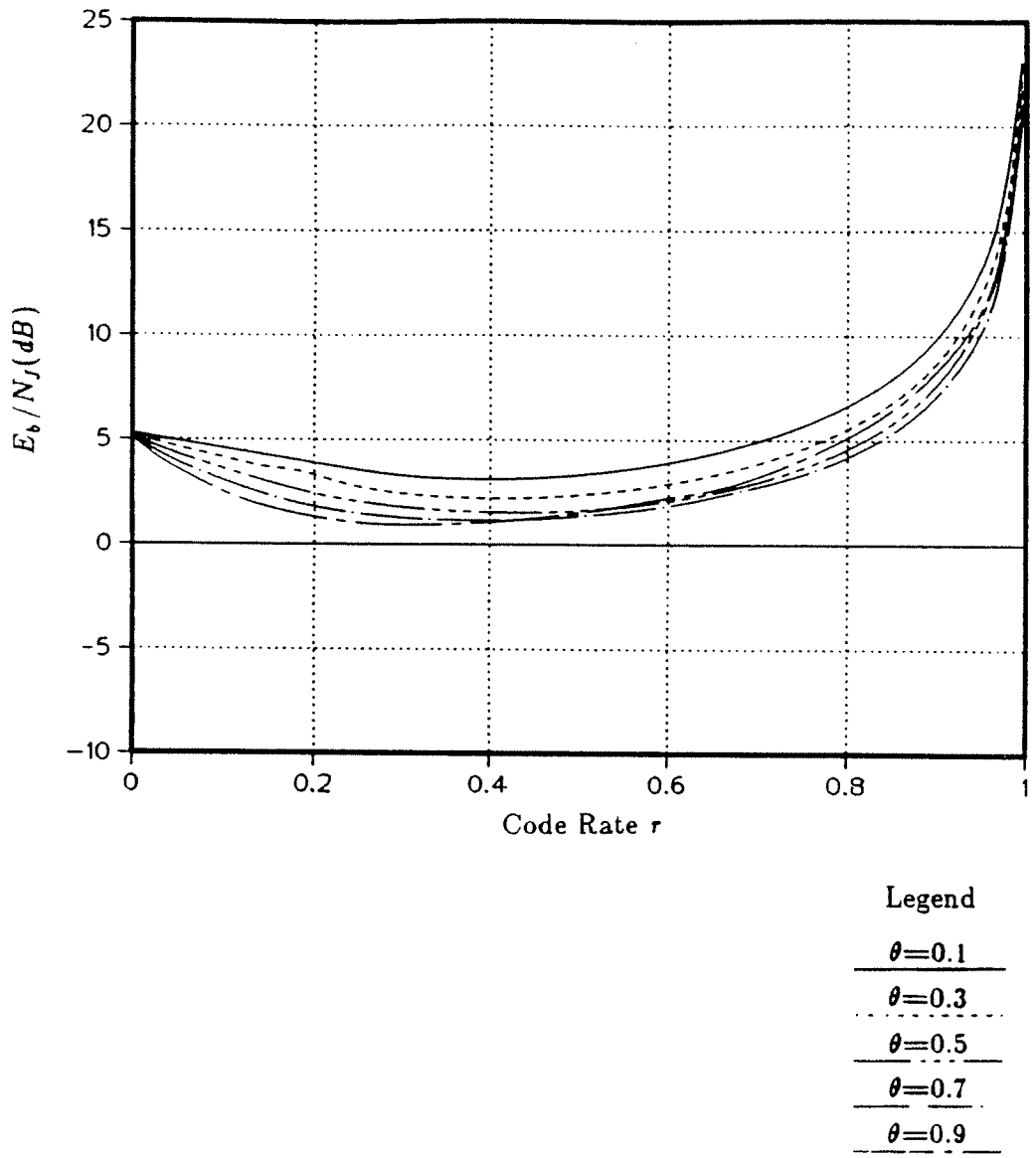


Figure 2.4.3.1. E_b/N_J needed for reliable communication at code rate r in four level random phase tone jamming and four level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

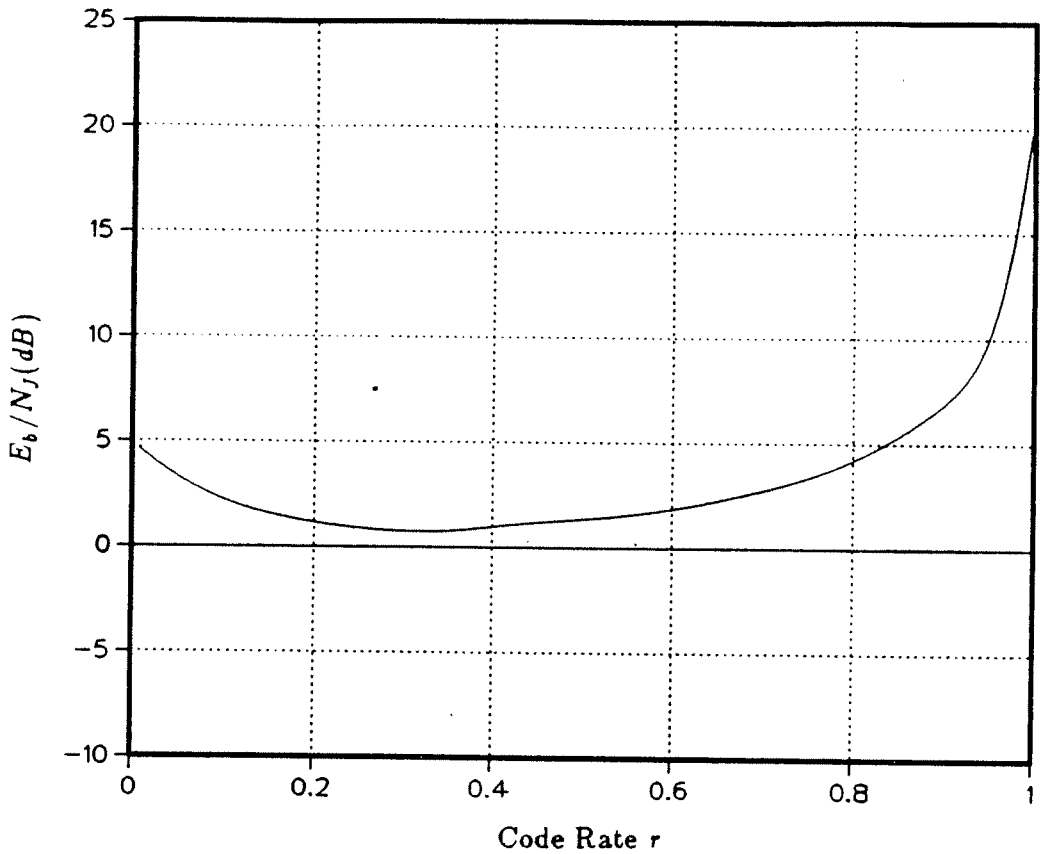


Figure 2.4.3.2. E_b/N_J needed for reliable communication at code rate r in four level random phase tone jamming and four level quantization system with optimal θ.

2.5. Partial-Band Gaussian Jamming

In this section we study the performance of the system in the presence of Gaussian jamming noise. This type of noise was defined in (2.2.9). The jamming strategies are to distribute Z subject to an average power constraint

$$E[Z^2]=1/2\lambda. \tag{2.5.1}$$

Many recent papers [CRE83][MCE81][HOU75] have been concerned with the worst-case partial band two level jamming. In our model we consider the multi-level jamming (two, three and four) and multi-level quantization.

In section 2.5.1 the channel capacity is calculated with two, three and four level quantization at the receiver under a two level partial band jammed environment. In section 2.5.2 and 2.5.3 we repeat the calculation for the case of three and four level jamming respectively.

2.5.1. Two Level Jamming

For two-level jamming strategies, the noise spectrum is flat over a fraction ρ of the spread signal band, where $0<\rho\leq 1$, and is zero elsewhere.

Two level jamming, Two level quantization

For a hard decision receiver, McEliece and Stark [MCE81] showed that the crossover probability (average error probability) of the binary symmetric channel is given by

$$\begin{aligned} p &= Pr\{y=1 \mid x=-1\} = Pr\{y=-1 \mid x=1\} \\ &= \rho\, Q\left(\sqrt{\frac{2E_c\rho}{N_J}}\right) \end{aligned} \tag{2.5.1.1}$$

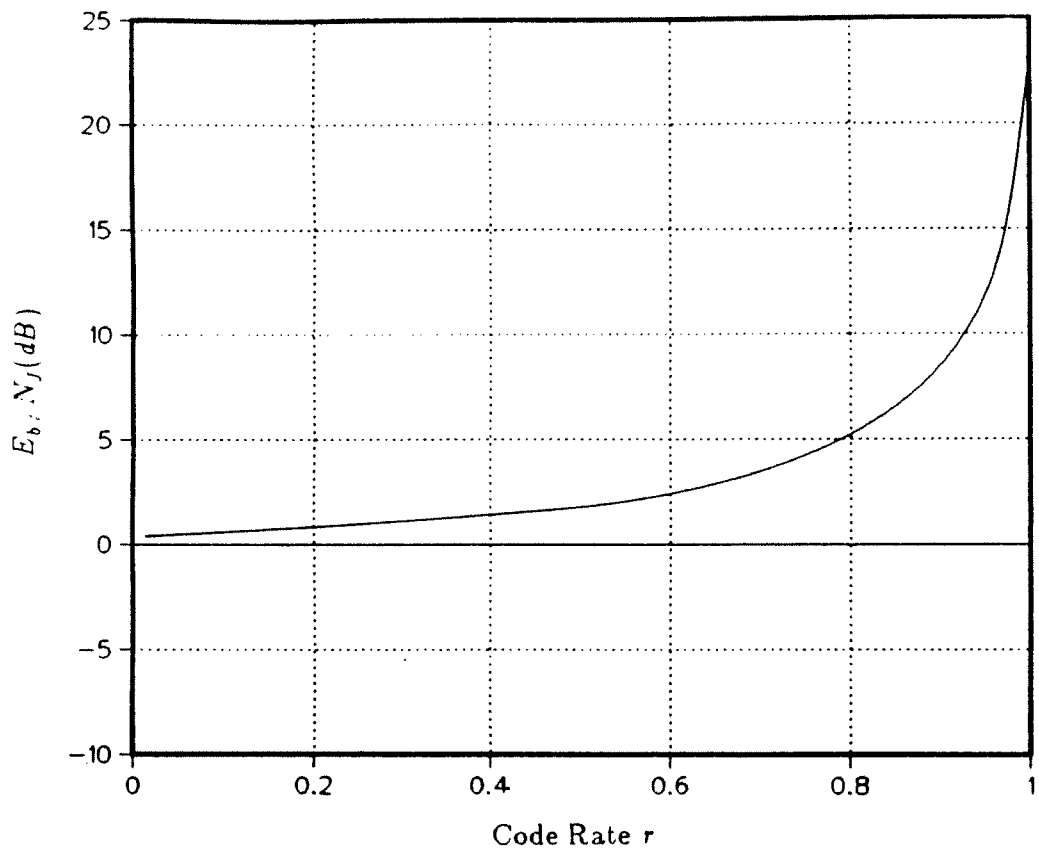


Figure 2.5.1.1. E_b/N_J needed for reliable communication at code rate r in two level Gaussian jamming and a hard decision system.

where $Q(x)=\int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$. The optimum ρ, denoted by ρ^*, minimizing the channel capacity C in (2.3.1.5) is

$$\rho^* = \begin{cases} 1 & \lambda \leq \gamma_0 \\ \gamma_0/\lambda & \lambda > \gamma_0 \end{cases} \tag{2.5.1.2}$$

where $\gamma_0=0.709$ and $\lambda=E_c/N_J$. The channel capacity is then given by

$$C = \begin{cases} 1-H_2(Q(\sqrt{2\lambda})) & \lambda \leq \gamma_0 \\ 1-H_2(\gamma_0 Q(\sqrt{2\gamma_0})/\lambda) & \lambda > \gamma_0. \end{cases} \tag{2.5.1.3}$$

The numerical result of this system is shown in Fig. 2.5.1.1. Notice in this case the optimum code rate (to minimize E_b/N_J necessary for reliable communication) is zero.

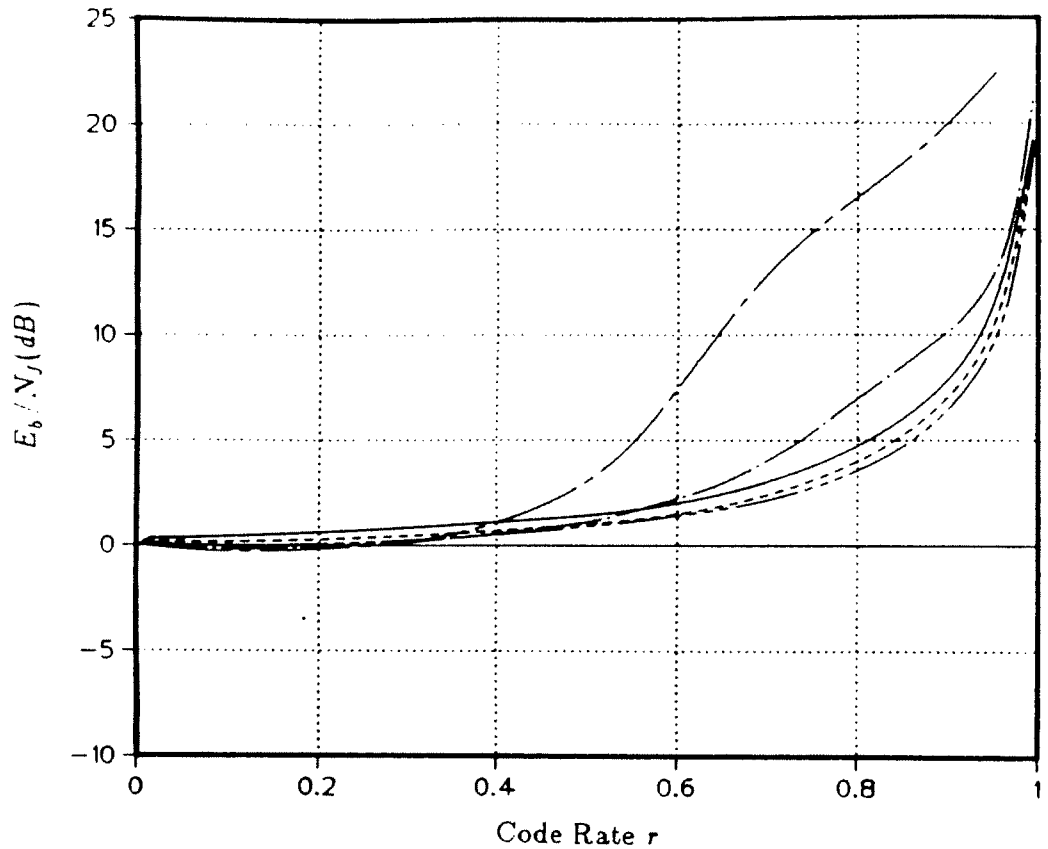
Two level jamming, three level quantization

With three level quantization the transition probabilities p_a, p_b and p_c are given by

$$\begin{aligned} p_a &= Q\Big((\theta-1)/\sigma\Big)\rho+\rho_0 \\ p_b &= \Big[Q\Big((1-\theta)/\sigma\Big)-Q\Big((1+\theta)/\sigma\Big)\Big]\rho \\ p_c &= Q\Big((1+\theta)/\sigma\Big)\rho \end{aligned} \tag{2.5.1.4}$$

where $\sigma^2 = N_J/2E_c\rho = 1/2\rho\lambda$ and $\rho_0=1-\rho$.

It is interesting to determine the optimum ρ is for sufficiently small signal-to-noise ratio. Let the symbol signal-to-noise ratio be λ. Since the channel capacity is a function of p_a and p_c , we rewrite those two transition probabilities as



Legend

θ=0.1

θ=0.3

θ=0.5

θ=0.7

θ=0.9

Figure 2.5.1.2. E_b/N_J needed for reliable communication at code rate r in two level Gaussian jamming and three level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

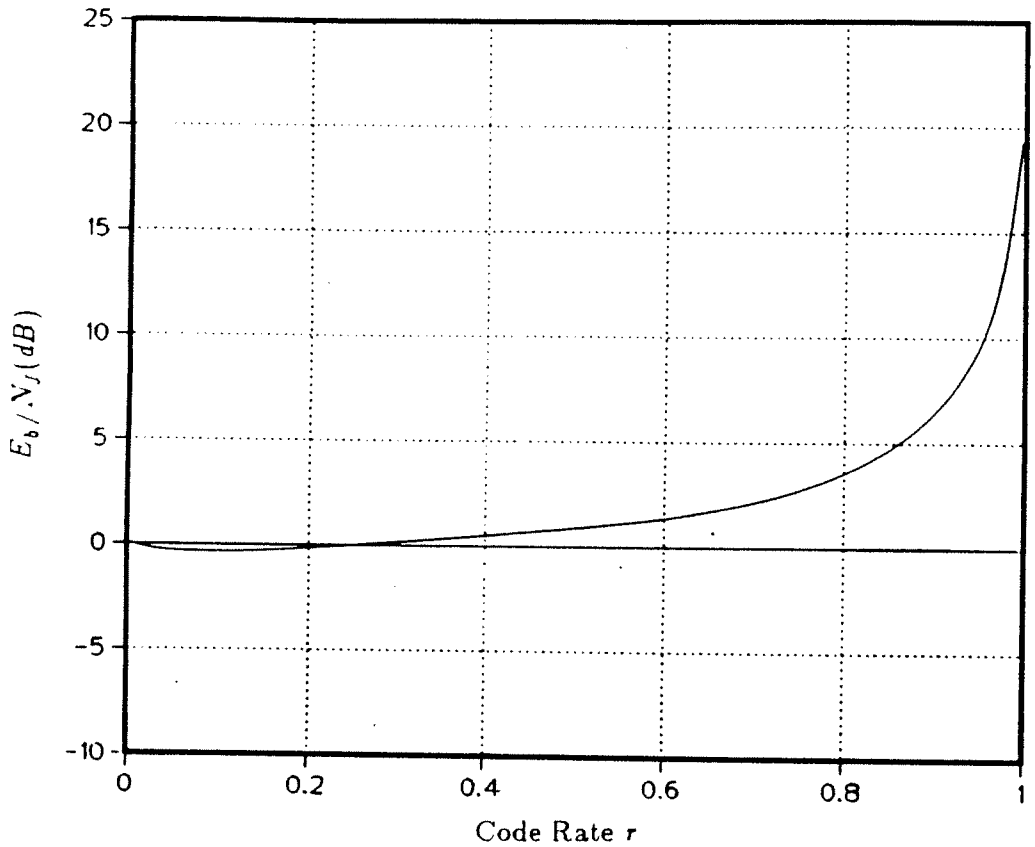


Figure 2.5.1.3. E_b/N_J needed for reliable communication at code rate r in two level Gaussian jamming and three level quantization system with optimal θ.

$$p_a = 1-Q\left((1-\theta)\sqrt{2\lambda\rho}\right)\rho$$
$$p_c = Q\left((1+\theta)\sqrt{2\lambda\rho}\right)\rho \ . \tag{2.5.1.5}$$

Before we perform the optimization of the channel capacity, we notice the following facts :

(1) For fixed λ, θ, consider p_a and p_c as a function of ρ. Then $p_c \leq p_a$ for all $0 \leq \rho \leq 1$.

(2) p_a is a convex $\cup$ function of ρ with minimum at

$$\rho_{\min} = \begin{cases} 1 & \text{if } \lambda \leq \gamma/(1-\theta)^2 \\ \dfrac{\gamma}{(1-\theta)^2\lambda} & \text{if } \lambda > \gamma/(1-\theta)^2 \end{cases} \tag{2.5.1.6}$$

where γ is the solution of

$$Q\left(\sqrt{2\gamma}\right) - \frac{1}{2\sqrt{\pi}}\sqrt{\gamma}\, e^{-\gamma} = 0 \tag{2.5.1.7}$$

and is 0.709. Also p_c is a concave $\cap$ function of ρ with maximum at

$$\rho_{\max} = \begin{cases} 1 & \text{if } \lambda \leq \gamma/(1+\theta)^2 \\ \dfrac{\gamma}{(1+\theta)^2\lambda} & \text{if } \lambda > \gamma/(1+\theta)^2 \ . \end{cases} \tag{2.5.1.8}$$

For fixed threshold θ, the minimum of C is obtained by the solution of

$$\frac{\partial}{\partial\rho} C(\theta,\rho,\lambda) = \frac{\partial p_c}{\partial\rho} A + \frac{\partial p_a}{\partial\rho} B = 0 \tag{2.5.1.9}$$

where

$$A = \log_2 \frac{2p_c}{p_a + p_c} \qquad B = \log_2 \frac{2p_a}{p_a + p_c} \ . \tag{2.5.1.10}$$

By fact (1), $p_c \leq p_a$, so $A \leq 0$ and $B \geq 0$. Moreover, if $\rho^*_{\theta,\lambda}$ is the solution of (2.5.1.9),

then $\rho_{\max} \leq \rho^*_{\theta,\lambda} \leq \rho_{\min}$, where $\rho_{\min}$ and $\rho_{\max}$ are defined in fact (2). Thus, given a fixed θ, for all $\lambda \leq 0.709/(1+\theta)^2$, the partial derivative of $C(\theta,\rho,\lambda)$ with respect to ρ is less than zero for all $0 \leq \rho \leq 1$. Therefore the worst case partial band jamming strategy for a small λ is full band jamming, i.e. $\rho=1$.

Since it is difficult to obtain the closed form for the optimum ρ for all λ, it is computed numerically. The resulting channel capacities are shown in Fig. 2.5.1.2 and Fig. 2.5.1.3. It is easy to see that for large θ and high rate, the optimal strategy of the jammer makes high signal power necessary. This is because the variance of Gaussian density is an inverse function of λ, and for large θ the jammer makes the communicator choose the erasure region with small power.

Two level jamming, four level quantization

With four level quantization it is also hard to obtain an analytical expression for the optimum ρ. Hence it is computed numerically. The channel transition probabilities are given by

$$\begin{aligned} p_a &= Q\Big((\theta-1)/\sigma\Big)\rho+\rho_0 \\ p_b &= \Big[Q\Big((1-\theta)/\sigma\Big)-Q\Big(1/\sigma\Big)\Big]\rho \\ p_c &= \Big[Q\Big(1/\sigma\Big)-Q\Big((1+\theta)/\sigma\Big)\Big]\rho \\ p_d &= Q\Big((1+\theta)/\sigma\Big)\rho \end{aligned} \tag{2.5.1.11}$$

where σ^2 is the same as that in (2.5.1.4). Fig. 2.5.1.4 and Fig. 2.5.1.5 show that the system performance at a high rate and large θ is much better than that with three level quantization. This is because that for large θ the erasure region becomes large as the variance of the received signal, σ^2, increases.

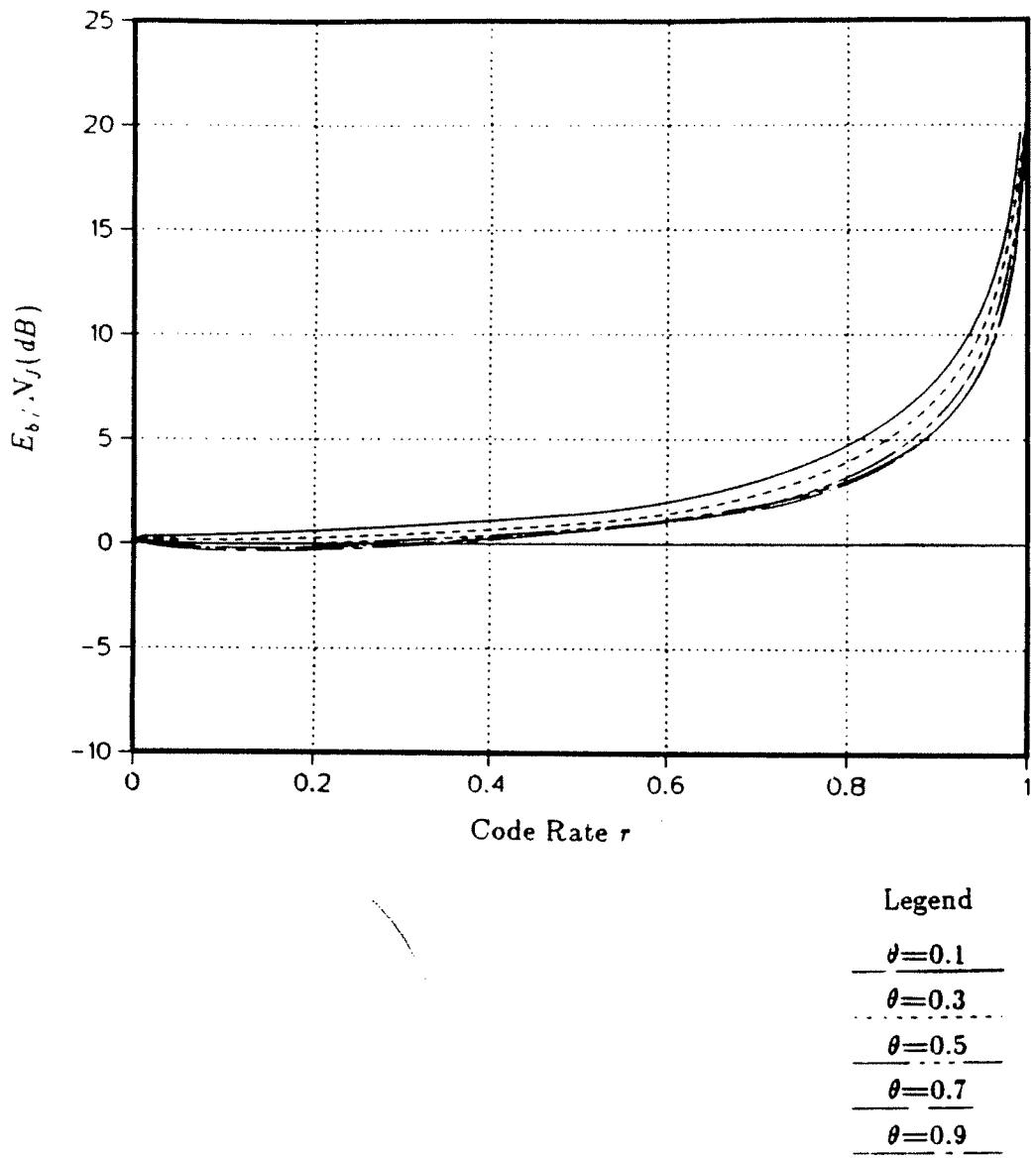


Figure 2.5.1.4. E_b/N_J needed for reliable communication at code rate r in two level Gaussian jamming and four level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

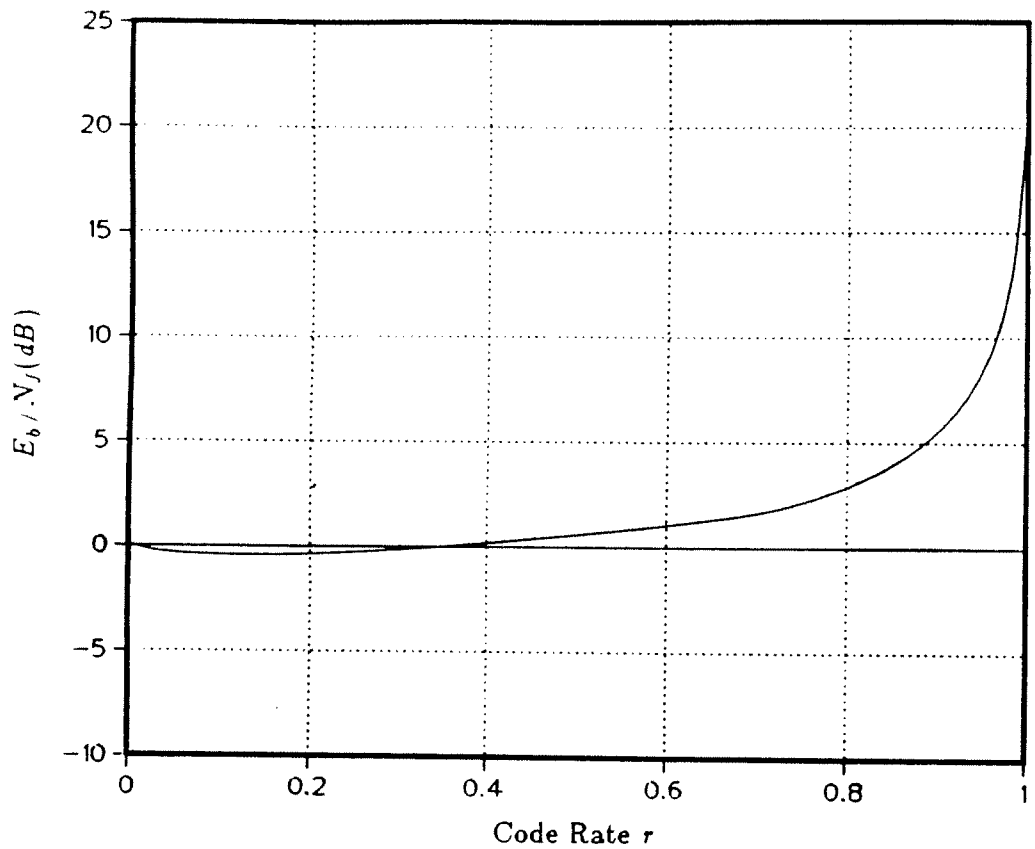


Figure 2.5.1.5. E_b/N_J needed for reliable communication at code rate r in two level Gaussian jamming and four level quantization system with optimal θ.

2.5.2. Three Level Jamming

In a three level jamming strategy, the jammer has at most three different power density levels over some fraction of transmitted signal bandwidth, that is, zero, $N_J\alpha_1/2$ and $N_J\alpha_2/2$ corresponding to the bandwidth fraction being jammed ρ_0, ρ_1 and ρ_2, respectively. The constraints to the jammer are (1) $\rho_0+\rho_1+\rho_2=1$ and (2) $\sum_{i=1}^{2}\alpha_i\rho_i=1$. The optimal values for the jammer noise levels and fractions being jammed are obtained numerically.

Three level jamming, Three level quantization

With three level quantization the expression for transition probabilities are given by

$$
\begin{aligned}
p_a &= \rho_0+\sum_{i=1}^{2} Q\Big((\theta-1)/\sigma_i\Big)\rho_i \\
p_b &= \sum_{i=1}^{2}\Big[Q\Big((1-\theta)/\sigma_i\Big)-Q\Big((1+\theta)/\sigma_i\Big)\Big]\rho_i \qquad (2.5.2.1)\\
p_c &= \sum_{i=1}^{2} Q\Big((1+\theta)/\sigma_i\Big)\rho_i
\end{aligned}
$$

where $\sigma_i^2=N_J\alpha_i/2E_c$, $i=1, 2$.

The numerical results are shown in Fig. 2.5.2.1 and Fig. 2.5.2.2 with fixed and optimal θ respectively. As discussed in the previous sections, for large θ, only a small amount of jamming power is needed to cause an erasure. Hence for large λ, the performance becomes worse as θ increases. It is very hard to obtain the optimal α_i and ρ_i analytically, but it is interesting to compare this system with two level Gaussian jammed system. In tone jamming the density is larger near the large (positive and negative values) than in the center or mean. In Gaussian jamming the

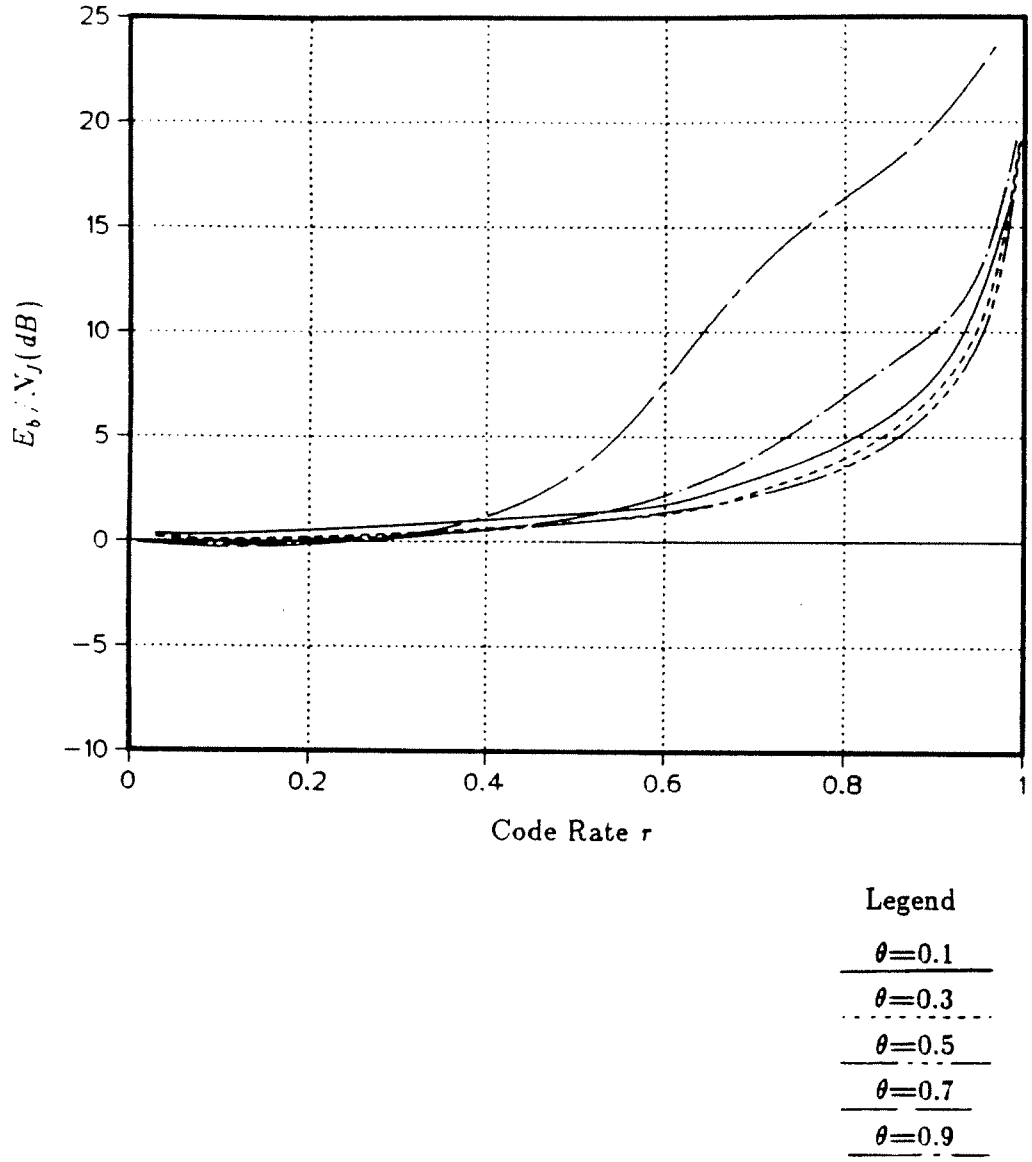


Figure 2.5.2.1. E_b/N_J needed for reliable communication at code rate r in three level Gaussian jamming and three level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

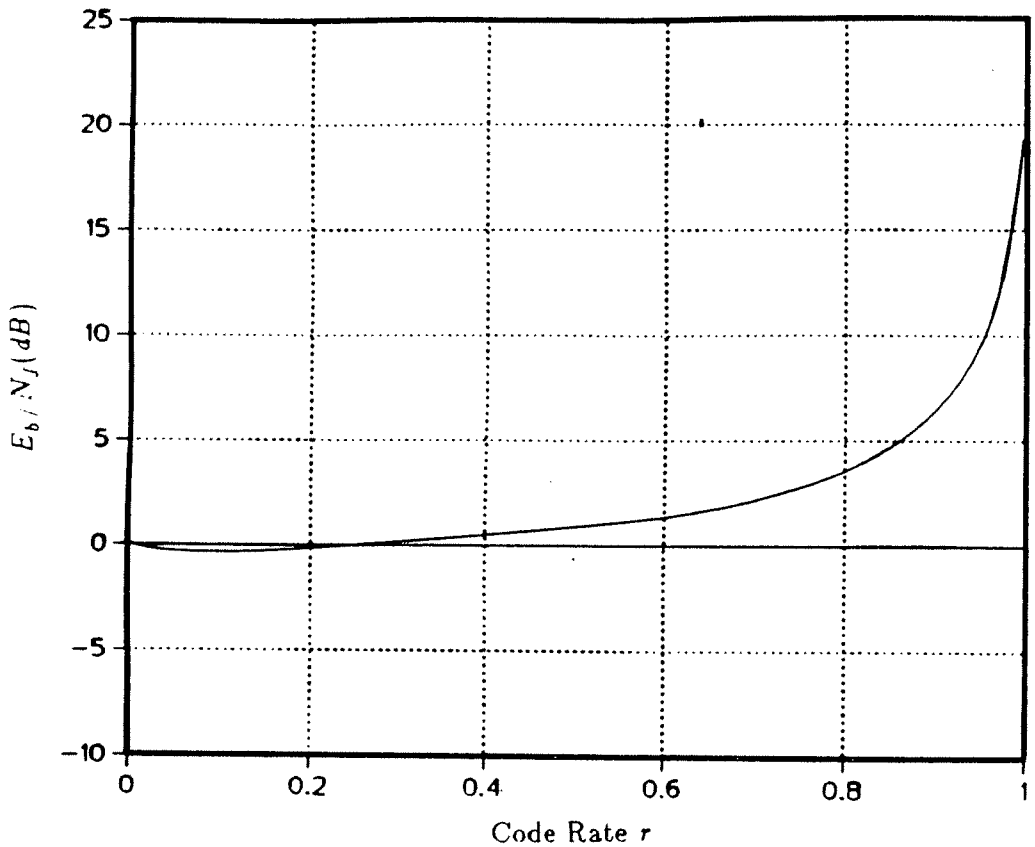


Figure 2.5.2.2. E_b/N_J needed for reliable communication at code rate r in three level Gaussian jamming and three level quantization system with optimal θ.

density is highly concentrated around the mean. Thus the performance of three level jamming is almost same as that of two level jamming, especially for small θ.

Three level jamming, four level quantization

With four level quantization the transition probabilities become

$$
\begin{aligned}
p_a &= \rho_0 + \sum_{i=1}^{2} Q\Big((\theta-1)/\sigma_i\Big)\rho_i \\
p_b &= \sum_{i=1}^{2} \Big[Q\Big((1-\theta)/\sigma_i\Big) - Q\Big(1/\sigma_i\Big)\Big]\rho_i \\
p_c &= \sum_{i=1}^{2} \Big[Q\Big(1/\sigma_i\Big) - Q\Big((1+\theta)/\sigma_i\Big)\Big]\rho_i \\
p_d &= \sum_{i=1}^{2} Q\Big((1+\theta)/\sigma_i\Big)\rho_i \ .
\end{aligned}
\tag{2.5.2.2}
$$

Fig. 2.5.2.3 and Fig. 2.5.2.4 show the performance for fixed θ and the optimal θ respectively. For large θ this system is considerable better than three level quantization. For example, at rate 0.6, θ=0.9, the receiver with four level quantization has about 6 dB gain. Three level jamming (for the optimal θ) does not degrade performance significantly compared with two level jamming.

2.5.3. Four Level Jamming

The optimal distribution of the jammer power is concentrated at most four values, and the corresponding power density levels are zero, $N_J\alpha_1/2$, $N_J\alpha_2/2$ and $N_J\alpha_3/2$, and the bandwidth fractions associated with the power density levels are ρ_0, ρ_1, ρ_2 and ρ_3, respectively. The jammer has an average power constraint and the optimal values for the jammer's parameters are calculated numerically.

The transition probabilities for the channel with four level quantization are given by

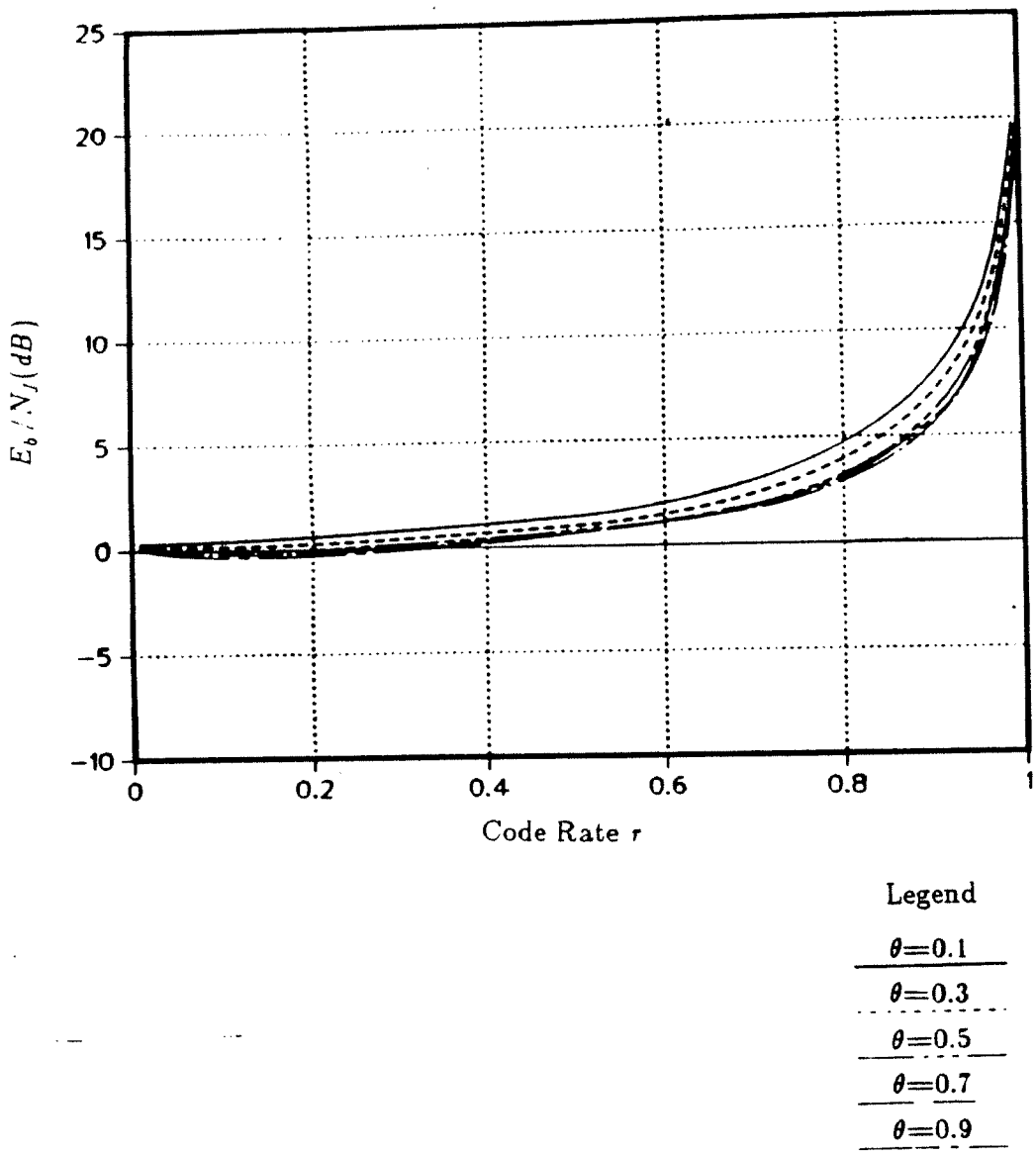


Figure 2.5.2.3. E_b/N_J needed for reliable communication at code rate r in three level Gaussian jamming and four level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

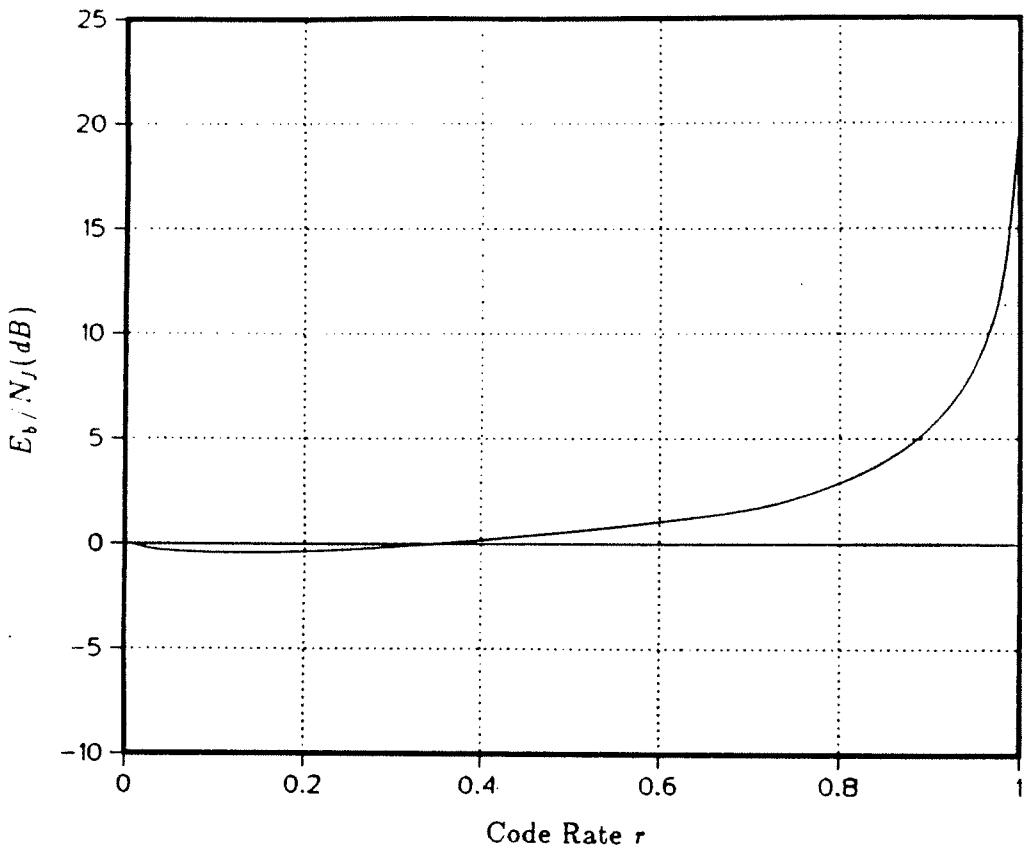


Figure 2.5.2.4. E_b/N_J needed for reliable communication at code rate r in three level Gaussian jamming and four level quantization system with optimal θ.

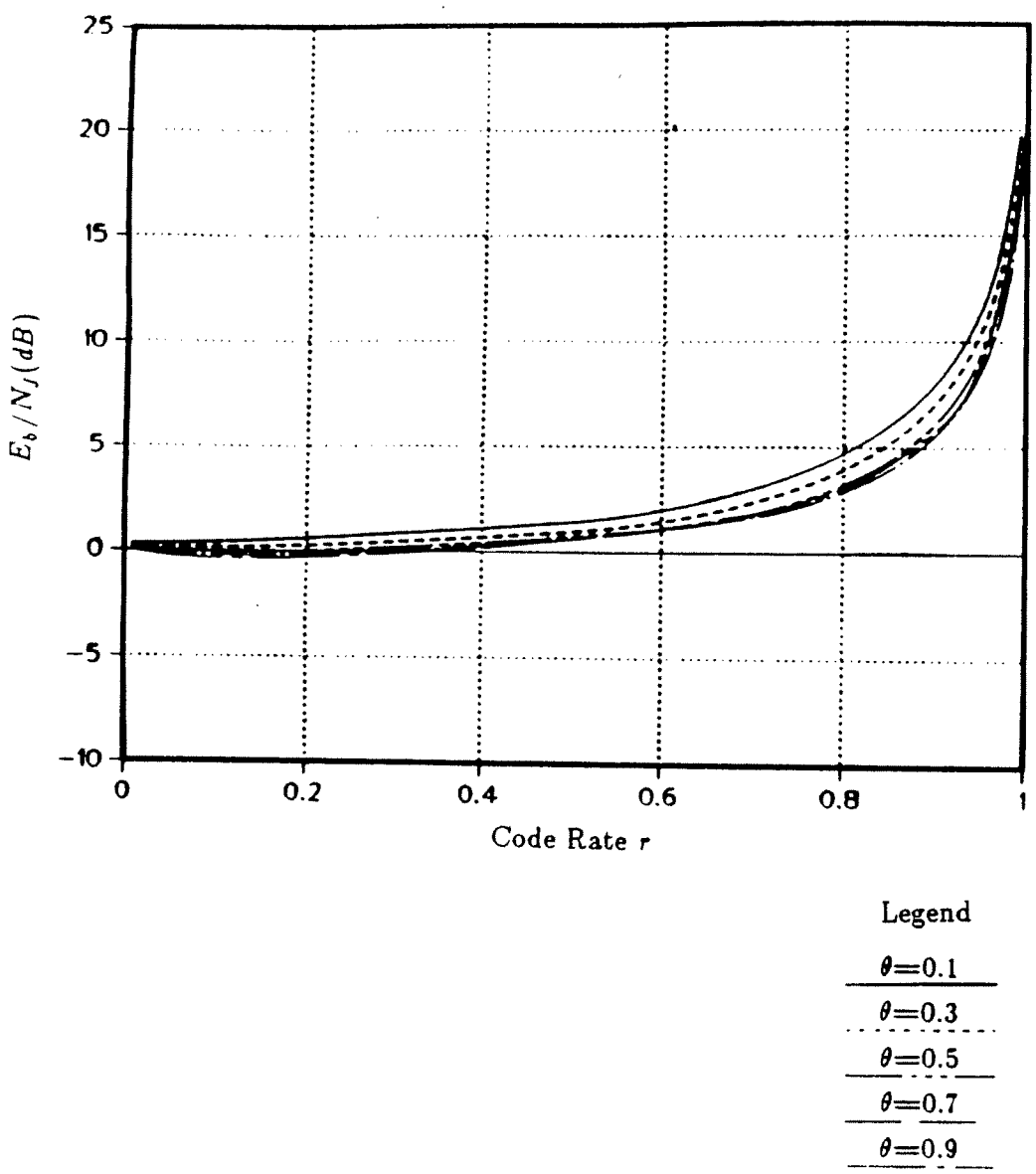


Figure 2.5.3.1. E_b/N_J needed for reliable communication at code rate r in four level Gaussian jamming and four level quantization system with θ=0.1, 0.3, 0.5, 0.7 and 0.9.

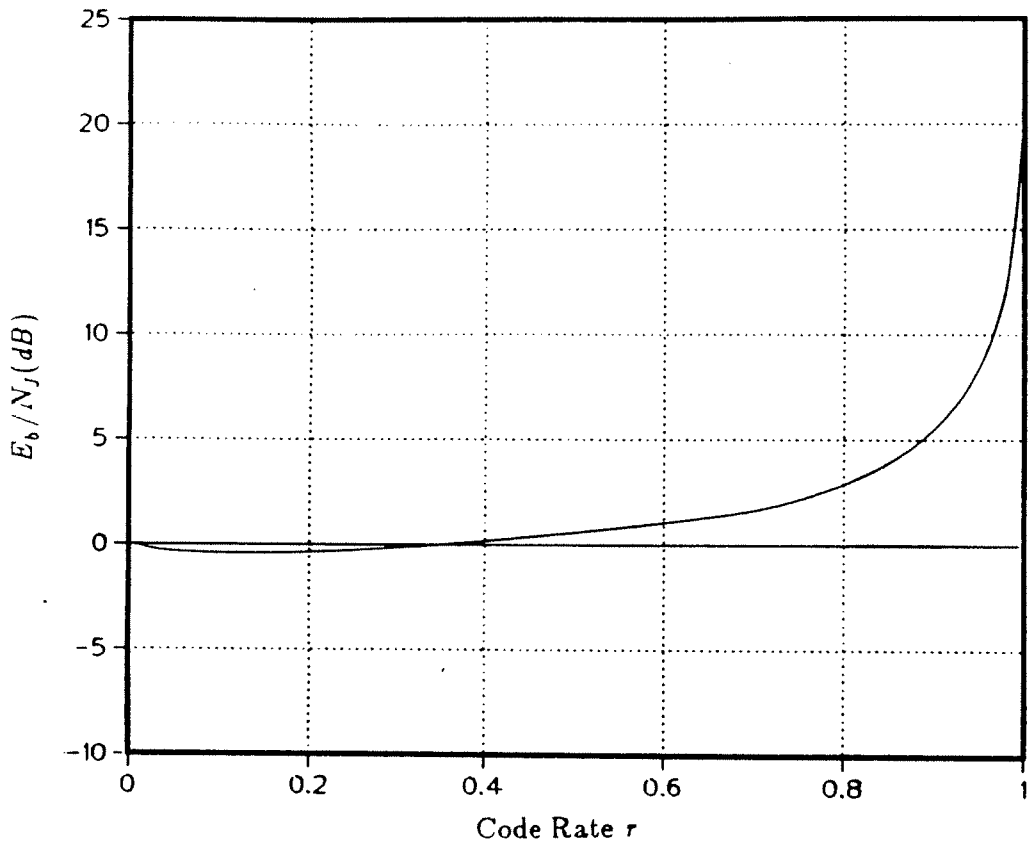


Figure 2.5.3.2. E_b/N_J needed for reliable communication at code rate r in four level Gaussian jamming and four level quantization system with optimal θ.

$$
\begin{aligned}
p_a &= \rho_0 + \sum_{i=1}^{3} Q\Big((\theta-1)/\sigma_i\Big)\rho_i \\
p_b &= \sum_{i=1}^{3} \Big[Q\Big((1-\theta)/\sigma_i\Big) - Q\Big(1/\sigma_i\Big)\Big]\rho_i \\
p_c &= \sum_{i=1}^{3} \Big[Q\Big(1/\sigma_i\Big) - Q\Big((1+\theta)/\sigma_i\Big)\Big]\rho_i \\
p_d &= \sum_{i=1}^{3} Q\Big((1+\theta)/\sigma_i\Big)\rho_i
\end{aligned}
\tag{2.5.3.1}
$$

where $\sigma_i^2 = N_J \alpha_i / 2E_c$, $i = 1, 2, 3$.

The numerical results of are shown in Fig. 2.5.3.1 and Fig. 2.5.3.2 for fixed θ and the optimal θ respectively. From the results we see that four level jamming does not degrade the performance significantly compared to two level jamming.

2.6. Discussion

In the previous sections 2.3, 2.4 and 2.5 we calculated the various capacities using different quantization schemes against several different jamming strategies.

For tone jamming and random phase tone jamming multilevel level jamming degrades the performance significantly compared to Gaussian jamming even though the receiver chooses the optimal threshold. In Gaussian jamming channel, since the noise density is concentrated around the mean of the received signal, multilevel jamming strategies do not degrade the system performance significantly compared to two level jamming. However with tone jamming significant degradation occurs when comparing multilevel jamming strategies to two level jamming strategies. We show the minimum required E_b/N_J at rate 1/2 against several jammer noise type when the optimal θ is used in table 2.6.1.

Level	Tone	Random Phase Tone	Gaussian
J2Q2	9.6	3.8	1.8
J2Q3	7.0	-0.5	0.9
J2Q4	3.6	-0.6	0.6
J3Q3	7.2	2.05	0.9
J3Q4	5.0	1.2	0.6
J4Q4	6.2	1.35	0.6

Table 2.6.1. $E_b/N_J(dB)$ necessary for reliable communication at rate 1/2 in coherent system. ($J2Q2$ is two level jamming and hard decision, $J2Q3$ is two level jamming and three level quantization, $J2Q4$ is two level jamming and four level quantization, $J3Q3$ is three level jamming and three level quantization, $J3Q4$ is three level jamming and four level quantization and $J4Q4$ is four level jamming and four level quantization).

In the next chapter 3 we discuss the noncoherent communication system with BFSK demodulation scheme.

CHAPTER III

CHANNELS WITH WORST-CASE JAMMING AND NONCOHERENT DEMODULATION

3.1. Introduction

In this chapter we consider worst-case jamming strategies for frequency-hopped spread-spectrum systems with binary frequency shift keying. Frequency-shift keying corresponds to changing the carrier frequency depending upon whether the input symbol is a +1 or a -1. The two modulated signals are orthogonal and thus the signal set dimensionality is two. The receiver is a noncoherent matched filter receiver (see Fig. 3.2.1).

As done in chapter 2, we consider different channels with two, three or four level jamming interference. The jammer has an average power constraint. For a hard decision channel the worst-case jamming strategy is to pulse between two values, one of them being zero [STA82a]. The ratio threshold technique introduced by Viterbi [VIT82] is used for three and four level quantization detector. In this technique the ratio of the outputs of the two noncoherent matched filters is quantized into a finite number of values. Throughout this chapter we analyze several different systems with two, three or four level quantization detector with one-dimensional or two-dimensional jamming.

In section 3.2 we explain the channel models and assumptions that we will make for this chapter. In section 3.3 we discuss the performance of the system in the presence of a jammer which places power in at most one of the two dimensions used by the transmitter. An error or erasure can occur only when jamming signal is in the opposite dimension of the transmitted signal. In section 3.4 we consider partial-band Gaussian noise jamming with an average power constraint. Finally in section 3.5 we compare the various systems considered in sections 3.3 and 3.4.

3.2. Channel Models

As discussed in chapter 2, we assume that there is a finite number of inputs (two) and a finite number of outputs of the channels (two, three or four). The jammer's strategies are to distribute the power level of a certain type of noise subject to an average power constraint. The types of noise to be discussed from the next section, are one-dimensional tone jamming and partial-band Gaussian jamming.

We use the same input and output alphabets introduced in section 2.2, i.e. input alphabet $A=\{+1,-1\}$ and output alphabet $B=\{\beta_1,\beta_2,\cdots,\beta_K\}$, where $K=2$, 3 and 4 depending upon the number of quantization levels.

The receiver model with noncoherent demodulation is shown in Fig. 3.2.1. Let the signal transmitted, $s_k(t)$, be

$$s_k(t) = \sqrt{\frac{2E_c}{T}}\cos(2\pi f_k t+\psi_k)\,, \qquad k=1,2 \tag{3.2.1}$$

where E_c is the channel symbol energy, T is the signal duration, f_k is carrier frequency corresponding to the input binary bit (i.e. $f_1=f_c+\Delta f$ and $f_2=f_c-\Delta f$ in (1.3.3.)) and ψ_k is uniformly distributed signal phase. The received signal $r(t)$ in

Fig. 3.2.1 contains the additive jamming signal and the transmitted signal. The jamming signal depends upon the noise type (i.e. tone jamming and Gaussian jamming). The output of the demodulator can be represented by a random vector $\mathbf{R}=(R_1,R_2)$. Each component of the random vector $\mathbf{R}$ is given by

$$R_k^2 = u_{ck}^2+u_{sk}^2 , \qquad k=1,2 \tag{3.2.2}$$

where u_{ck}^2, u_{sk}^2 are in-phase component and quadrature-phase component of the received signal respectively. The output of the square law combining receiver due to the jamming noise signal given by (1.4.6) can be represented by the random vector $\mathbf{I}=(I_1,I_2)$ where

$$I_k^2=n_{ck}^2+n_{sk}^2 , \qquad k=1, 2 \tag{3.2.3}$$

and

$$n_{ck} = \int_0^T \sqrt{2/E_c T} \cos(2\pi f_k t)\, j(t)\, dt \tag{3.2.4a}$$

$$n_{sk} = \int_0^T \sqrt{2/E_c T} \sin(2\pi f_k t)\, j(t)\, dt . \tag{3.2.4b}$$

In (3.2.4) the noise signal $j(t)$ depends on the jamming signal type.

One-Dimensional Tone Jamming

For one-dimensional tone jamming the noise signal $j(t)=Z_{i,0}\,\hat{j}(t)$ given by (1.4.7), where $\hat{j}(t)$ is given by (1.4.5). The random vector $\mathbf{I}=(I_1,I_2)$ in our model has one of two forms: either $\mathbf{I}=(Z,0)$ or $\mathbf{I}=(0,Z)$ where $Z=Z_{i,0}/\sqrt{\lambda}$ depending on if $V_0=+1$ or -1. As in the previous chapter $Z_{i,0}$ is the amplitude of the jamming signal in frequency slot i. We allow the jammer to optimize the distribution of $Z_{i,0}$ subject to an average power constraint $E[Z_{i,0}^2]=1$, or equivalently optimize the

distribution of Z subject to $E[Z^2]=1/\lambda$.

Gaussian Jamming

For Gaussian jamming the noise signal $j(t)$ is $Z_{i,0}\,\hat{j}(t)$ where $\hat{j}(t)$ is Gaussian random process with power spectral density $N_J/2$ centered at f_c. In (3.2.4) n_{ck} and n_{sk}, $k=1, 2$ have the same form given by

$$\begin{aligned} n_{ck} &= Z\ N_c \\ n_{sk} &= Z\ N_s \end{aligned} \tag{3.2.5}$$

where N_c and N_s are independent Gaussian random variables with mean 0 and variance 1, and Z is random variable with average power $E[Z^2]=1/2\lambda$. The jamming strategies are then to distribute Z with an average power $1/2\lambda$.

The channel transition probabilities are then given by

$$p(y \mid x) = \int_z^{\infty} p(y \mid x,z)\ dP(z) \tag{3.2.6}$$

where $p(y \mid x,z)$ is a conditional transition probability given $Z=z$, $x \in A$ and $y \in B$ and $P(z)$ is the distribution function of Z. For each noise type of jammer mentioned earlier in this section, the jammer has two, three or four levels subject to an average power constraint.

The channel output Y in Fig. 3.2.1 is then given by

$$Y = Q_L(R_1^2, R_2^2) \tag{3.2.7}$$

where Q_L is a two dimensional L level quantizer. Since the channel is symmetric we may assume $+1$ was sent for analysis. For a hard decision receiver the decision rule is given by

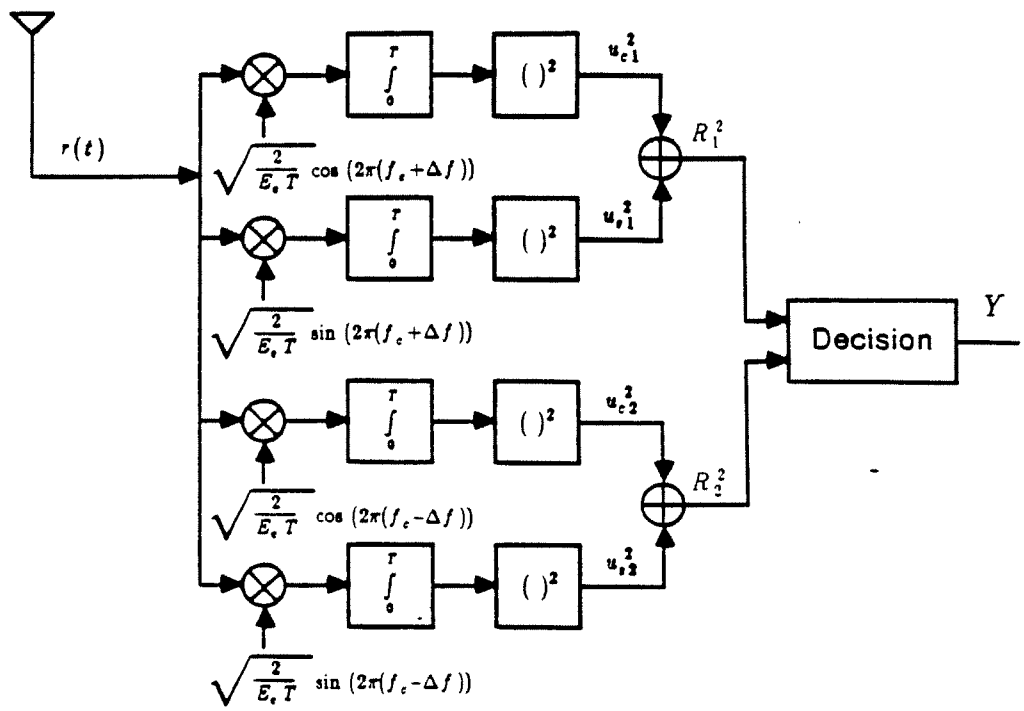


Figure 3.2.1. Receiver model for noncoherent system.

$$Q_2(R_1^2, R_2^2) = \begin{cases} 1 & R_2^2 \le R_1^2 \\ -1 & R_2^2 > R_1^2 \end{cases} . \tag{3.2.8}$$

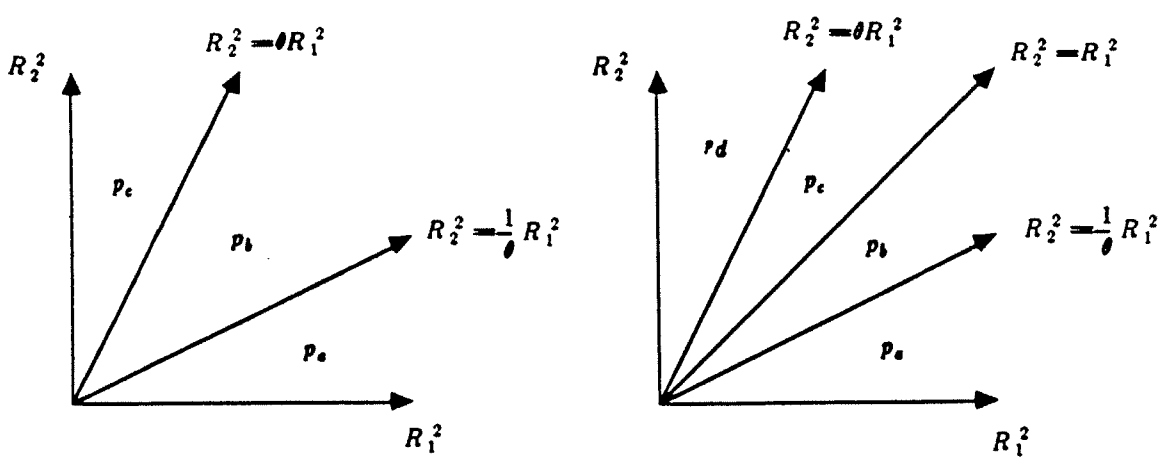
The three level quantizer considered is given by

$$Q_3(R_1^2, R_2^2) = \begin{cases} 1 & R_2^2 \le R_1^2/\theta \\ ? & R_1^2/\theta < R_2^2 \le R_1^2 \cdot \theta \\ -1 & R_2^2 > R_1^2 \cdot \theta \end{cases} \tag{3.2.9}$$

and the four level quantizer considered is given by

$$Q_4(R_1^2, R_2^2) = \begin{cases} 1, \text{ good} & R_2^2 \le R_1^2/\theta \\ 1, \text{ bad} & R_1^2/\theta < R_2^2 \le R_1^2 \\ -1, \text{ bad} & R_1^2 < R_2^2 \le R_1^2 \cdot \theta \\ -1, \text{ good} & R_2^2 > R_1^2 \cdot \theta \,. \end{cases} \tag{3.2.10}$$

In (3.2.9) and (3.2.10) we assume θ is between one and infinity. Fig. 3.2.2 shows the signal spaces and the decision regions for three and four level quantization receivers. The transition probabilities for the channel then can be written, for two level quantization, as



(a) Three level quantization (b) Four level quantization

Figure 3.2.2 Decision regions in three and four level quantization receiver.

$$p(y \mid x) = \begin{cases} 1-p & x=1,\ y=1 \\ p & x=1,\ y=-1. \end{cases} \tag{3.2.11}$$

For three level quantization the transition probabilities are

$$p(y \mid x) = \begin{cases} p_a & x=1,\ y=1 \\ p_b & x=1,\ y=? \\ p_c & x=1,\ y=-1. \end{cases} \tag{3.2.12}$$

For four level quantization the transition probabilities are

$$p(y \mid x) = \begin{cases} p_a & x=1,\ y=1, \text{ good} \\ p_b & x=1,\ y=1, \text{ bad} \\ p_c & x=1,\ y=-1, \text{ bad} \\ p_d & x=1,\ y=-1, \text{ good.} \end{cases} \tag{3.2.13}$$

The transition probabilities depend upon the jammer power distribution and the type of the noise.

3.3. One-Dimensional Tone Jamming

The jammer signal is added to the transmitted signal over the channel and this perturbing signal appears only in one dimension, that is, the jamming noise has the form : $\mathrm{I} = (Z,0)$ or $(0,Z)$ with probability one half each where Z is the random variable with an average power

$$E[Z^2] = 1/\lambda. \tag{3.3.1}$$

The jammer strategies are to distribute Z subject to an average power constraint $1/\lambda$.

3.3.1. Two Level Jamming

In this section we consider three different detectors, one hard decision detector (i.e. two level quantization) and two soft decision detectors (i.e. three level and four level quantizations).

For a hard decision channel the worst-case jamming strategy is to concentrate Z on two or fewer power levels: one just large enough to overcome the signal and the other zero. Assume the jammer has three levels, z_1, z_2 and 0, and $z_1=1$ with some probability greater than zero. If $0 \leq z_2 < 1$ then the capacity does not depend on z_2. If $z_2 > 1$ then the capacity is the same as if $z_2=1$. Thus two level jamming is the worst-case jamming strategy.

Two level jamming, two level quantization

With a hard decision detector (two level quantization) the crossover probability of the channel is given by

$$p = 0.5\rho_1 \tag{3.3.1.1}$$

where ρ_1 is the probability being jammed. The worst-case distribution of Z the jammer employs is then

$$Pr\{Z=z\} = \begin{cases} 1 & z=\sqrt{1/\lambda} \\ 0 & \text{otherwise} \end{cases} \tag{3.3.1.2}$$

for $\lambda \leq 1$ and

$$Pr\{Z=z\} = \begin{cases} 1/\lambda & z=1 \\ 1-1/\lambda & z=0 \end{cases} \tag{3.3.1.3}$$

for $\lambda > 1$. We can rewrite the crossover probability of the channel, p_b shown in

(3.3.1.1), as

$$p = \begin{cases} 0.5 & \lambda \leq 1 \\ 0.5/\lambda & \lambda > 1. \end{cases} \tag{3.3.1.4}$$

The channel capacity is then given by

$$C = 1 - H_2(p) = \begin{cases} 0 & \lambda \leq 1 \\ 1 - H_2(0.5/\lambda) & \lambda > 1 \end{cases} \tag{3.3.1.5}$$

where $H_2(x) = -x\log_2 x - (1-x)\log_2(1-x)$. The minimum bit signal-to-noise ratio needed to obtain the channel capacity as a code rate r is shown in Fig. 3.3.1.1.

Two level jamming, three level quantization

With three level quantization the jammer concentrates his power on two values, that is, zero and a value greater than or equal to $1/\sqrt{\theta}$. As done in chapter 2, we determine the optimum strategy and the resulting capacity.

For $\lambda \leq 1/\theta$ the jammer has enough power so that $Z = \sqrt{\theta}$ with probability one. For this strategy the crossover probability of the resulting binary symmetric channel is one half, and the resulting channel capacity is zero.

For $1/\theta < \lambda \leq \theta$ the jammer no longer has enough power to cause an error with probability one half. The jamming strategies in this region are

$$Z = \begin{cases} \sqrt{\theta} & w.p. \ \ 1/\lambda\theta \\ 0 & w.p. \ \ 1 - 1/\lambda\theta \end{cases} \tag{3.3.1.6}$$

or

$$Z = \begin{cases} 1/\sqrt{\theta} & w.p. \ \ 1 \\ 0 & w.p. \ \ 0. \end{cases} \tag{3.3.1.7}$$

The first strategy yields capacity $1 - H_2(1/2\lambda\theta)$ and the second strategy yields

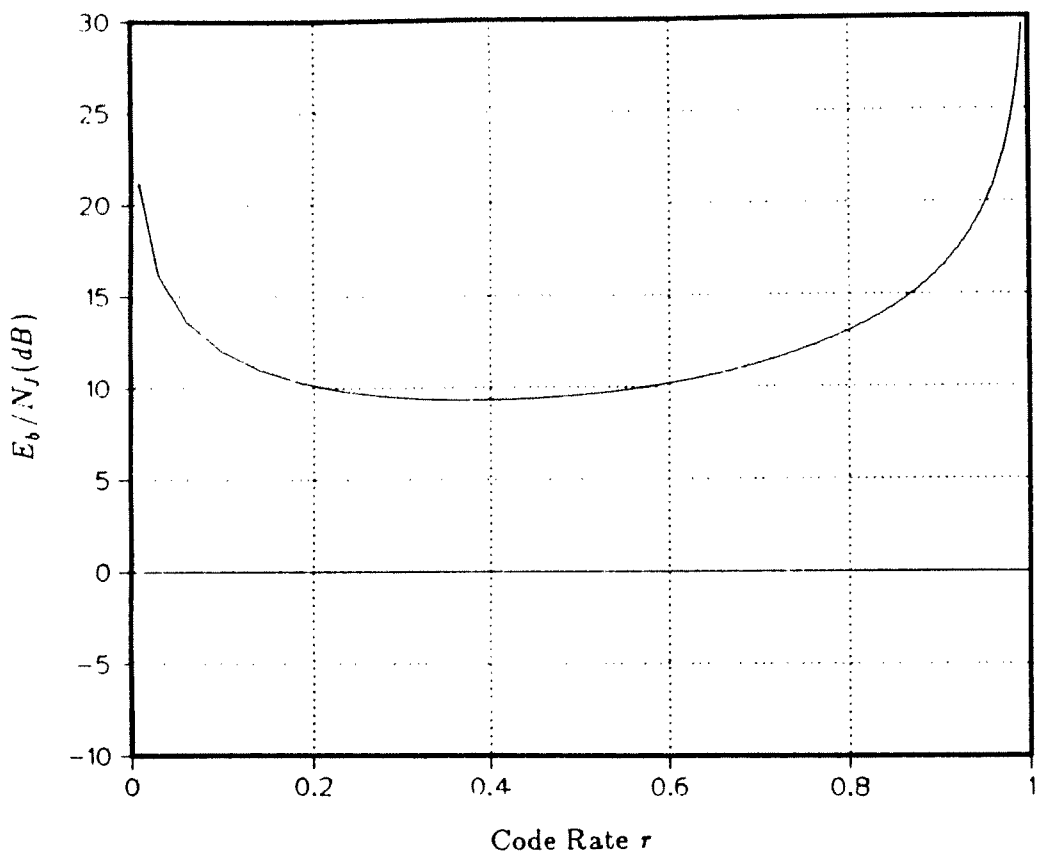


Figure 3.3.1.1 E_b/N_J needed for reliable communication at code rate r in one-dimensional two level tone jamming and hard decision system.

capacity one half. Therefore the capacity in this region is is the minimum of the two capacities:

$$C = \min(1/2,\ 1-H_2(1/2\lambda\theta)). \tag{3.3.1.8}$$

If $1/2\lambda\theta<0.11$ the capacity is 1/2. Thus for $1/\theta<\lambda\leq\theta$ the capacity is given by

$$C = \begin{cases} 1-H_2(1/2\lambda\theta) & 1/\theta<\lambda\leq 4.545/\theta \\ 1/2 & 4.545/\theta<\lambda\leq\theta. \end{cases} \tag{3.3.1.9}$$

For $\lambda>\theta$ the feasible jamming strategies are

$$Z = \begin{cases} \sqrt{\theta} & w.p.\ \ 1/\lambda\theta \\ 0 & w.p.\ \ 1-1/\lambda\theta \end{cases} \tag{3.3.1.10}$$

$$Z = \begin{cases} 1/\sqrt{\theta} & w.p.\ \ \theta/\lambda \\ 0 & w.p.\ \ 1-\theta/\lambda. \end{cases} \tag{3.3.1.11}$$

The resulting capacities are $C=1-H_2(1/2\lambda\theta)$ and $C=1-\theta/2\lambda$ respectively. Thus the overall channel capacity is given by

$$C = \begin{cases} 0 & 0\leq\lambda\leq 1/\theta \\ 1-H_2(1/2\lambda\theta) & 1/\theta<\lambda\leq 4.545/\theta \\ 1/2 & 4.545/\theta<\lambda\leq\theta \\ \min(1-\theta/2\lambda,\ 1-H_2(1/2\lambda\theta)) & \lambda>\theta. \end{cases} \tag{3.3.1.12}$$

Notice for $\theta<2.13$, $4.545/\theta>\theta$ so that the third region in (3.3.1.12) is empty. For $\theta>2.13$ this region is nonempty and the capacity is a constant independent of λ.

In Fig. 3.3.1.2 we show the minimum bit signal-to-noise ratio needed for reliable communication at a code rate r for thresholds $\theta = 1.5$, 2, 3, 5 and 10. For small λ large θ makes the performance better. The bit signal-to-noise ratio vs. code rate minimized over θ is shown in Fig. 3.3.1.3. As θ approaches infinity the optimal

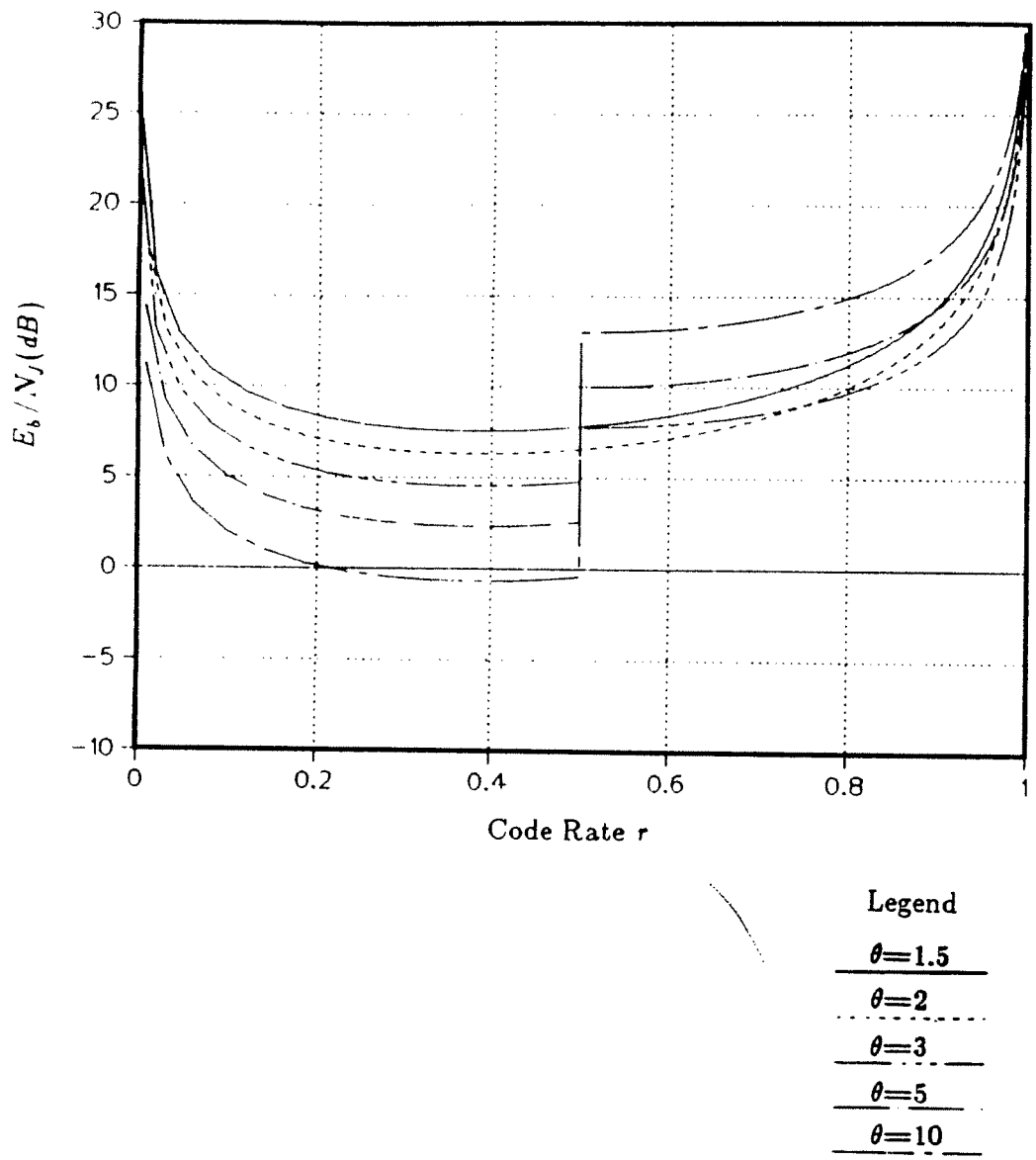


Figure 3.3.1.2. E_b/N_J needed for reliable communication at code rate r in one-dimensional two level tone jamming and three level quantization system with θ=1.5, 2, 3, 5, and 10.

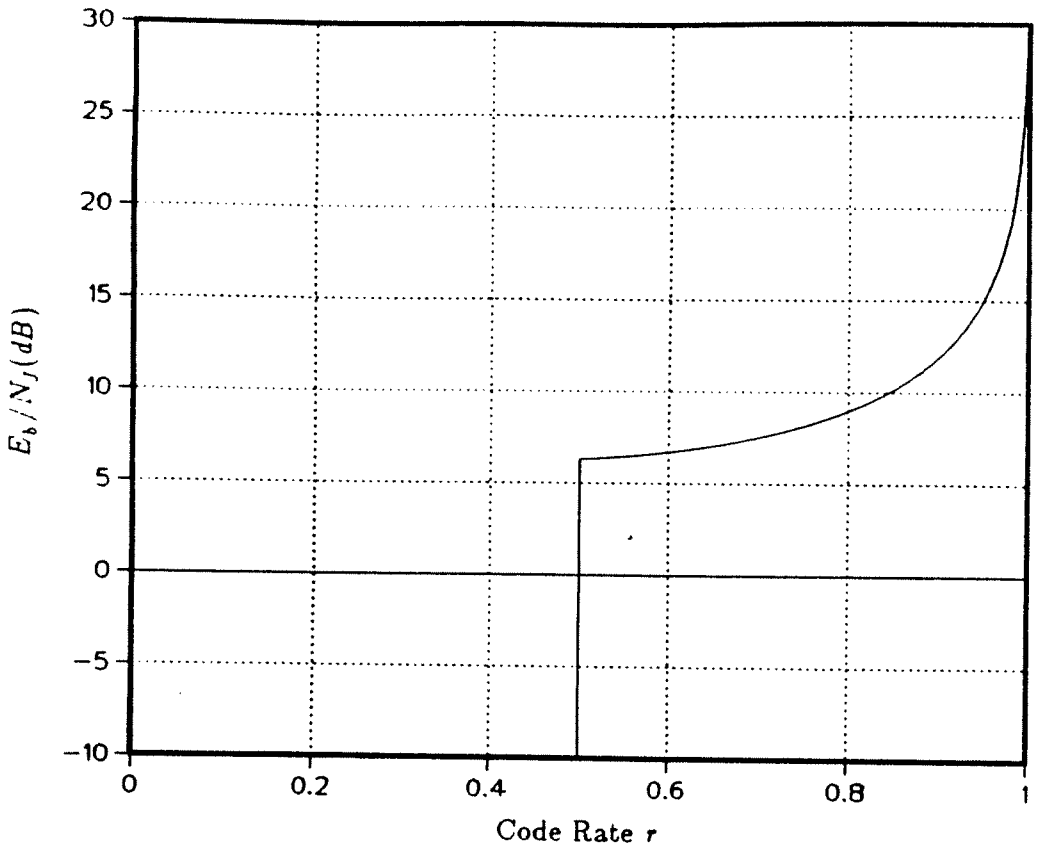


Figure 3.3.1.3. E_b/N_J needed for reliable communication at code rate r in one-dimensional two level tone jamming and three level quantization system with optimal θ.

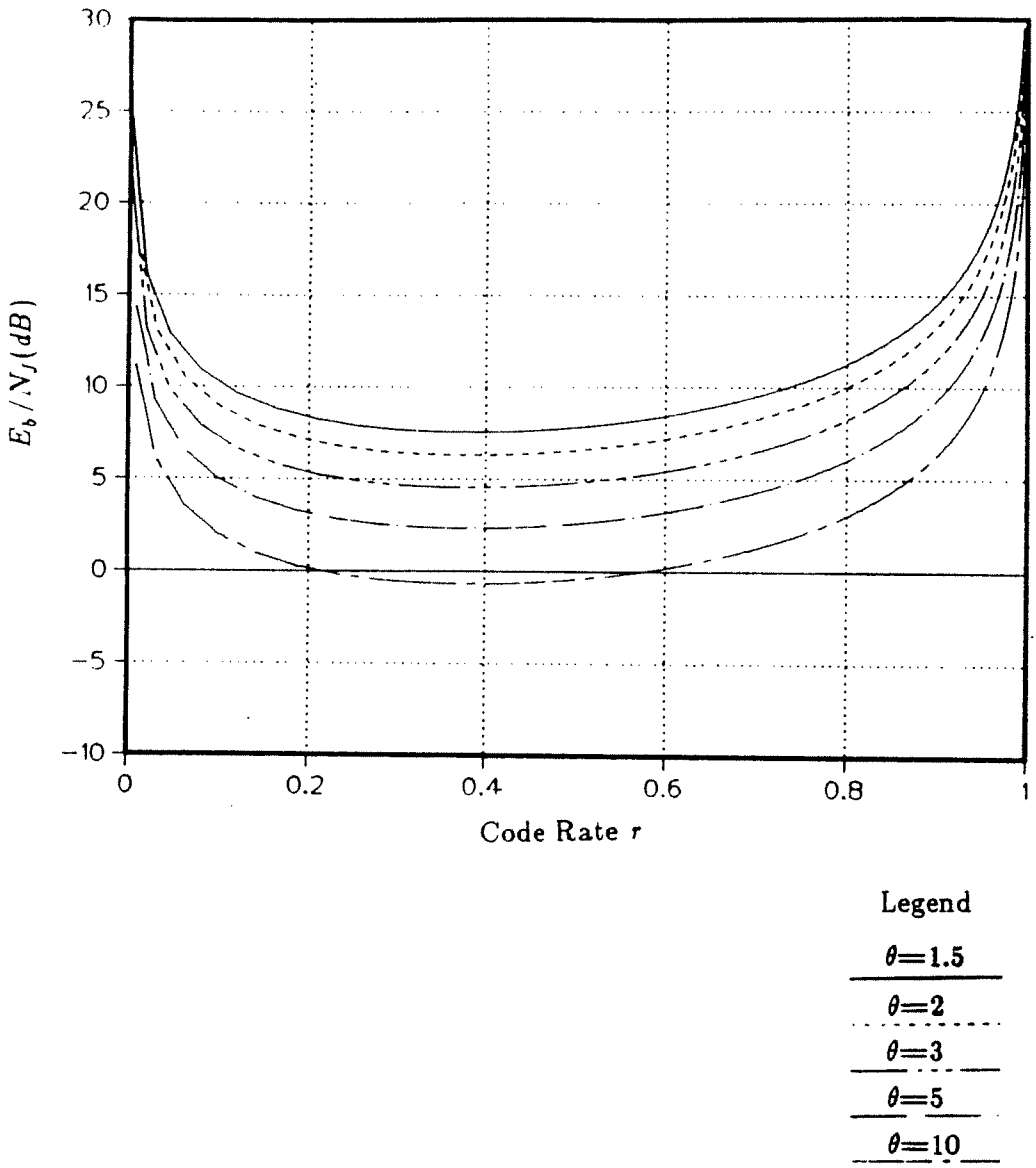


Figure 3.3.1.4. E_b/N_J needed for reliable communication at code rate r in one-dimensional two level tone jamming and four level quantization system with θ=1.5, 2, 3, 5 and 10.

channel capacity can not be less than one half regardless of the jammer power.

Two level jamming, four level quantization

With four level quantization and two level jamming the optimal jamming strategy is given by

$$Z = \begin{cases} \sqrt{\theta} & w.p. \quad 1/\lambda\theta \\ 0 & w.p. \quad 1-1/\lambda\theta. \end{cases} \tag{3.3.1.13}$$

The resulting capacity is then given by

$$C = \begin{cases} 0 & \lambda \leq 1/\theta \\ 1-H_2(1/2\lambda\theta) & \lambda > 1/\theta. \end{cases} \tag{3.3.1.14}$$

In Fig. 3.3.1.4 we show the minimum bit signal-to-noise ratio needed to obtain code rate given ratio threshold θ= 1.5, 2, 3, 5 and 10. As θ increases, the performance becomes better. Notice that the channel capacity is close to one as θ approaches infinity regardless of the jamming power.

3.3.2. Three Level Jamming

In this section we discuss a system with three level and four level quantization under one-dimensional three level jamming environment.

The optimal strategy for the jammer is to concentrate Z on three or fewer power levels : one zero and the others (one or two) just large enough to overcome the signals considered for detection at the receiver.

Three level jamming, Three level quantization

With three level quantization the optimal distribution of Z is concentrated at three points, namely zero, $1/\sqrt{\theta}$ and $\sqrt{\theta}$.

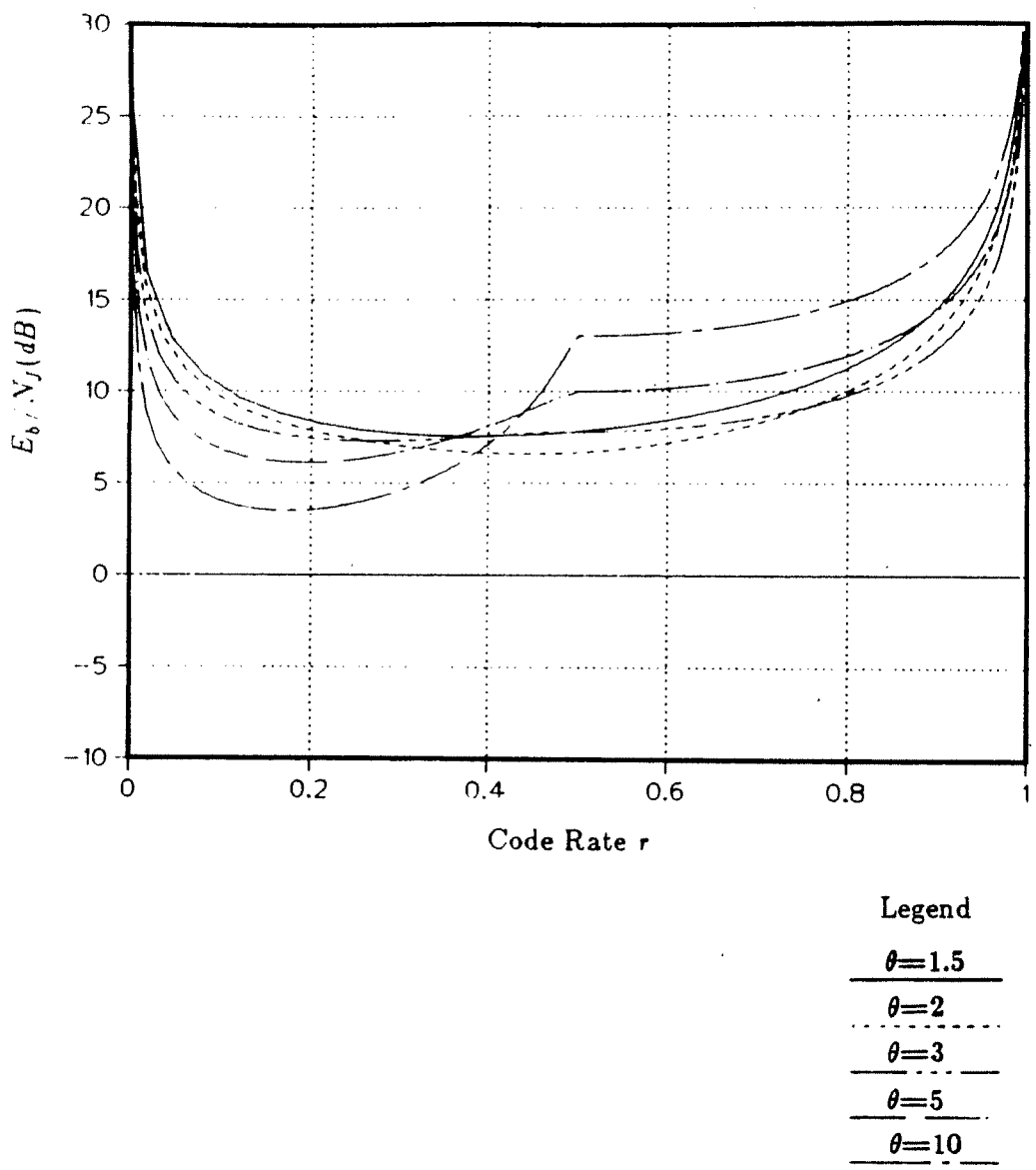


Figure 3.3.2.1. E_b/N_J needed for reliable communication at code rate r in one-dimensional three level tone jamming and three level quantization system with θ=1.5, 2, 3, 5 and 10.

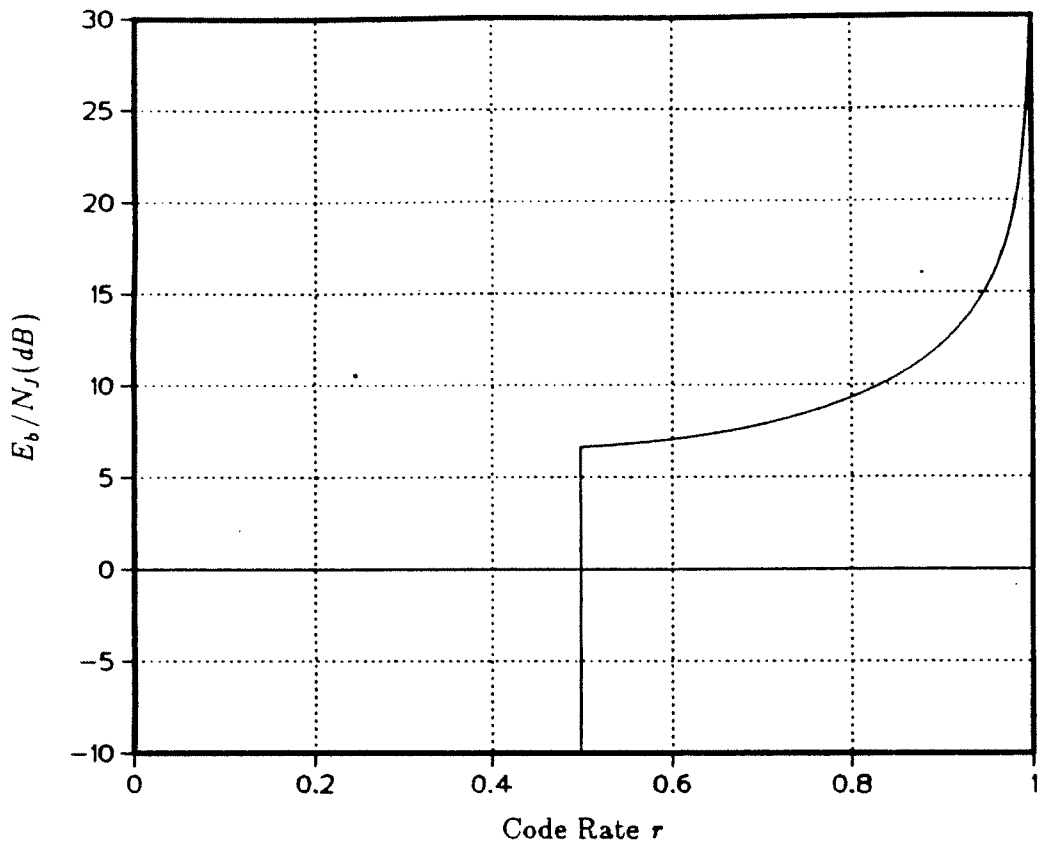


Figure 3.3.2.2. E_b/N_J needed for reliable communication at code rate r in one-dimensional three level tone jamming and three level quantization system with optimal θ.

For some λ and θ, the worst-case distribution the jammer employs is the same as (3.3.1.6) for $\lambda \leq 1/\theta$ and

$$Pr\{Z=z\} = \begin{cases} \rho_1 & z=z_1 \\ \rho_2 & z=z_2 \\ 1-\rho_1-\rho_2 & z=0 \end{cases} \tag{3.3.2.1}$$

for $\lambda > 1/\theta$, where $z_1 = 1/\sqrt{\theta}$ and $z_2 = \sqrt{\theta}$, and the transition probabilities p_a, p_b and p_c are as follows :

$$\begin{aligned} p_a &= 1-0.5(\rho_1+\rho_2) \\ p_b &= 0.5\rho_1 \\ p_c &= 0.5\rho_2 \ . \end{aligned} \tag{3.3.2.2}$$

The channel capacity is given by

$$C = p_a \log_2 p_a + p_c \log_2 p_c - (p_a + p_c)\log_2(p_a + p_c) + p_a + p_c \ . \tag{3.3.2.3}$$

The minimum bit signal-to-noise ratios vs code rates are shown in Fig. 3.3.2.1 with several fixed θ, and the performance of the system with the optimal θ is shown in Fig.3.3.2.2. With the optimal θ, the capacity can not be less than one half independent of the average jammer power.

Three level jamming, four level quantization

With four level quantization the distribution of Z is concentrated at three points, that is, zero, $z_1 \geq 1/\sqrt{\theta}$, $z_2 \geq 1$, where $Pr\{Z=z_1\}=\rho_1$ and $Pr\{Z=z_2\}=\rho_2$. There are three possible situations depending upon z_1 and z_2 for analysis :

(1) If $z_1 \in [1/\sqrt{\theta}, 1)$ and $z_2 \in [1, \sqrt{\theta})$ then

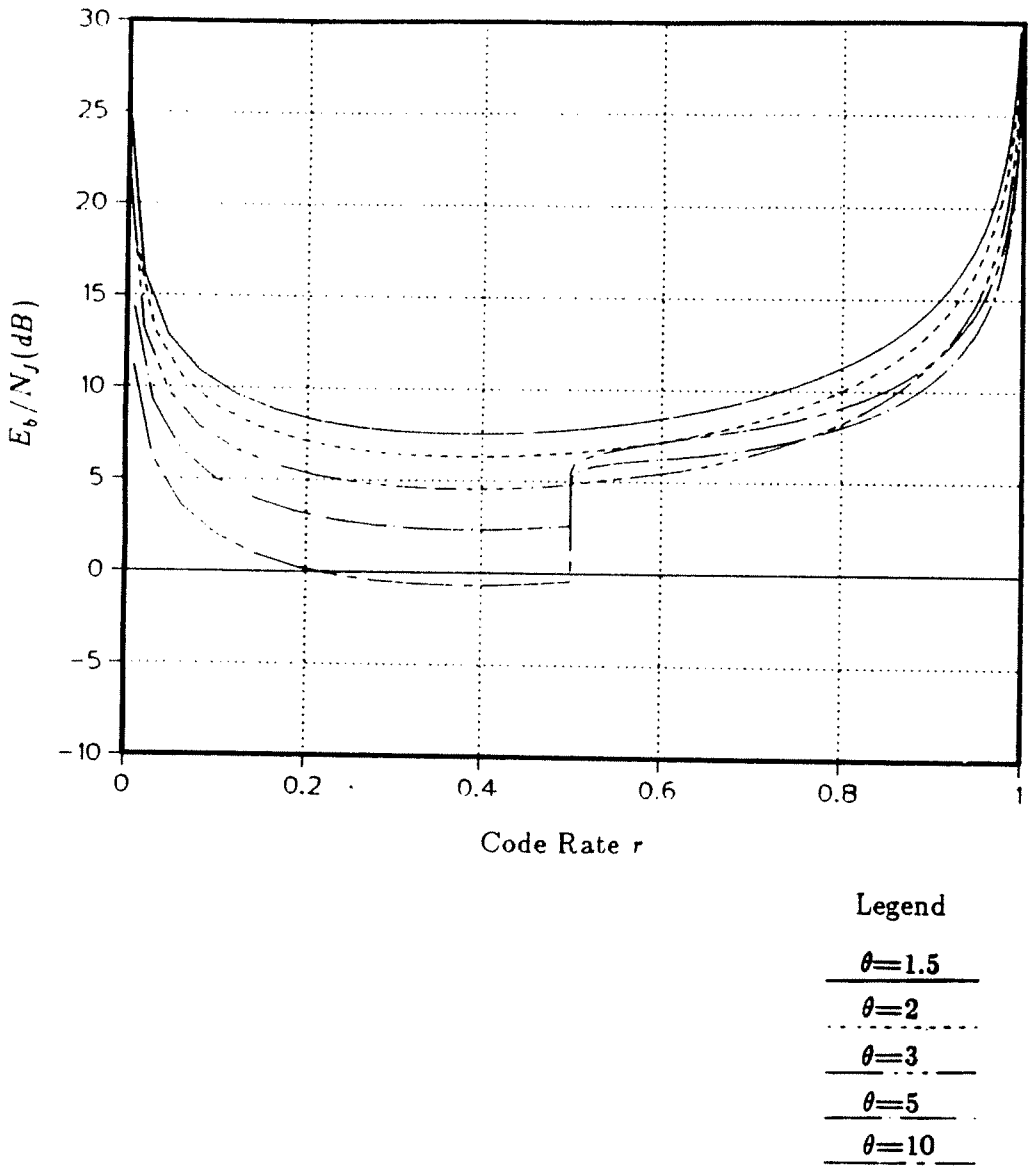


Figure 3.3.2.3. E_b/N_J needed for reliable communication at code rate r in one-dimensional three level tone jamming and four level quantization system with $\theta=$ 1.5, 2, 3, 5 and 10.

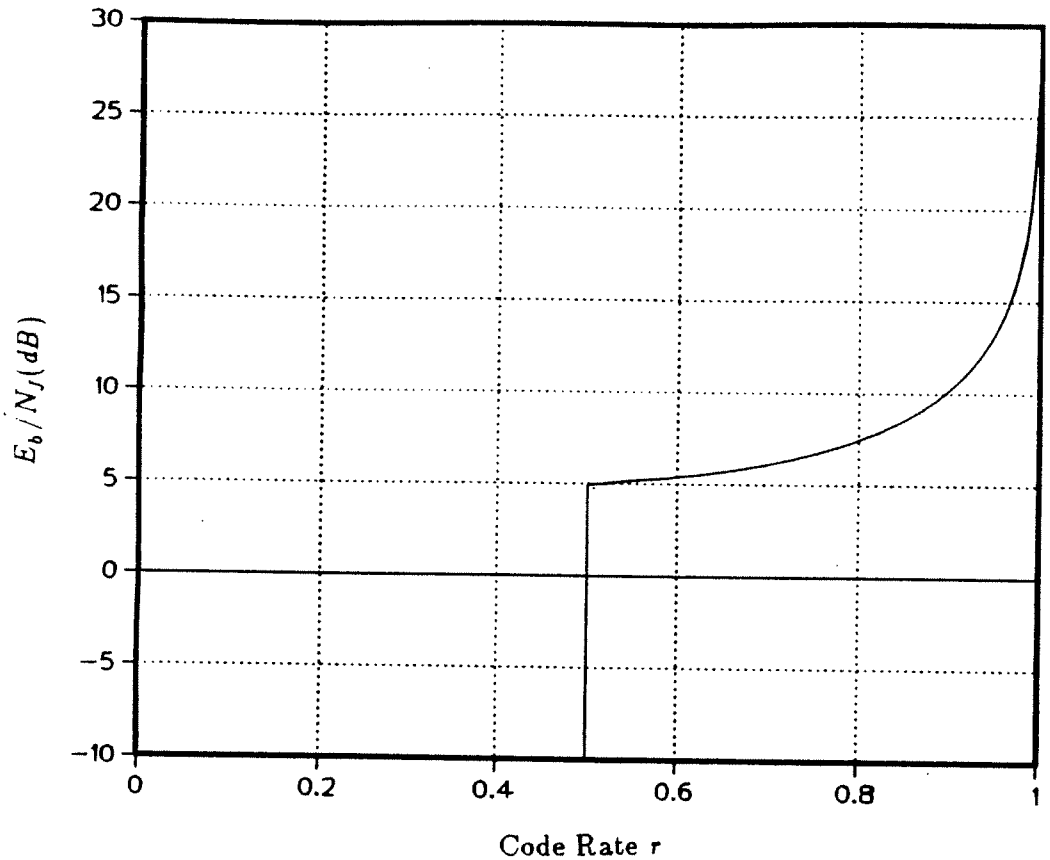


Figure 3.3.2.4. E_b/N_J needed for reliable communication at code rate r in one-dimensional three level tone jamming and four level quantization system with optimal θ.

$$\begin{aligned} p_a(z_1) &= 0.5 & p_a(z_2) &= 0.5 \\ p_b(z_1) &= 0.5 & p_b(z_2) &= 0 \\ p_c(z_1) &= 0 & p_c(z_2) &= 0.5 \\ p_d(z_1) &= 0 & p_d(z_2) &= 0. \end{aligned} \tag{3.3.2.4}$$

(2) If $z_1 \in [1/\sqrt{\theta}, 1)$ and $z_2 \in [\sqrt{\theta}, \infty)$ then

$$\begin{aligned} p_a(z_1) &= 0.5 & p_a(z_2) &= 0.5 \\ p_b(z_1) &= 0.5 & p_b(z_2) &= 0 \\ p_c(z_1) &= 0 & p_c(z_2) &= 0 \\ p_d(z_1) &= 0 & p_d(z_2) &= 0.5. \end{aligned} \tag{3.3.2.5}$$

(3) If $z_1 \in [1, \sqrt{\theta})$ and $z_2 \in [\sqrt{\theta}, \infty)$ then

$$\begin{aligned} p_a(z_1) &= 0.5 & p_a(z_2) &= 0.5 \\ p_b(z_1) &= 0 & p_b(z_2) &= 0 \\ p_c(z_1) &= 0.5 & p_c(z_2) &= 0 \\ p_d(z_1) &= 0 & p_d(z_2) &= 0.5. \end{aligned} \tag{3.3.2.6}$$

The transition probabilities are then given by

$$\begin{aligned} p_a &= \sum_{i=1}^{2} p_a(z_i)\rho_i + \rho_0 \\ p_b &= \sum_{i=1}^{2} p_b(z_i)\rho_i \\ p_c &= \sum_{i=1}^{2} p_c(z_i)\rho_i \\ p_d &= \sum_{i=1}^{2} p_d(z_i)\rho_i \ . \end{aligned} \tag{3.3.2.7}$$

The channel capacity is then given by

$$\begin{aligned} C = {} & p_a \log_2 p_a + p_b \log_2 p_2 + p_c \log_2 p_c + p_d \log_2 p_d + 1 \\ & -(p_a + p_d)\log_2(p_a + p_d) - (p_b + p_c)\log_2(p_b + p_c). \end{aligned} \tag{3.3.2.8}$$

Fig. 3.3.2.3 shows the minimum bit signal-to-noise ratio needed for reliable communication at code rate r with various θ. Fig. 3.3.2.4 also shows the minimum bit

signal-to-noise ratio as a function of code rate when the optimal θ is chosen.

3.3.3. Four Level Jamming

In this section we discuss the four level jamming and four level quantization at the receiver. The distribution of Z is concentrated at four points, that is, zero, $1/\sqrt{\theta}$, 1 and $\sqrt{\theta}$.

For $\lambda \leq 1/\theta$, the optimal strategy is given by

$$Pr\{Z=z\} = \begin{cases} 1 & z=\sqrt{1/\lambda} \\ 0 & \text{otherwise.} \end{cases} \tag{3.3.3.1}$$

In this case the capacity is zero. For $\lambda > 1$, $Pr\{Z=z\}$ is given by

$$Pr\{Z=z\} = \begin{cases} \rho_1 & z=z_1 \\ \rho_2 & z=z_2 \\ \rho_3 & z=z_3 \\ 1-\rho_1-\rho_2-\rho_3 & z=0 \end{cases} \tag{3.3.3.2}$$

where $z_1=1/\sqrt{\theta}$, $z_2=1$ and $z_3=\sqrt{\theta}$, and ρ_1, ρ_2 and ρ_3 are probabilities being jammed corresponding to z_1, z_2 and z_3, respectively. The transition probabilities p_a, p_b, p_c and p_d are then given by

$$\begin{aligned} p_a &= 1-0.5(\rho_1+\rho_2+\rho_3) \\ p_b &= 0.5\rho_1 \\ p_c &= 0.5\rho_2 \\ p_d &= 0.5\rho_3 \end{aligned} \tag{3.3.3.3}$$

and the channel capacity is determined using (3.3.2.8). The numerical results showing the relation of bit signal-to-noise and code rate given θ are shown in Fig. 3.3.3.1. The minimum bit signal-to-noise ratios needed to obtain code rates with the optimal θ is shown in Fig. 3.3.3.2..

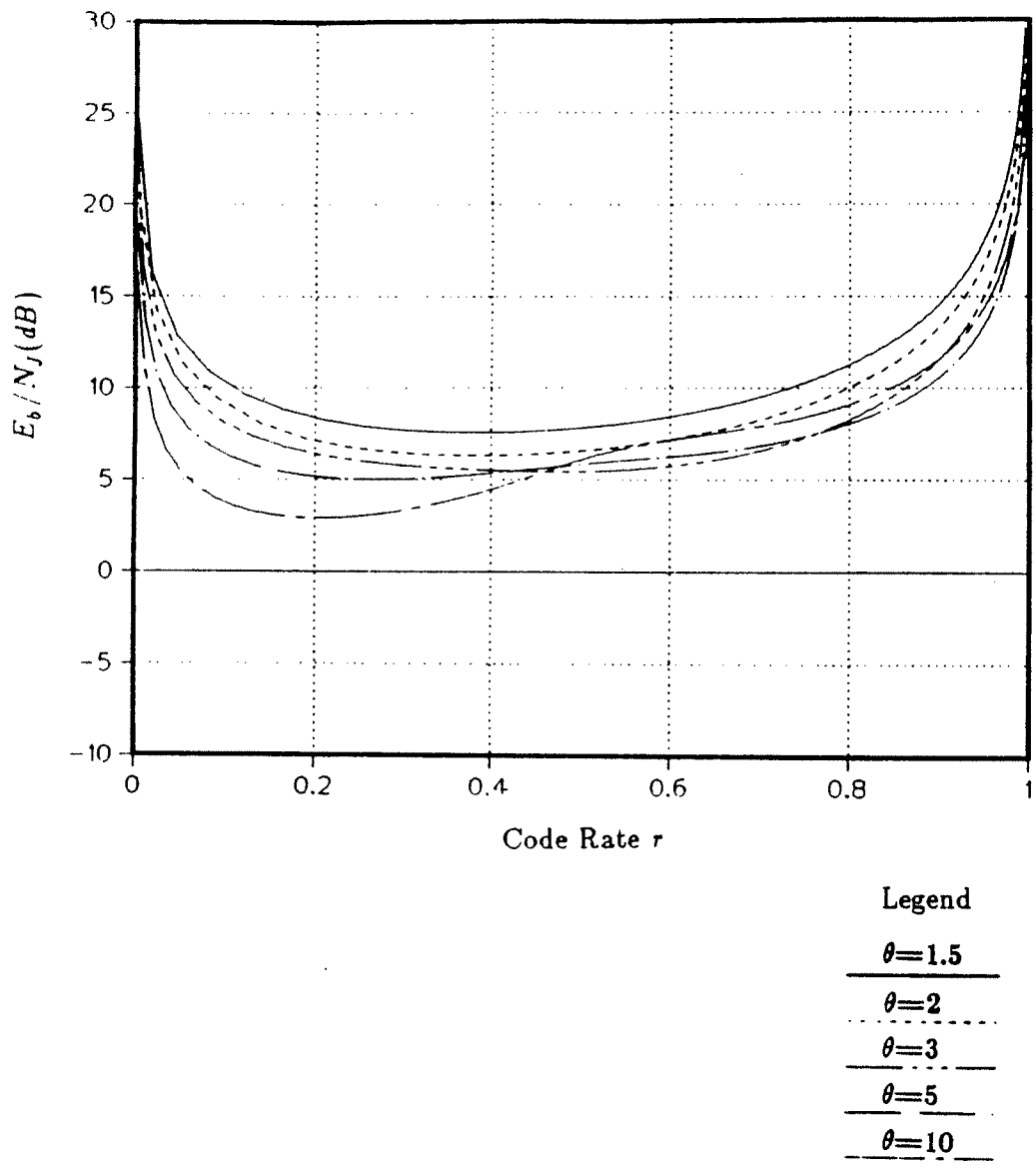


Figure 3.3.3.1. E_b/N_J needed for reliable communication at code rate r in one-dimensional four level tone jamming and four level quantization system with θ= 1.5, 2, 3, 5 and 10.

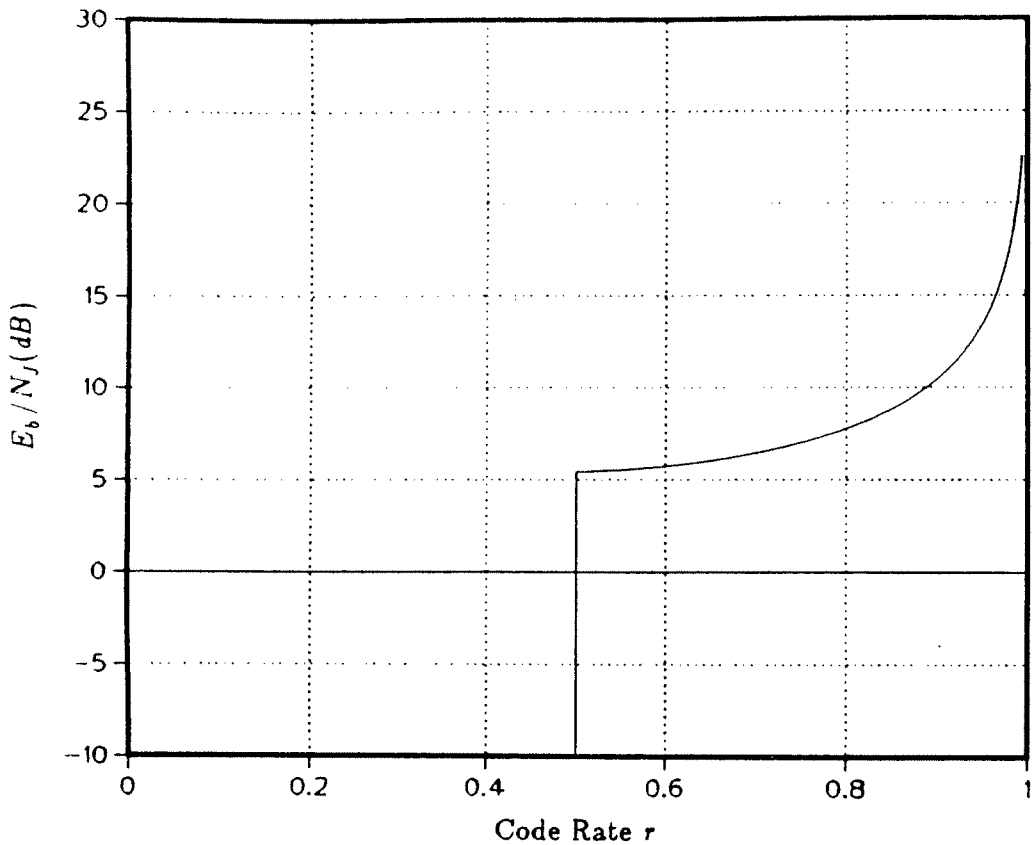


Figure 3.3.3.2. E_b/N_J needed for reliable communication at code rate r in one-dimensional four level tone jamming and four level quantization system with optimal θ.

3.4. Partial-Band Gaussian Jamming

In this section we study the performance of a noncoherent communication system in the presence of Gaussian jamming noise. For this type of jamming, each component in (3.2.4) is given by $I=ZN$ where N is a Gaussian random variable with mean 0 and variance 1, and Z is a random variable with average power $1/2\lambda$. The jamming strategies are then to distribute Z subject to an average power $1/2\lambda$.

The worst-case partial band jamming strategies have been discussed in many papers [HOU75,VIT75] when a hard decision receiver is used. In our model we consider multi-level Gaussian jamming and multi-level quantization. We find that in the Gaussian jamming channel more than two level jamming strategy does not significantly degrade the performance (over two level jamming) because the jammer's power density is highly concentrated around the mean which is zero.

In section 3.4.1 the channel capacity is calculated with two, three and four level quantization at the receiver under two level Gaussian jamming environment. In section 3.4.2 and 3.4.3 we repeat the calculation for the case of three and four level jamming.

3.4.1. Two Level Jamming

In two level Gaussian jamming environment, ρ, $0\leq\rho\leq 1$, denotes the fraction of the spread signal band is jammed. The jammer has a power density $N_J/2\rho$ with probability ρ and zero power density with probability $1-\rho$.

Two level jamming, two level quantization

For a hard decision receiver, Viterbi [VIT75] showed that the crossover probability of the binary symmetric channel is given by

$$p = \frac{\rho}{2} e^{-\lambda\rho/2} \tag{3.4.1.1}$$

where λ is the symbol signal-to-noise ratio. Since the channel capacity is decreasing function of the crossover probability, the jammer can minimize the capacity by choosing the fraction ρ which maximizes (3.4.1.1). The optimum ρ, denoted ρ^*, is then given by

$$\rho^* = \begin{cases} 1 & \lambda \leq 2 \\ 2/\lambda & \lambda > 2. \end{cases} \tag{3.4.1.2}$$

The channel capacity is then given by

$$C = \begin{cases} 1-H_2(e^{-\lambda/2}/2) & \lambda \leq 2 \\ 1-H_2(e^{-1}/\lambda) & \lambda > 2. \end{cases} \tag{3.4.1.3}$$

Figure 3.4.1.1 shows the minimum bit signal-to-noise ratio needed to obtain code rate under worst-case two level Gaussian jamming.

Two level jamming, three level quantization

In three level quantization system we employ the ratio threshold θ for making a decision. The transition probabilities in (3.2.12) corresponding to the decision rule in (3.2.9) are then given by

$$\begin{aligned} p_a &= \left(1-\frac{\theta}{1+\theta} e^{-1/2\sigma^2(1+\theta)}\right)\rho+\rho_0 \\ p_b &= \left(\frac{\theta}{1+\theta} e^{-1/2\sigma^2(1+\theta)}-\frac{1}{\theta} e^{-\theta/2\sigma^2(1+\theta)}\right)\rho \\ p_c &= \frac{\rho}{\theta} e^{-\theta/2\sigma^2(1+\theta)} \end{aligned} \tag{3.4.1.4}$$

where $\sigma^2=1/2\rho\lambda$ and $\rho_0=1-\rho$. The optimum ρ, denoted by ρ^*, is given by $\rho^*=1$

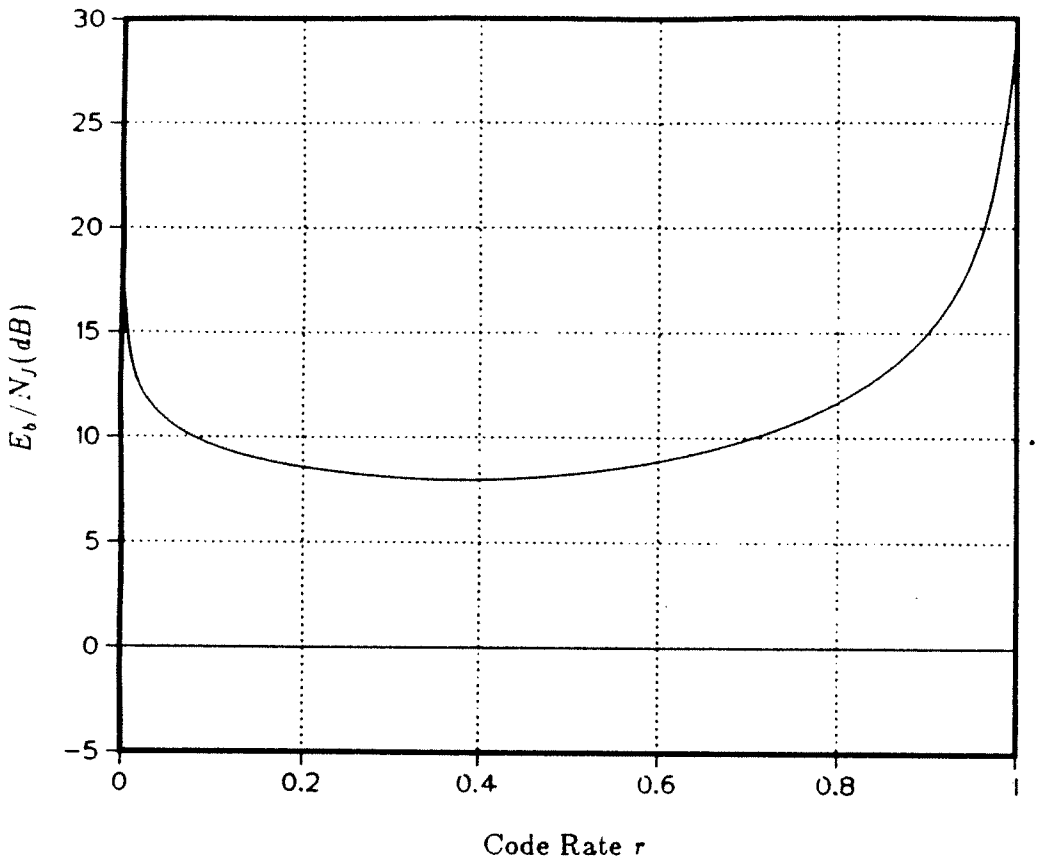


Figure 3.4.1.1. E_b/N_J needed for reliable communication at code rate r in two level Gaussian jamming and a hard decision system.

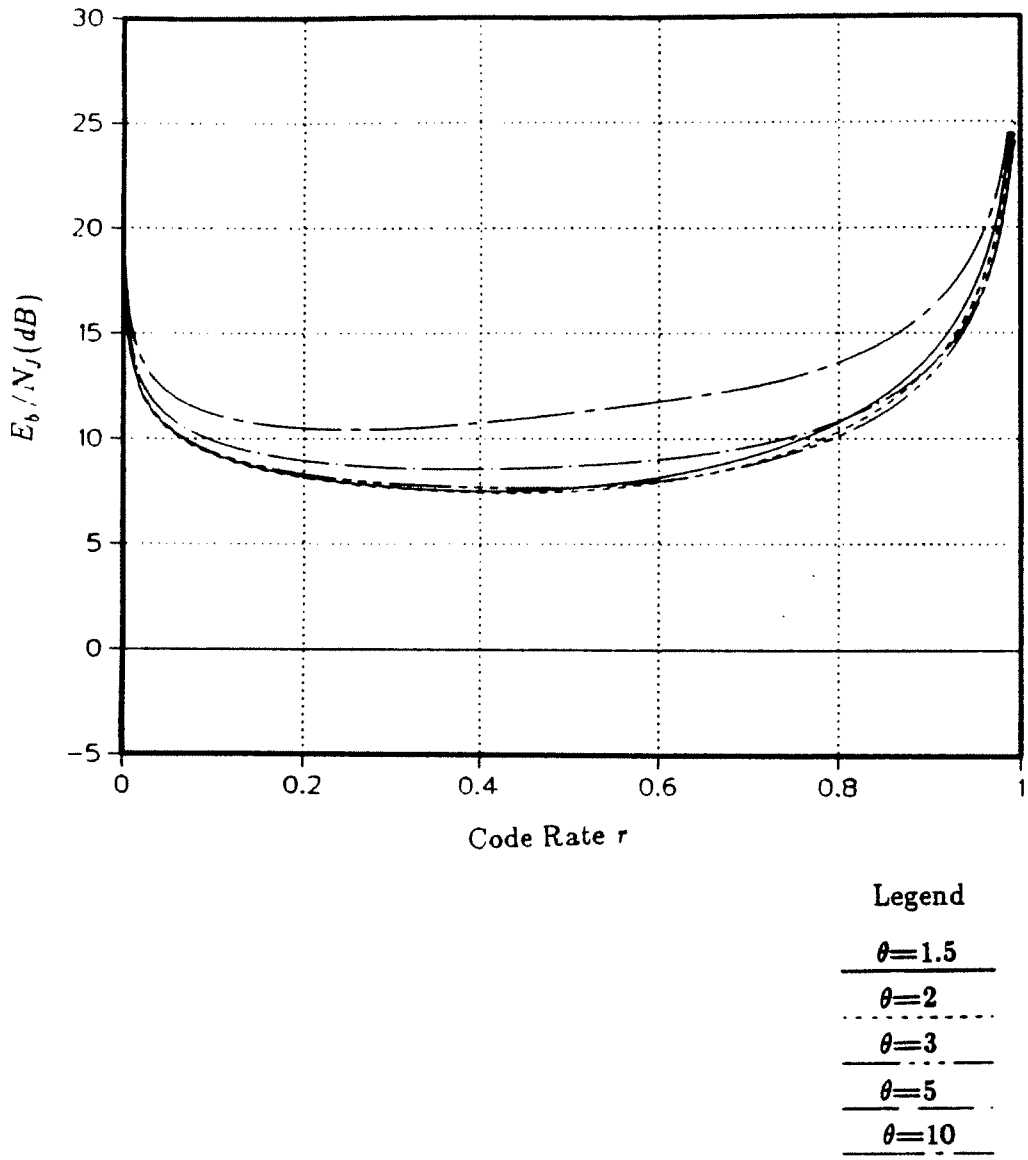


Figure 3.4.1.2. E_b/N_J needed for reliable communication at code rate r in two level Gaussian jamming and three level quantization system with θ=1.5, 2, 3, 5 and 10.

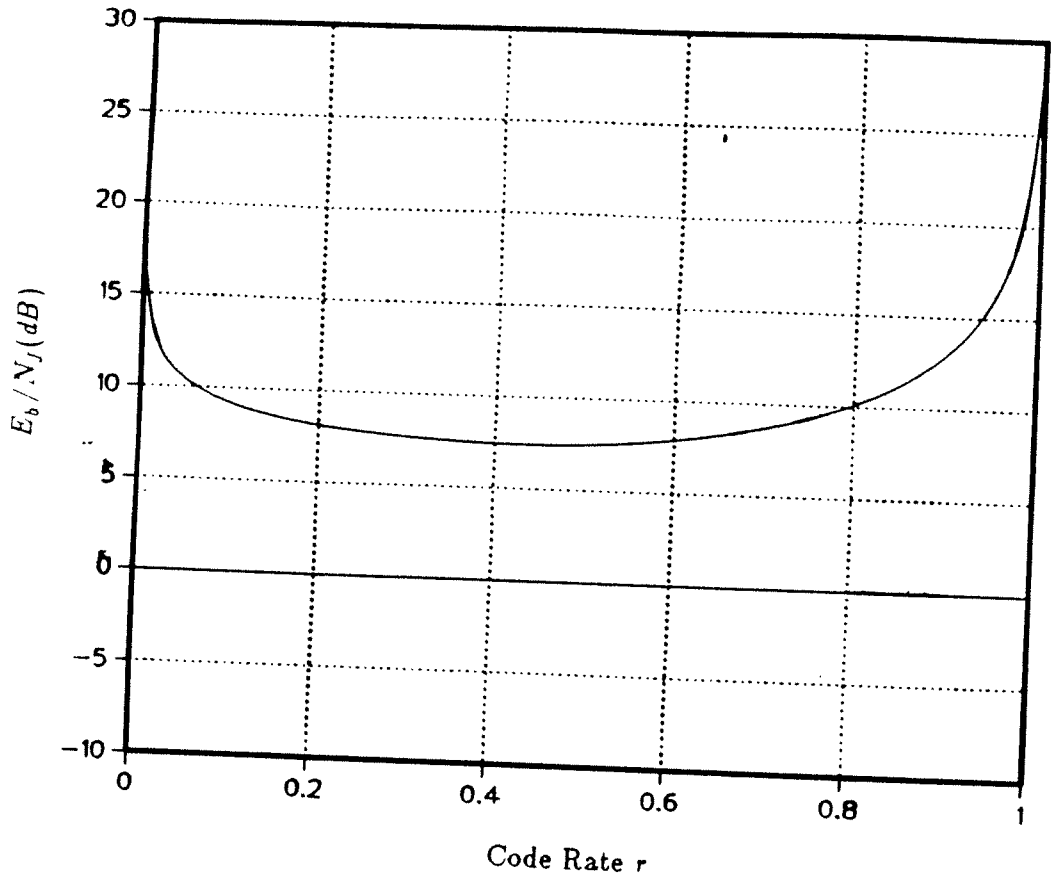


Figure 3.4.1.3. E_b/N_J needed fore reliable communication at code rate r in two level Gaussian jamming and three level quantization system with optimal θ.

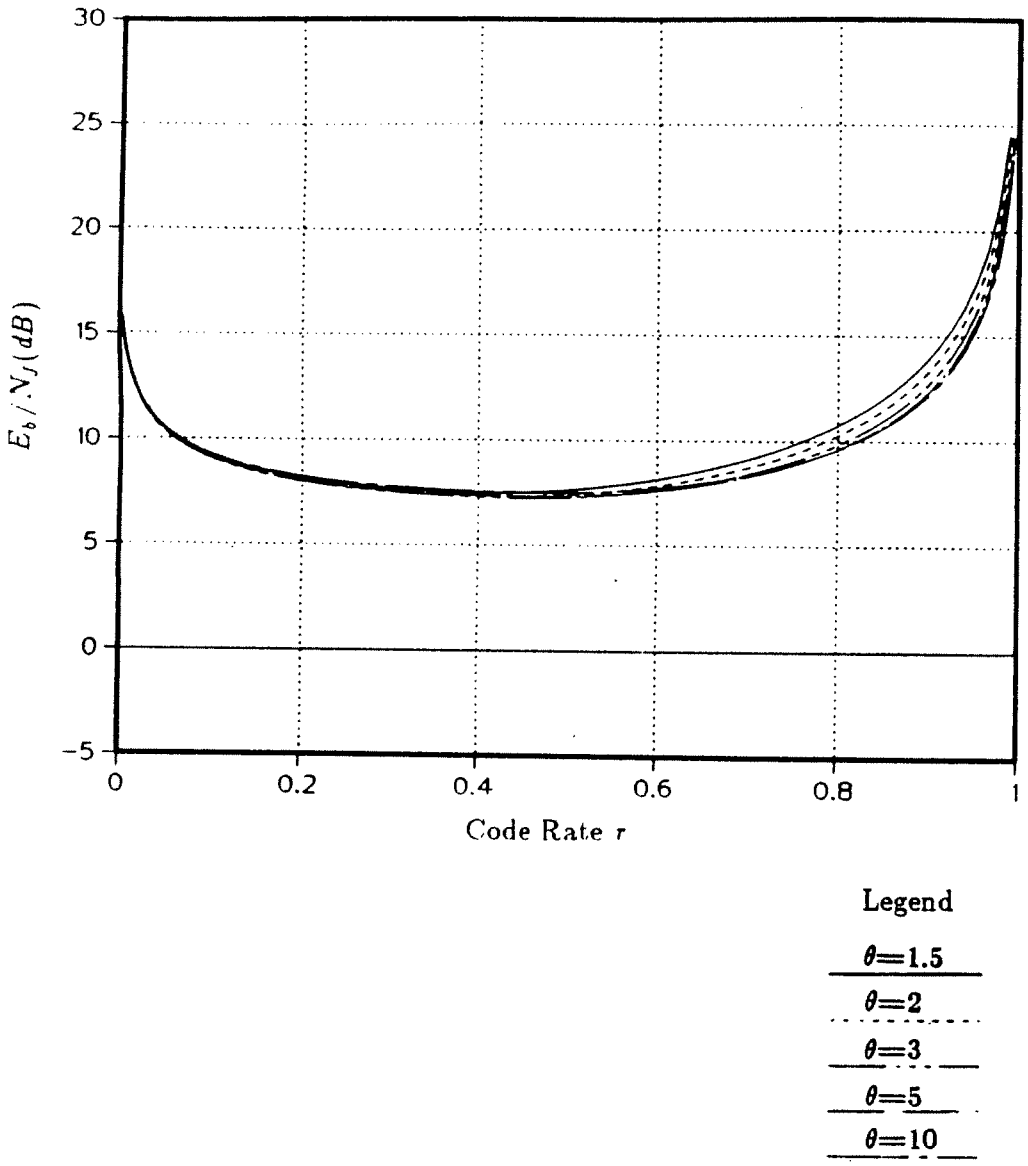


Figure 3.4.1.4. E_b/N_J needed for reliable communication at code rate r in two level Gaussian jamming and four level quantization system with θ=1.5, 2, 3, 5 and 10.

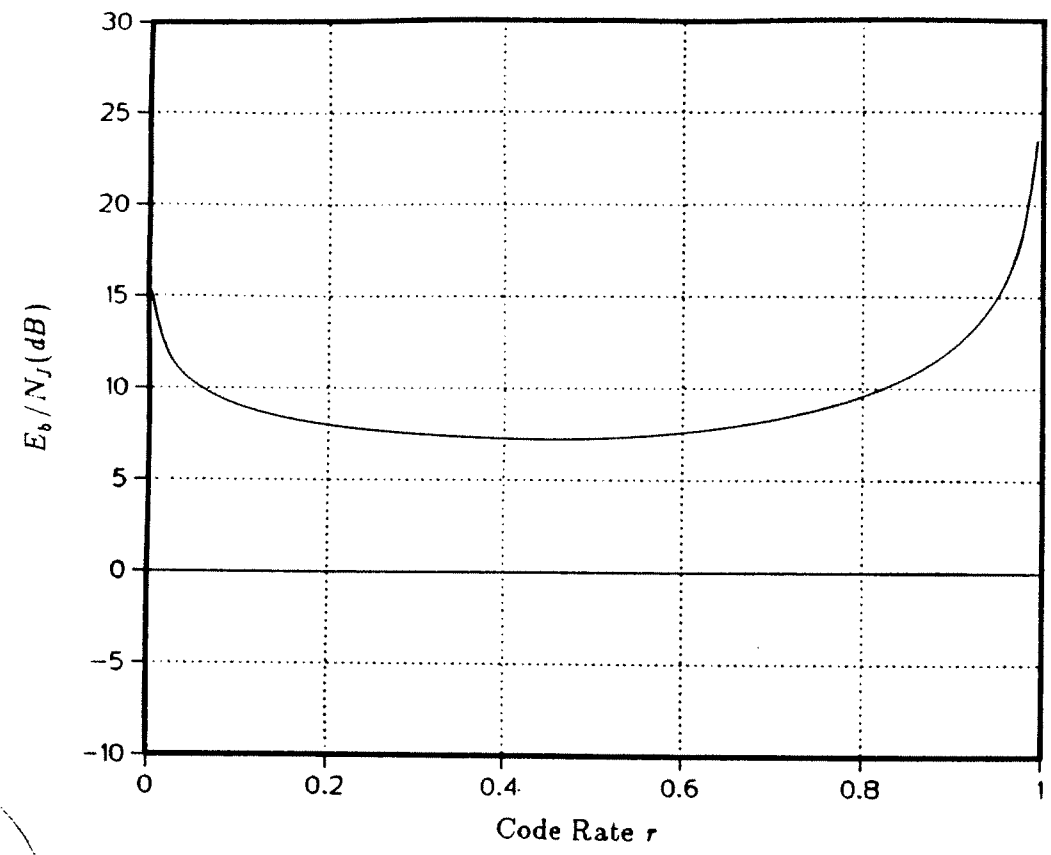


Figure 3.4.1.5. E_b/N_J needed for reliable communication at code rate r in two level Gaussian jamming and four level quantization system with optimal θ.

for λ sufficiently small [CHA85], that is, if $\lambda \leq \lambda_\theta$ then $\rho^* = 1$, where λ_θ is the solution of the partial derivative of the channel capacity with respect to ρ being equal to zero evaluated at $\rho = 1$. However it is very hard to obtain the closed form for optimum ρ for all λ. This has been computed numerically and the resulting channel capacity is given by (3.3.2.3). The relations of the bit signal-to-noise ratio and code rate are shown in Fig. 3.4.1.2 and Fig. 3.4.1.3.

Two level jamming, four level quantization

With the four level quantization we split the erasure symbol into two different channel output symbols, namely +1, bad channel state and -1, bad channel state. For this system the channel transition probabilities defined in (3.2.13) corresponding to the decision rule in (3.2.10) are given by

$$\begin{aligned} p_a &= \left(1 - \frac{\theta}{1+\theta} e^{-1/2\sigma^2(1+\theta)} \right)\rho + \rho_0 \\ p_b &= \left(\frac{\theta}{1+\theta} e^{-1/2\sigma^2(1+\theta)} - \frac{1}{2} e^{-1/4\sigma^2} \right)\rho \\ p_c &= \left(\frac{1}{2} e^{-1/4\sigma^2} - \frac{1}{1+\theta} e^{-\theta/2\sigma^2(1+\theta)} \right)\rho \\ p_d &= \frac{\rho}{1+\theta} e^{-\theta/2\sigma^2(1+\theta)} \end{aligned} \tag{3.4.1.5}$$

Since it is hard to obtain the closed form for optimum ρ for all λ, the performance has been calculated numerically and is shown in Fig. 3.4.1.4 and Fig. 3.4.1.5 with fixed θ and the optimal θ respectively.

3.4.2. Three Level Jamming

In three level jamming the jammer has at most three power density levels over some fraction of the signal bandwidth, that is, zero, $N_J\alpha_1/2$ and $N_J\alpha_2/2$

corresponding to the bandwidth fraction being jammed ρ_0, ρ_1 and ρ_2, respectively.

Three level jamming, three level quantization

With three level quantization the transition probabilities defined in (3.2.12) of the symmetric channel corresponding to the decision rule in (3.2.9) are given by

$$
\begin{aligned}
p_a &= \rho_0 + \sum_{i=1}^{2} \left(1 - \frac{\theta}{1+\theta} e^{-1/2\sigma_i^2(1+\theta)} \right) \rho_i \\
p_b &= \sum_{i=1}^{2} \left(\frac{\theta}{1+\theta} e^{-1/2\sigma_i^2(1+\theta)} - \frac{1}{\theta} e^{-\theta/2\sigma_i^2(1+\theta)} \right) \rho_i \\
p_c &= \sum_{i=1}^{2} \left(\frac{1}{\theta} e^{-\theta/2\sigma_i^2(1+\theta)} \right) \rho_i
\end{aligned}
\tag{3.4.2.1}
$$

where $\sigma_i^2 = N_J \alpha_i / 2E_c$, $i=1,2$. In Fig 3.4.2.1 we show the minimum E_b/N_J to obtain the code rate r with several θ. In Gaussian jamming channel the performance degradation is not large, even though the number of jammer levels increase. This is because the Gaussian density is highly concentrated around the mean.

Three level jamming, four level quantization

With four level quantization the transition probabilities defined in (3.2.13) corresponding to the decision rule in (3.2.10) are given by

$$
\begin{aligned}
p_a &= \rho_0 + \sum_{i=1}^{2} \left(1 - \frac{\theta}{1+\theta} e^{-1/2\sigma_i^2(1+\theta)} \right) \rho_i \\
p_b &= \sum_{i=1}^{2} \left(\frac{\theta}{1+\theta} e^{-1/2\sigma_i^2(1+\theta)} - \frac{1}{2} e^{-1/4\sigma_i^2} \right) \rho_i \\
p_c &= \sum_{i=1}^{2} \left(\frac{1}{2} e^{-1/4\sigma_i^2} - \frac{1}{\theta} e^{-\theta/2\sigma_i^2(1+\theta)} \right) \rho_i \\
p_d &= \sum_{i=1}^{2} \left(\frac{1}{\theta} e^{-\theta/2\sigma_i^2(1+\theta)} \right) \rho_i
\end{aligned}
\tag{3.4.2.2}
$$

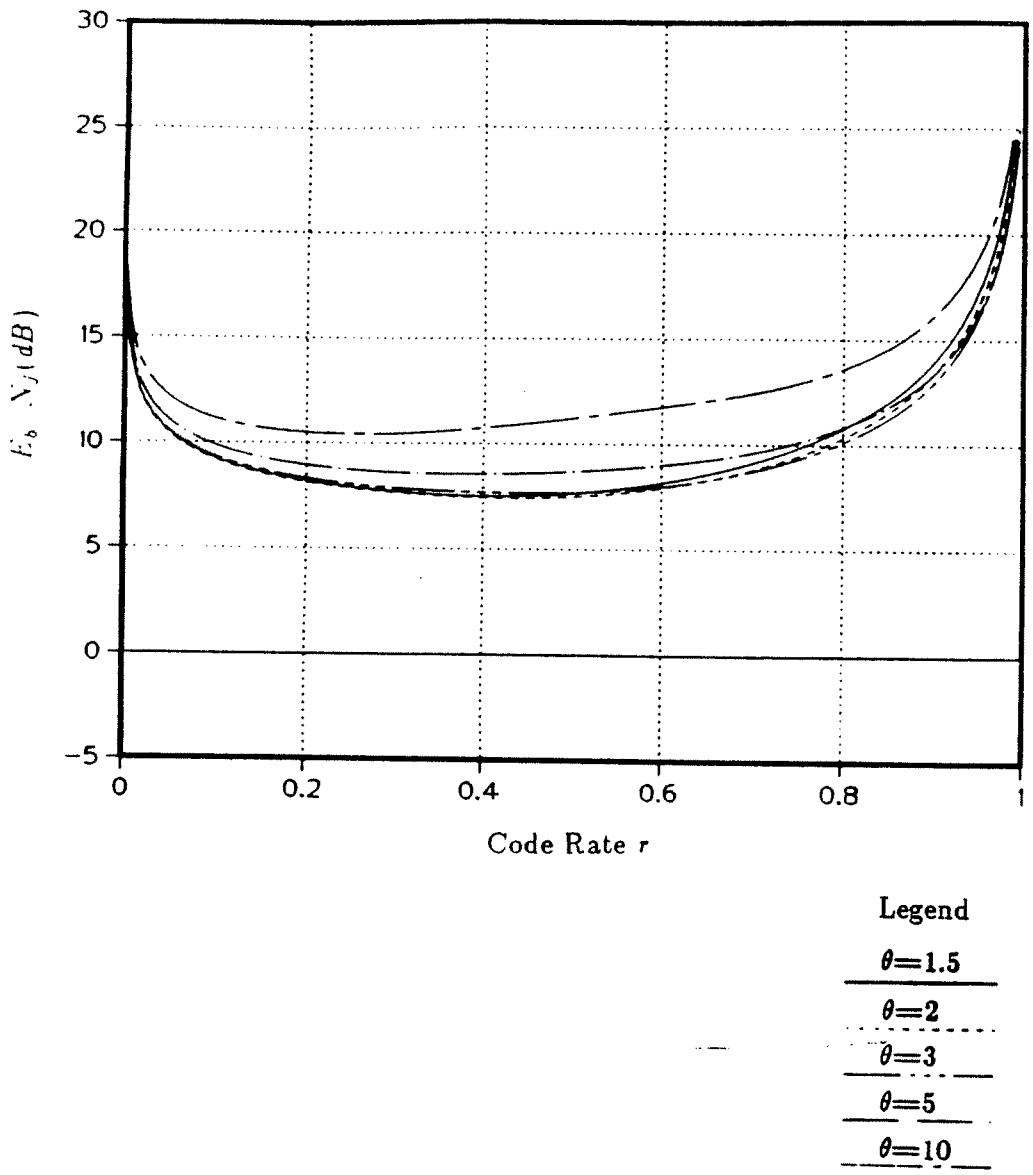


Figure 3.4.2.1. E_b/N_J needed for reliable communication at code rate r in three level Gaussian jamming and three level quantization system with θ=1.5, 2, 3, 5 and 10.

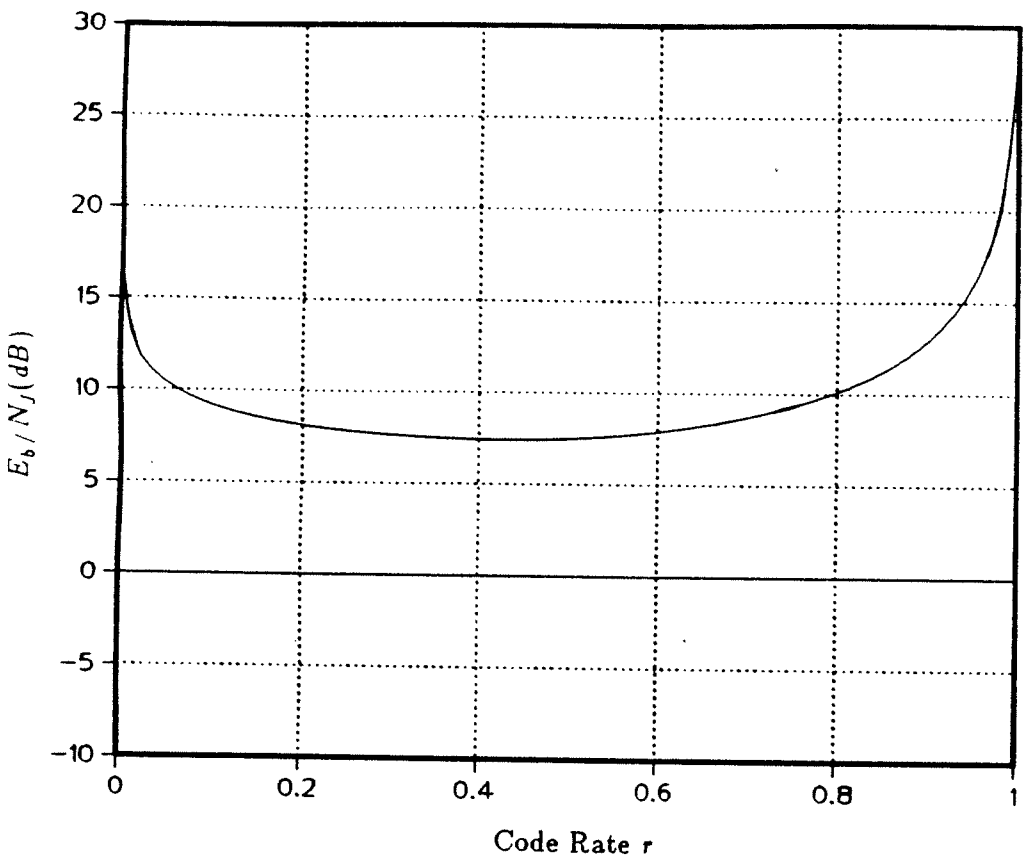


Figure 3.4.2.2. E_b/N_J needed for reliable communication at code rate r in three level Gaussian jamming and three level quantization system with optimal θ.

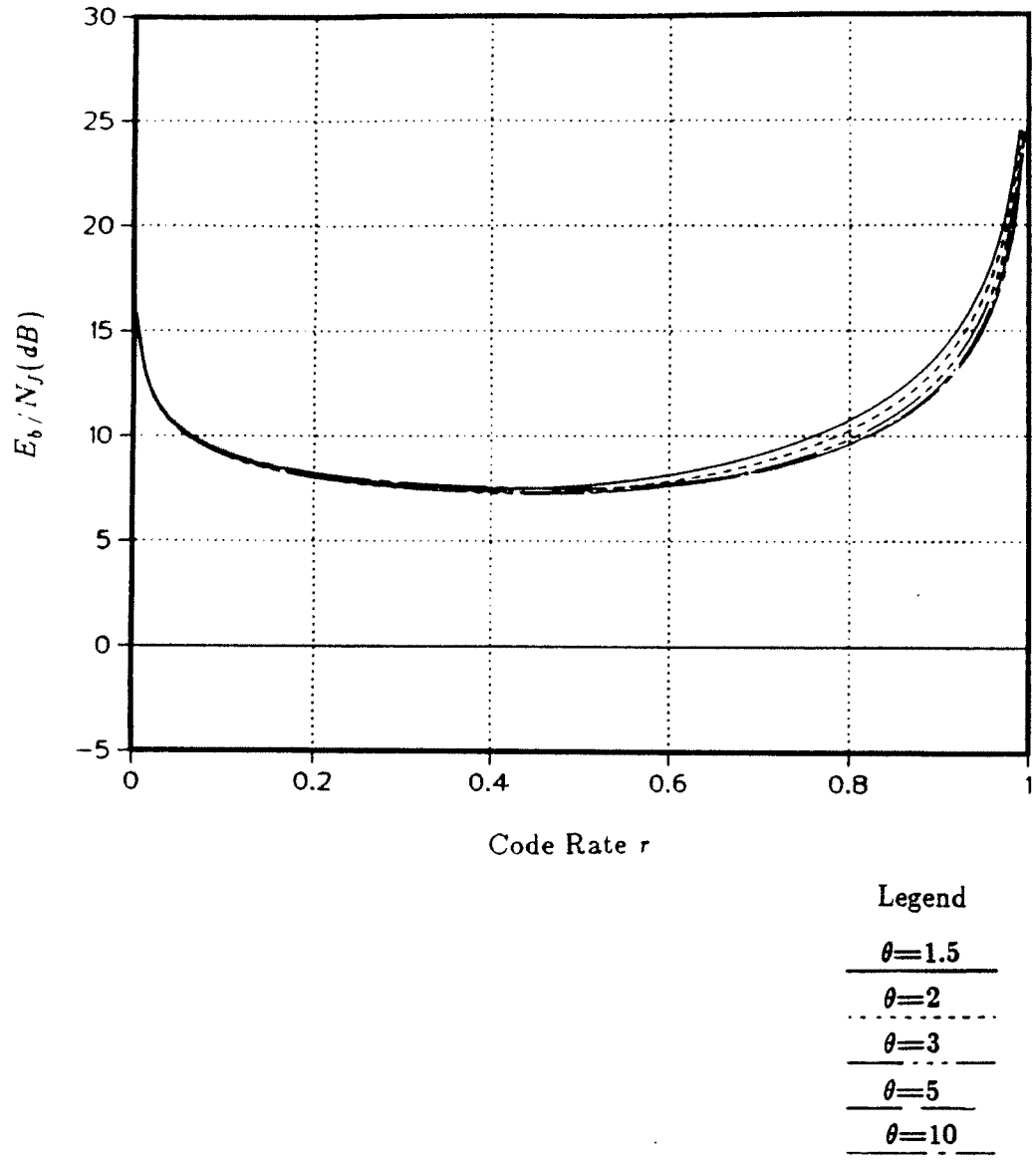


Figure 3.4.2.3. E_b/N_J needed for reliable communication at code rate r in three level Gaussian jamming and four level quantization system with θ=1.5, 2, 3, 5 and 10.

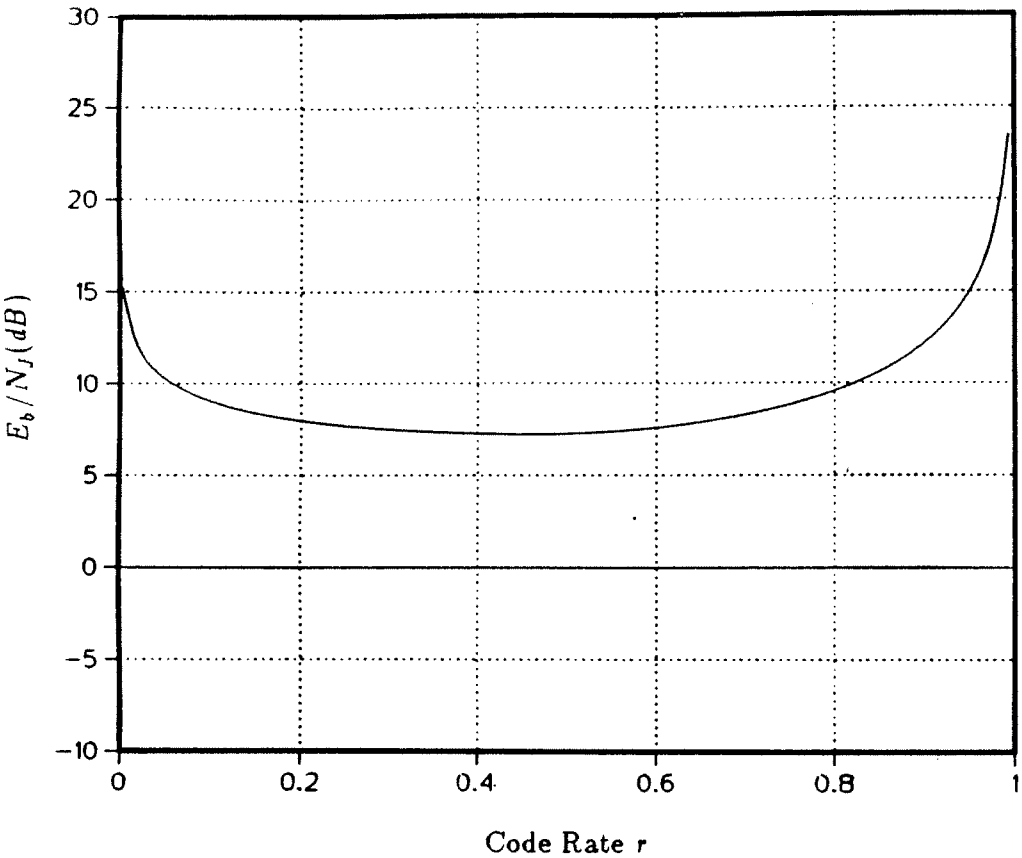


Figure 3.4.2.4. E_b/N_J needed for reliable communication at code rate r in three level Gaussian jamming and four level quantization system with optimal θ.

where $\sigma_i^2 = N_J \alpha_i / 2E_c$, $i=1,2$. As seen in the Gaussian channel the performance is almost same as the two level jamming and resulting performance is shown in Fig. 3.4.2.3 and Fig. 3.4.2.4.

3.4.3. Four Level Jamming

The optimal distribution of the jammer power is concentrated at most four points, that is, the power density levels are denoted by zero, $N_J \alpha_1 / 2$, $N_J \alpha_2 / 2$ and $N_J \alpha_3 / 2$, and the bandwidth fractions corresponding to the spectral densities are ρ_0, ρ_1, ρ_2 and ρ_3, respectively. The jammer has an average power constraint. The optimal distribution of the jammer power is computed numerically. The resulting performance of the four level jamming is nearly the same as that of the three level jamming.

The transition probabilities defined in (3.2.13) for the channel with four level quantization rule given by (3.2.10) are given by

$$
\begin{aligned}
p_a &= \rho_0 + \sum_{i=1}^{3} \left(1 - \frac{\theta}{1+\theta} e^{-1/2\sigma_i^2(1+\theta)} \right) \rho_i \\
p_b &= \sum_{i=1}^{3} \left(\frac{\theta}{1+\theta} e^{-1/2\sigma_i^2(1+\theta)} - \frac{1}{2} e^{-1/4\sigma_i^2} \right) \rho_i \\
p_c &= \sum_{i=1}^{3} \left(\frac{1}{2} e^{-1/4\sigma_i^2} - \frac{1}{\theta} e^{-\theta/2\sigma_i^2(1+\theta)} \right) \rho_i \\
p_d &= \sum_{i=1}^{3} \left(\frac{1}{\theta} e^{-\theta/2\sigma_i^2(1+\theta)} \right) \rho_i
\end{aligned}
\tag{3.4.3.1}
$$

where $\sigma_i^2 = N_J \alpha_i / 2E_c$, $i=1,2,3$. In Fig. 3.4.3.1 and Fig. 3.4.3.2 we show the minimum E_b/N_J needed to obtain the code rate r with fixed θ and the optimal θ, respectively.

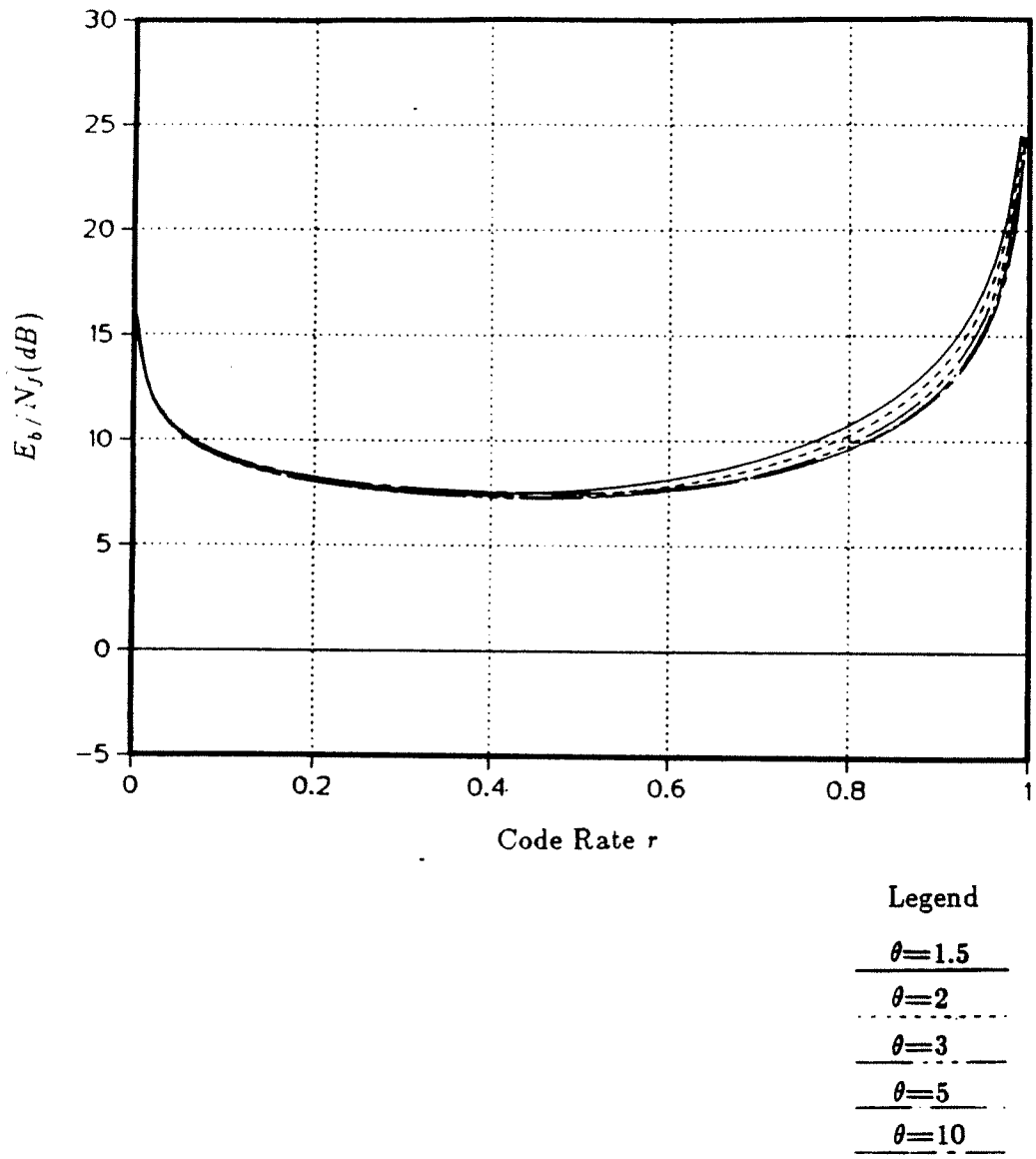


Figure 3.4.3.1. E_b/N_J needed for reliable communication at code rate r in four level Gaussian jamming and four level quantization system with θ=1.5, 2, 3, 5 and 10.

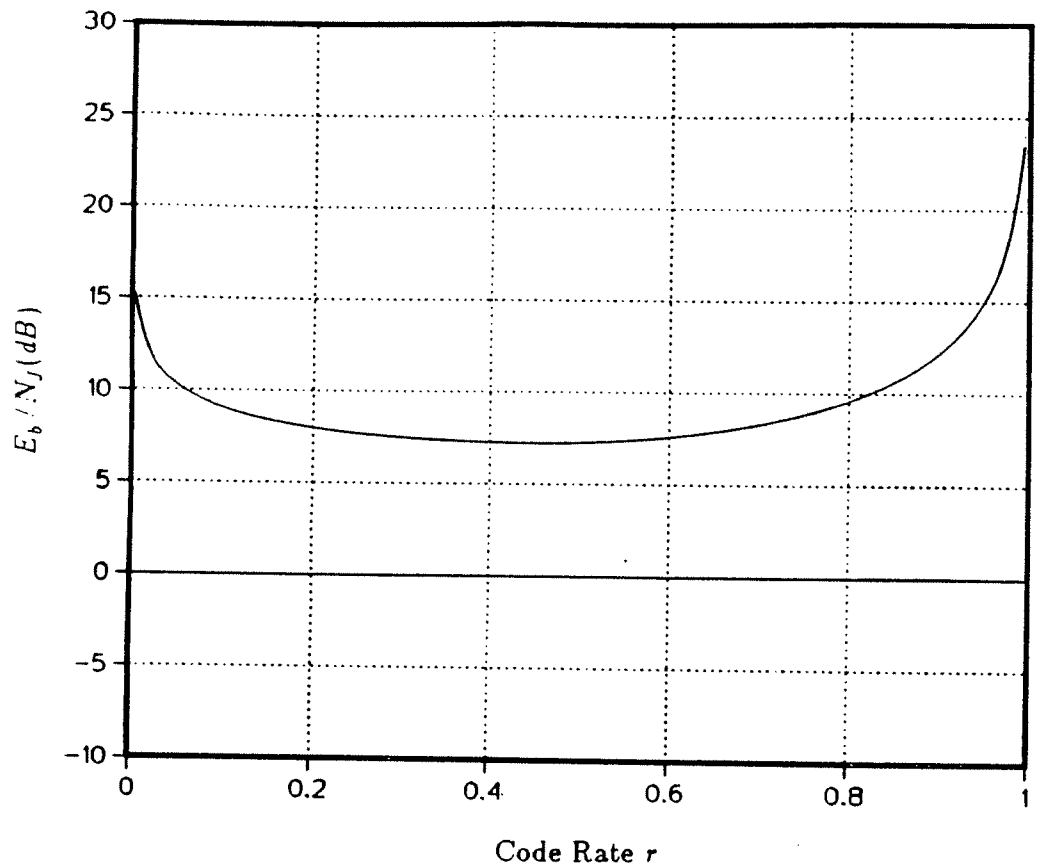


Figure 3.4.3.2. E_b/N_J needed for reliable communication at code rate r in four level Gaussian jamming and four level quantization system with optimal θ.

3.5. Discussion

In section 3.3 and 3.4 we discussed the performance of the system with different quantization levels against several jamming strategies.

In one dimensional tone jamming if the communicator chooses the optimal ratio threshold θ then the jammer can not make the capacity less than one half even though the jammer increases the power. Higher level jamming makes the performance worse than lower level jamming. In Gaussian jamming, however, higher level jamming strategies do not degrade the performance significantly.

We show the minimum required information bit signal-to-noise ratio at rate 1/2 against several jammer noise type when the optimal θ is used in Table 3.5.1.

Level	One-dim Tone	Gaussian
J2Q2	9.6	8.25
J2Q3	6.0	7.45
J2Q4	$-\infty$	7.27
J3Q3	6.7	7.45
J3Q4	4.5	7.27
J4Q4	5.4	7.27

Table 3.5.1. $E_b/N_J(dB)$ necessary for reliable communication at rate 1/2 in non-coherent system. ($J2Q2$ is two level jamming and hard decision, $J2Q3$ is two level jamming and three level quantization, $J2Q4$ is two level jamming and four level quantization, $J3Q3$ is three level jamming and three level quantization, $J3Q4$ is three level jamming and four level quantization and $J4Q4$ is four level jamming and four level quantization).

CHAPTER IV

CONCLUSIONS

In this thesis we have analyzed frequency-hopped spread-spectrum systems with two, three and four level quantization in the presence of worst-case jamming. In coherent systems, we have considered tone jamming, phase mismatched tone jamming and partial-band Gaussian jamming as the noise type of the jammer. In noncoherent systems, the jammer noise types considered were one-dimensional tone jamming and partial-band Gaussian jamming. For both coherent and noncoherent system, two, three and four level quantization were employed at the receiver. As a jamming strategy, we considered two, three and four different power levels subject to an average power constraint. Unlike tone jamming, the performance of multilevel (more than two level) Gaussian jamming is very close to two level Gaussian jamming. In noncoherent systems with one-dimensional tone jamming and optimum ratio-threshold the capacity can not be less than one half independent of the jammer's power. We have determined the minimum bit signal-to-noise ratio under the worst-case jamming environment when L level quantizers are used for $L=2$, 3 and 4.

One interesting extension is to employ cutoff rate as the performance measure. Cutoff rate is considered as a practical limit of code rate. It will be interesting to

compare channel capacity and cutoff rate in the presence of worst-case jamming and to find the worst-case jamming strategy. Another interesting problem is to consider the performance of specific error correction codes with worst-case (multi-level) jamming. The error correction codes of interest are repetition codes, convolutional codes and Reed-Solomon codes. The analysis with additive background noise would also be interesting. The results in this thesis can be used to obtain a bound in this case by considering another system with a jammer with average power being the sum of the background noise power and the original jammer power and no background noise. This is a bound since the feasible strategies for the jammer with in the later system include those strategies in the former system.

APPENDICES

APPENDIX A

WORST-CASE DISTRIBUTION FOR PARTIAL-BAND JAMMING

In this appendix we prove that two levels is optimum for binary PSK based on the exact bit error probability.

Let Z be a nonnegative random variable with mean N_J and distribution $P_Z(z)$. If $Z=z$ represents the jammer having noise level z then the average error probability p_e is

$$p_e = E\left[Q\left(\sqrt{\frac{2E_c}{Z}} \right) \right] \tag{A.1}$$

where E_c is the received energy of the transmitted signal and $Q(x)=\int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$. Define the function $f_1(z)$ by

$$f_1(z)=\begin{cases} Q(\sqrt{2E_c/z}) & z>0 \\ 0 & z=0. \end{cases} \tag{A.2}$$

This function is shown in Fig. A.1.

We will show that $f_1(z)$ has a single point of inflection:

$$\begin{aligned} f'_1(t) &= \sqrt{E_c/4\pi}\, e^{-E_c/z} z^{-3/2} \\ f''_1(z) &= \sqrt{E_c/4\pi}\, e^{-E_c/z} z^{-5/2} \left(\frac{E_c}{z} - \frac{3}{2} \right). \end{aligned} \tag{A.3}$$

Since $e^{-E_c/z} z^{-5/2}$ is always nonnegative the only point of inflection occurs at $z=2E_c/3$. For $z<2E_c/3$, $f_1(z)$ is a convex function while for $z>2E_c/3$, $f_1(z)$ is a concave function. Define the functin $f_2(z)$ by

$$f_2(z)=\begin{cases}\gamma_0\ Q(\sqrt{2\gamma_0})\ z/E_c & 0\leq z\leq E_c/\gamma_0\\ f_1(z) & z>E_c/\gamma_0\end{cases} \tag{A.4}$$

where γ_0 is the solution of

$$Q(\sqrt{2\gamma_0})-\frac{\sqrt{\gamma_0}}{\sqrt{4\pi}}\ e^{-\gamma_0}=0. \tag{A.5}$$

The value of γ_0 is 0.709. This function is shown in Fig. A.1. From the above we have

$$f_1(z)\leq f_2(z) \qquad z\geq 0 \tag{A.6}$$

with equality if $z>E_c/\gamma_0$. From (A.1) we have

$$p_e=E[f_1(z)]\leq E[f_2(z)] \tag{A.7}$$

with equality if $z>E_c/\gamma_0$ or if the distribution of Z is concentrated at the two points $z=0$ and $z=E_c/\gamma_0$. Now since $f_2(z)$ is a concave function by Jensen's inequality we have

$$p_e\leq E[f_2(Z)]\leq f_2[E(Z)]=f_2(N_J). \tag{A.8}$$

Equality can be achieved in (A.8) if z is concentrated on two points or less. Notice that

$$f_2(N_J)=\begin{cases}\gamma_0\ Q\left(\sqrt{2\gamma_0}\right)\dfrac{N_J}{E_c} & E_c/N_J\geq\gamma_0\\ Q\left(\sqrt{2E_c/N_J}\right) & E_c/N_J<\gamma_0.\end{cases} \tag{A.9}$$

Thus we have shown that two levels is a worst-case distribution for binary PSK with error probability as the performance measure.

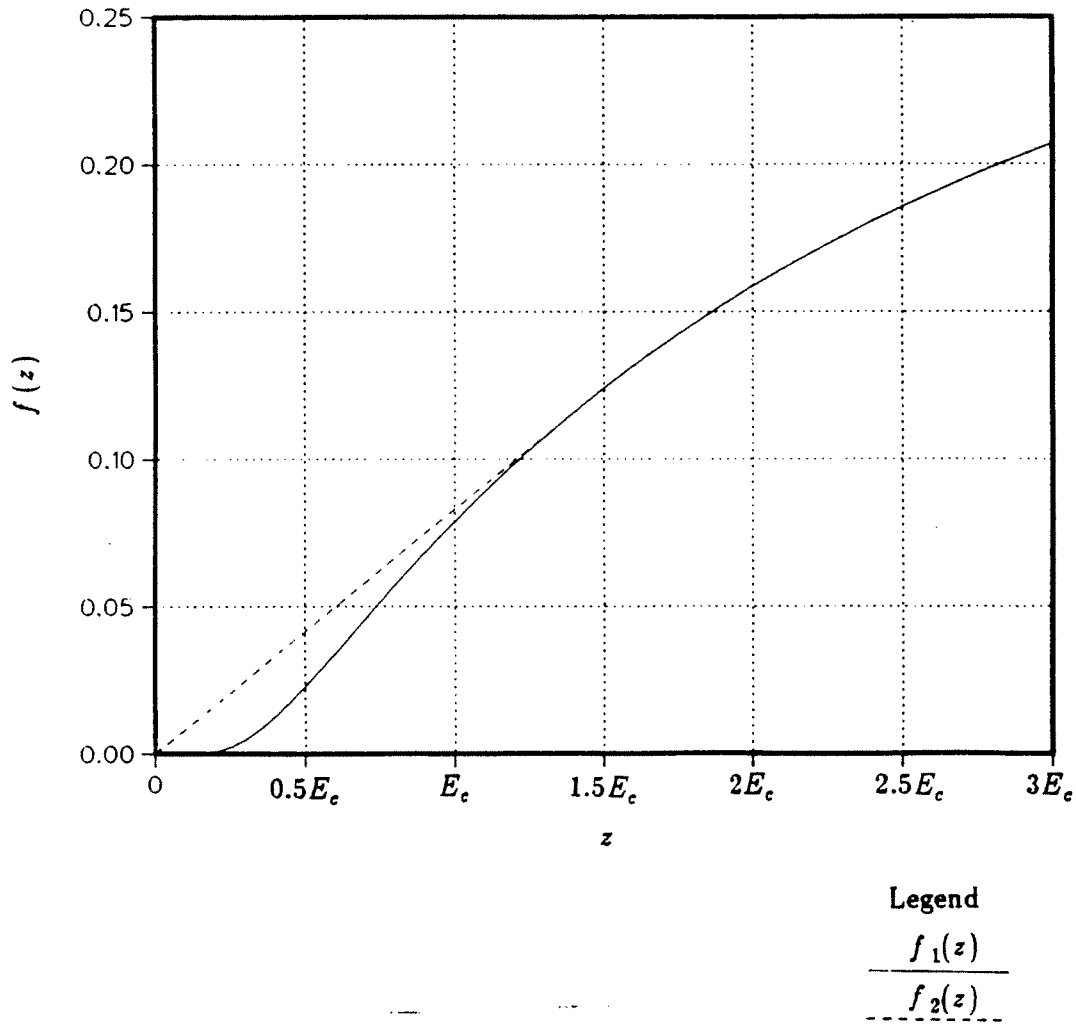


Figure A.1. The functions $f_1(t)$ and $f_2(t)$

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