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Achievable Rates for Multiple Access Channels with Correlated Messages

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Abstract

An achievable rate region for multiple access channels with correlated messages is presented. We consider bipartite graphs with parameters $(\theta_1, \theta_2, \theta_1', \theta_2')$ to represent correlated messages where θ_1 and θ_2 denote the sizes of the messages of the two transmitters and θ_1' and θ_2' characterize the correlation structure of messages. It is shown that such correlated messages, represented as a graph can be sent with arbitrarily small error probability over the multiple access channel, given by $p(y|x_1, x_2)$, by using correlated codewords represented as a graph, if the sizes of the messages and the correlation structure of the messages satisfy certain conditions. We prove this by using the random coding argument with jointly typical sequence decoding. We also compare two different coding schemes for the Gaussian multiple access channel with jointly Gaussian channel input. One is the separate source and channel coding scheme which uses independent codewords after using Slepian-Wolf coding on correlated messages, and the other is the proposed method of correlated codewords. The result says that we can send the same amount of information over the multiple access channel with less power by using correlated codewords.

1 Introduction

Consider a set of transmitters wishing to accomplish reliable simultaneous communication with a single receiver using a multiple access channel [1, 3, 2, 4, 5]. The transmitters do not communicate among themselves. Each transmitter among a set, has some independent information, and together they wish to communicate their information to a joint receiver. This channel was first studied by Ahlswede in [1] and by Liao in [3], where they obtained the capacity region.

Around the same time, another multiterminal communication problem involving separate (distributed) encoding of correlated information sources was formulated and the corresponding optimum rate region was obtained by Slepian and Wolf in [6] (also see [13]). In this problem, the goal is to represent remotely two (for example) correlated information sources using a pair of index sets to be transmitted to a joint receiver who wishes to get a noiseless reproduction of the corresponding sources. In this paper we also refer to this system as Slepian-Wolf source coding.

Thus a natural extension of Shannon's point-to-point communication paradigm involving source coding and channel coding to multiple terminals was first considered by Slepian and Wolf in a subsequent work [7]. They considered a special class of such problems where there are two transmitter terminals sharing three independent information sources. The first transmitter has access to the first and the second source, and the second transmitter has access to the second and the third source. In other words, the sources of information accessed by these terminals have a common part [16, 17, 18]. In [7], a characterization of set of such information sources that can be transmissible reliably over a given multiple access channel was given (with direct and converse parts).

In a multiple access channel with correlated sources, generally there could be two ways of sending these sources over the given channel. One is separate source coding and channel coding, which first applies the Slepian-Wolf source coding to the correlated sources in order to minimize the sum rate of the messages representing the sources, and then applies the standard multiple access channel coding on these independent messages. The other is joint source-channel coding which may reduce both delay and complexity, where the sources are directly mapped into the channel input. But designing such a scheme is generally a more difficult optimization problem. In the latter case, what one can say is whether a given set of sources can be reliably transmitted over a given multiple access channel. One of the results that can be deduced from [7] is that separate source and channel coding is not optimal in this multiterminal communication problem.

Later, Cover, El Gamal and Salehi [8] obtained a more general coding theorem for multiple access channels with correlated sources, which includes the results of [1], [3], [6] and [7] as special cases. In this work, they consider direct mapping of source symbols into channel inputs. However, Dueck [9] showed, by an example that the approach of [8] only gives sufficient conditions for the transmissibility of correlated sources over a given multiple access channel, but not necessary conditions. Cover, El Gamal and Salehi [8] also gave another interesting example (to be illustrated here in the next section) that shows that separate source and channel coding is not optimal for multiple access channels with correlated sources. Further work related to joint source-channel coding in this multiterminal setting can be found in [10, 11, 12].

In separate source and channel coding (in two- or multiterminal problems) [20], as inspired from point-to-point communication, *bit* is the digital interface between source coding and channel coding modules. That is, the source coding output is represented as bits (messages) and these become the input to the channel coding module. Here, in the process of removing all the redundancy during source coding, we lose the correlation structure present in the sources which may help the transmission in combating the interference and channel noise. In order to get any gain from the existing correlation between the sources, one should encode these sources into messages which preserve the correlation, and in effect, should translate the correlation in the messages into the channel input.

To do this, in this work, we propose to use graphs as digital interface between the above multiuser source coding and channel coding problems. In this case, the multiterminal sources can be mapped independently into a message set characterized by a graph which preserves a predetermined amount of correlation between the sources, and this message set (which we refer to as message-graph) becomes the input to the channel encoder. In general, with a message-graph, one need not associate any stochastic characterization. This will be made more clear later.

The work of Slepian and Wolf [7] is along this direction, where, as mentioned above, they considered two correlated messages with a common part [17] as inputs to the two encoders to be transmitted over a multiple access channel. However, it was shown by Gács and Körner [16] and Witsenhausen [17] that the common part of two dependent random variables is zero in most cases. Rather, in this work, we consider a more general class of correlated messages where they need not have a common part. Towards this goal of improving the performance of transmission of correlated sources over multiple access channels, in this work we first address the channel coding part with an associated message-graph (to be defined shortly). The complementary problem of mapping correlated sources into such graphs will be pursued in another work.

We discuss efficient channel coding and achievable rates on the assumption that we are given a message-graph with a predetermined correlation structure. We consider channel coding for multiple access channels with correlated messages, which exploits the existing correlation structure between the messages. We show that, by adopting correlated codewords represented as a graph, referred to as code-graph, the achievable rate region for multiple access channel with correlated messages can be larger than the capacity region for multiple access channels with independent codewords. We also prove the achievability part of the theorem by using random coding arguments with jointly typical sequence decoding.

As an example, we consider the achievable rate region for the Gaussian multiple access channel with jointly Gaussian channel input. We also compare our new coding scheme with the separate source and channel coding scheme which involves conventional multiple access channel coding preceded by Slepian-Wolf source coding ([6], [13]) of correlated messages. The result says that we can send the same amount of information over the multiple access channel with less power by adopting correlated codewords. In other words, we can send more information through a multiple access channel with the same given power. We will discuss a method of generation of different codebooks by permutation and relabeling of the indices of a given codebook.

2 Preliminaries

In this section, we briefly outline the results available in the literature on the multiple access channel and Slepian-Wolf source coding.

2.1 Multiple Access Channel Capacity with Independent Messages

We summarize the well-known results [19] of the multiple access channel capacity in this section.

We are given a multiple access channel characterized by a conditional distribution $p(y|x_1, x_2)$ for a two-transmitter problem, with finite input alphabets $\mathcal{X}_1$, $\mathcal{X}_2$ respectively and a finite output alphabet $\mathcal{Y}$. The channel is assumed to be memoryless and stationary.

A transmission system with parameters $(n, \Delta_1, \Delta_2, \tau)$ for the given multiple access channel would involve

- A codebook given by $\mathbb{C} = \mathbb{C}_1 \times \mathbb{C}_2$, where

- $\mathbb{C}_1 = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{\Delta_1}\}$, $\quad \mathbb{C}_2 = \{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{\Delta_2}\}$,
- $\forall i \in \{1, 2 \ldots, \Delta_1\}$, $\mathbf{v}_i \in \mathcal{X}_1^n$ and $\forall j \in \{1, 2 \ldots, \Delta_2\}$, $\mathbf{u}_j \in \mathcal{X}_2^n$

- A decoder mapping $g : \mathcal{Y}^n \to \mathbb{C}$

- A performance measure, given by the average probability of error:
  $\tau = \sum_{\mathbf{v}_i, \mathbf{u}_j \in \mathbb{C}} \frac{1}{|\mathbb{C}|} \, Pr\,[g(\mathbf{Y}) \neq (\mathbf{v}_i, \mathbf{u}_j) | \mathbf{X}_1 = \mathbf{v}_i, \mathbf{X}_2 = \mathbf{u}_j]$.

The goal is to find the closure $\mathcal{R}_{MA}$ of the set of all achievable rate pairs (R_1, R_2), where a rate pair (R_1, R_2) is said to be achievable for the given multiple access channel if $\forall \epsilon > 0$, and for sufficiently large n, there exists a transmission system as defined above with parameters $(n, \Delta_1, \Delta_2, \tau)$ with $\frac{1}{n} \log \Delta_i > R_i - \epsilon$ for $i = 1, 2$ and a corresponding decoder with the average probability of error $\tau < \epsilon$.

The solution [1, 3] is given by the following information-theoretic characterization: $\mathcal{R}_{MA}$ is equal to the closure of the set of all (R_1, R_2) where there exists a product distribution on the input $p_1(x_1)p_2(x_2)$ such that

$$R_1 \leq I(X_1; Y | X_2), \quad R_2 \leq I(X_2; Y | X_1), \quad R_1 + R_2 \leq I(X_1, X_2; Y). \tag{1}$$

2.2 Noiseless Encoding of Correlated Sources

We are given a pair of correlated sources (for a two-source problem), with a joint distribution $p(s_1, s_2)$ with finite alphabets $\mathcal{S}_i$ for $i = 1, 2$ respectively. The sources are assumed to be memoryless and stationary.

A transmission system with parameters $(n, \Delta_1, \Delta_2, \tau)$, for representing the given correlated sources would involve

- A set of mappings $\{f_1, f_2, g\}$ where
  - $f_i : \mathcal{S}_i^n \to \{1, 2, \ldots, \Delta_i\}$ for $i = 1, 2$, and
  - $g : \{1, 2, \ldots, \Delta_1\} \times \{1, 2, \ldots, \Delta_2\} \to \mathcal{S}_1^n \times \mathcal{S}_2^n$

- A performance measure given by the probability of error
  $\tau = Pr\,[(\mathbf{S}_1, \mathbf{S}_2) \neq g(f_1(\mathbf{S}_1), f_2(\mathbf{S}_2))]$.

The goal is to find the closure $\mathcal{R}_{SW}$ of set of achievable rate pairs (R_1, R_2) where a rate pair (R_1, R_2) is said to be achievable for the given correlated sources if $\forall \epsilon > 0$ and for sufficiently large n, exists a transmission system as defined above with parameters $(n, \Delta_1, \Delta_2, \tau)$ with $\frac{1}{n} \log \Delta_i < R_i + \epsilon$ for $i = 1, 2$ and the probability of error $\tau < \epsilon$.

The solution [6] is given by the following information theoretic characterization: $\mathcal{R}_{SW}$ is equal to the set of all (R_1, R_2) such that

$$R_1 \geq H(S_1 | S_2), \quad R_2 \geq H(S_2 | S_1), \text{ and } R_1 + R_2 \geq H(S_1, S_2). \tag{2}$$

2.3 Example of Correlated Sources over Multiple Access Channel

Let us consider an interesting example given in [8], which shows the advantage of encoders that exploit the correlation between sources. Consider the transmission of a set of correlated sources (S_1, S_2) with the joint distribution $p(s_1, s_2)$ given by

(s_1, s_2)	$p(s_1, s_2)$
(0, 0)	1/3
(0, 1)	0
(1, 0)	1/3
(1, 1)	1/3

over a multiple access channel defined by $\mathcal{X}_1 = \mathcal{X}_2 = \{0, 1\}$, $\mathcal{Y} = \{0, 1, 2\}$, $Y = X_1 + X_2$. Here $H(S_1, S_2) = \log 3$ =1.58 bits. On the other hand, if X_1 and X_2 are independent,

$$\max_{p(x_1)p(x_2)} I(X_1, X_2\ ; Y) \ = \ 1.5 \text{ bits.}$$

Thus $H(S_1, S_2) > I(X_1, X_2\ ; Y)$ for all $p_1(x_1)p_2(x_2)$. Consequently there is no way, even with the use of Slepian-Wolf source coding of S_1 and S_2, to use the standard multiple access channel to send S_1 and S_2 reliably. However, it is easy to see that with the choice $X_1 \equiv S_1$ and $X_2 \equiv S_2$, error-free transmission of the sources over the channel is possible. This example shows that separate source and channel coding described above is *not* optimal — the partial information that each of the random variables S_1 and S_2 contains about the other is destroyed in this separation. In our proposed approach (to be discussed next), we allow our codes to depend statistically on the source outputs. This induces some dependence between the codewords, which will help combat the adversities of the channel more effectively.

3 Problem Formulation

3.1 Multiple Access Communication System with Correlated Messages

Consider the block diagram of a communication system shown in Figure 1. There are two senders and one receiver. Here we assume that the multiple access channel is stationary and memoryless.

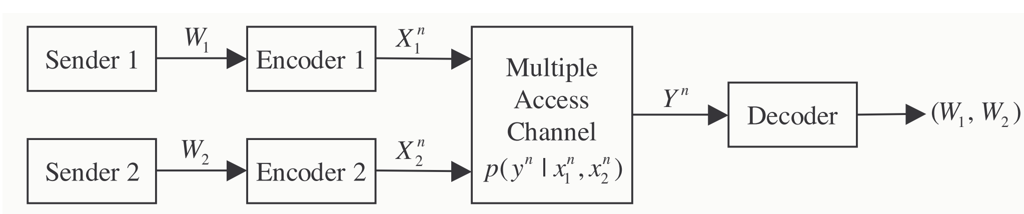

Figure 1: The multiple access channel: the messages W_1 and W_2 of the encoders can not be chosen independently.

Two senders have integer message sets $\mathcal{W}_1 = \{1, 2, \ldots, \Delta_1\}$ and $\mathcal{W}_2 = \{1, 2, \ldots, \Delta_2\}$ respectively. We assume that there is some kind of "correlation"[1] between two message sets, i.e., messages from each sender can not

[1] Note this is an abuse of notation. In general there may not be any stochastic characterization of the messages of the two users. See Remark 1 for more elaboration at the end of this section.

be chosen independently. If the messages of the senders can be chosen independently, then all possible pairs (W_1, W_2) in the set $\mathcal{W}_1 \times \mathcal{W}_2$ can occur jointly. On the other hand, if they are related, only some pairs (W_1, W_2) in the set occur and the other pairs do not. We will use bipartite graphs to describe "correlated" messages of the two senders. In this subsection, our exposition is at times informal, though precise definitions follow immediately.

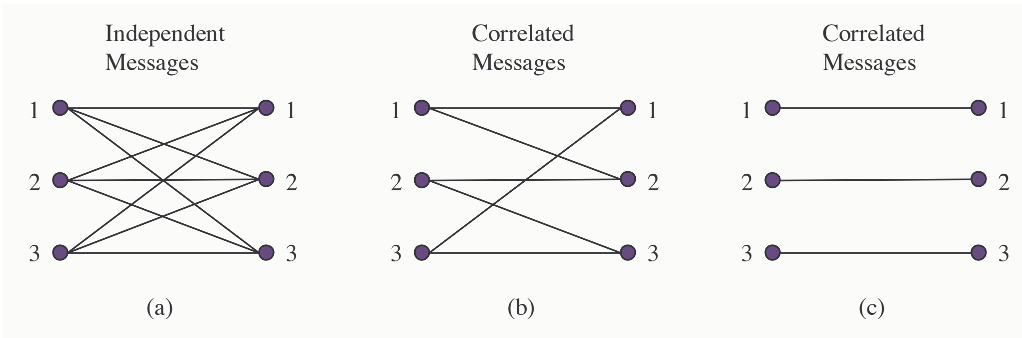

Figure 2: "Independent" and "correlated" messages: the number of allowed message pairs decreases from (a) to (c).

As an example, let us consider the simple case as shown in Figure 2. In this case, two senders have $\mathcal{W}_1 = \mathcal{W}_2 = \{1, 2, 3\}$. The vertices in the bipartite graph denote messages in the message sets, and an edge between two vertices imply that the message pairs can occur jointly. The complete bipartite graph of Figure 2(a) corresponds to the case for which two messages from each sender can be chosen independently, so all the possible pairs can occur. Figure 2(b) and Figure 2(c) show the case for which two messages are "correlated". In case of Figure 2(b), message pairs (1, 1), (1, 2), (2, 2), (2, 3), (3, 3), and (3, 1) can occur, but (1, 3), (2, 1) and (3, 2) can not occur. Similarly, only three message pairs (1, 1), (2, 2) and (3, 3) can occur in case of Figure 2(c), which also means that the two messages are exactly the same; in other words, they are perfectly "correlated". The messages of Figure 2(c) have higher "correlation" than those of Figure 2(b).

Definition: A bipartite graph G, and a message-graph $E(G)$ characterized by G with parameters $(\theta_1, \theta_2, \theta_1', \theta_2')$ are defined as follows.

1. G has two sets of vertices where the number of vertices are θ_1 and θ_2 respectively, and a vertex in one set can only be connected with the vertices in the other set.

2. A vertex in one set represents an element in the message set $\mathcal{W}_1 = \{1, 2, \ldots, \theta_1\}$, and similarly a vertex in the other set represents an element in $\mathcal{W}_2 = \{1, 2, \ldots, \theta_2\}$.

3. An edge in the graph, denoted by $(i - j)$, represents a message pair (i, j), $i \in \mathcal{W}_1$ and $j \in \mathcal{W}_2$, which can occur jointly. The set of all the message pairs represented as edges in the graph G is denoted by $E(G)$, i.e., $E(G) = \{(i, j) : \exists\ (i - j) \text{ in } G,\ i \in \mathcal{W}_1, j \in \mathcal{W}_2\}$. We refer to $E(G)$ as the message-graph. This is an

abuse of notation. Note G is just a way of representing the message set given by $E(G)$. The distinction between a message-graph $E(G)$ and the graph G associated with it should be clear from now on.

4. θ'_1 is the number of the messages in $\mathcal{W}_1$ which can occur jointly with a fixed message in $\mathcal{W}_2$, and similarly θ'_2 is the number of the messages in $\mathcal{W}_2$ which can occur jointly with a fixed message in $\mathcal{W}_1$. We assume that the number of edges going out of any vertex in a given set is the same.

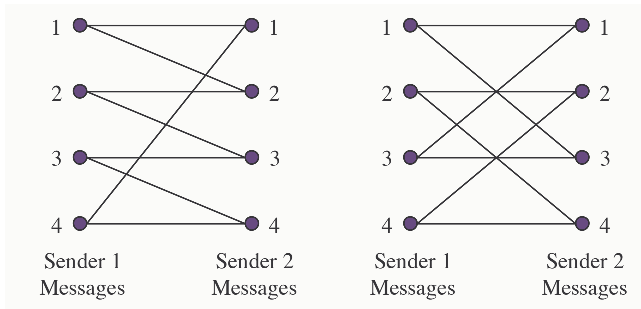

Figure 3: Examples of bipartite graphs with parameters (4, 4, 2, 2): the message-graph characterized by the graph on the right side can be decomposed into 3 independent messages, with both encoders sharing a common message. This can be seen by renaming $1, 2, 3$ and 4 as $11, 21, 12$ and 22 respectively.

Figure 3 illustrates two bipartite graphs with parameters (4, 4, 2, 2) as examples. An example of the transmission system considered by [7], where the messages of the two users have a "common part" can be represented by the graph on the right side. In the other words, the graph on the right side can be represented as a set of three independent messages of length 2, with the first user having the first and the second message sets, and the second user having the the second and the third message sets. This can be seen by renaming $1, 2, 3$ and 4 as $11, 21, 12$ and 22 respectively. Now each message of each user has two labels. As can be seen from the graph, for any valid message pair of the two users, the corresponding first labels are the same. Now if we consider each label as a message, then the first label of both users corresponds to the common message. Such graphs form a subset of all incomplete graphs as given in the above definition.

Note that $E(G)$ is always a subset of $\mathcal{W}_1 \times \mathcal{W}_2$. If the messages belonging to $\mathcal{W}_1$ and $\mathcal{W}_2$ are "independent", then $\theta_1 = \theta'_1$ and $\theta_2 = \theta'_2$ since all the message pairs can occur jointly. Thus in this case $E(G) = \mathcal{W}_1 \times \mathcal{W}_2$. As the "correlation" between two message sets becomes higher, θ'_1 and θ'_2 will decrease since there will be less edges in G. A bipartite graph G_2 is said to cover G_1 if $E(G_1) \subseteq E(G_2)$. *In the sequel for ease of exposition, we consider a symmetric system with bipartite graphs with parameters* $(\theta_1, \theta_1, \theta'_1, \theta'_1)$.

Remark 1: The goal of the encoders of the two users is to transmit these pairs of messages (or edges in the graph associated with the messages) reliably over the channel. As an analogy, the conventional Shannon's channel coding theorem can be interpreted as finding the maximum number of codewords (colors, if each codeword has a different color) that are distinguishable at the noisy receiver. Note that this number will not change even if the frequency of occurrence of these colors is not the same. In conventional multiple access channel, the goal is to distinguish pairs of colors at the noisy receiver, where the first color can come from one set and the second color can come from another set, and all possible combination of pairs in the two sets are allowed. A natural question to ask is: if only a fraction of all possible combination of pairs of colors is permitted, what is the maximum size of the sets of these colors for which reliable distinguishability can be guaranteed at the receiver. In this work we obtain a partial answer to this question. Since we do not associate any stochastic characterization with these messages, the terms like correlation and independence are written in quotes.

3.2 Channel Codes for "Correlated" Messages

As shown in the previous example in Section 2.3, if we can design special codes which translate the existing "correlation" between messages of two senders into the channel input instead of adopting conventional codes which do not consider any "correlation" between the messages, we might achieve higher transmission rates than is bounded by the conventional codes where all the codeword pairs can occur jointly.

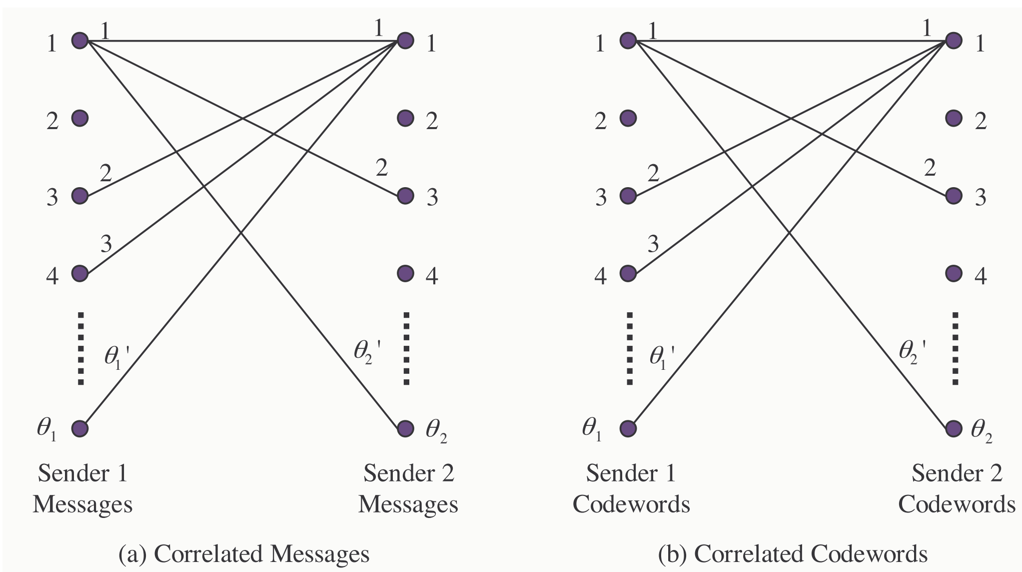

Figure 4: "Correlated" messages and "correlated" codewords: an edge in the graph on the left side denotes a pair of messages that can occur jointly. An edge in the graph on the right side denotes a pair of codewords that satisfies a property (relation).

Now let us consider a coding scheme which translates predetermined "correlation" between the messages into the codewords to be transmitted over the channel as illustrated in Figure 4. Figure 4(a) shows the graph associated with the messages. Note that in Figure 4(a), the message pairs (1, 1), (1, 3) can occur jointly but (1, 2), (1, 4) can not occur jointly. Our goal is to translate this "correlation" between messages into a set of codewords as shown in Figure 4(b). We want codeword pairs (1, 1) and (1, 3) to have a special property, which is not present in codeword pairs (1, 2) and (1,4).

Thus, with a pair of sets of codewords, we can associate a bipartite graph with respect to a property (or relation) defined on a pair of codewords, one from each set. The vertices in the graph denote the codewords. A pair of vertices is connected by an edge if the corresponding pair of codewords have this relation. Such a pair of sets of codewords is referred to as a code-graph. Similar to a message-graph, we emphasize the distinction between a code-graph and the graph characterizing it. A relation associated with a code-graph is desirable if it aids in distinguishing of different codeword pairs allowed by the graph reliably at the receiver. Our goal is to construct a code-graph that covers a given message-graph, and the relation associated with the code-graph is desirable. These ideas are made precise in the next section. Before, we present our main results, we take a detour discussion of some properties of these bipartite graphs.

The motivation of this is the important issue of matching message-graphs with code-graphs. As we have seen, there could be large number of graphs with the same parameters $(\theta, \theta, \theta', \theta')$ for some θ and θ'. This begs the question of transmissibility of a message-graph by a code-graph with both having the same parameters. The essential conclusions of this discussion are summarized in Remark 2 in the next subsection.

3.3 Code Generation by Permutation and Relabeling

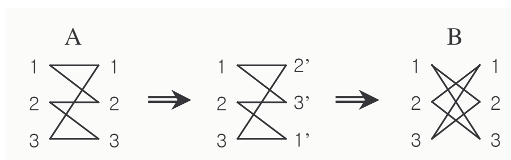

Figure 5: Example of permutation and relabeling.

Suppose we are given a "correlated" message-graph, characterized by A and a "correlated" code-graph characterized by B in Figure 5. In this case, it is not efficient to use the code-graph characterized by B to send the message-graph associated with A as B does not cover A. However, there exists an interesting property between the two graphs. Note that if we permute right vertices of A, $(1,2,3)$ into $(2',3',1')$, relabel $(2',3',1')$ as $(2,3,1)$, and then move right vertices together with their connected edges in natural order $(1,2,3)$, then we get graph B. This implies that we can use the code-graph characterized by B to send the message-graph characterized by A after permuting and relabeling. This procedure is illustrated in Figure 5. Clearly, we can also get graph A from graph B similarly.

In general, it may not be easy to construct a specific "correlation" structure in the codebook which can be used to send a given message-graph. But this example motivates us to study the structure of bipartite graphs and its relation to permutation and relabeling. To get insight into this issue, we will discuss some interesting properties of some simple bipartite graphs, and the possibility of obtaining graphs with a desired "correlation" structure from a given graph by permutation and relabeling the indices.

Let us consider a set bipartite graphs, denoted by $K_{n,a}$, $n \in \mathbb{Z}^+$ where $\mathbb{Z}^+$ is the set of positive integers, and $a \in \{1, 2, \ldots, n\}$. Each graph in $K_{n,a}$ has n vertices on both sides, and each vertex on the left has a different edges to the right, and similarly each vertex on the right has exactly a different edges to the left. For example, Figure 6 illustrates all the elements of $K_{3,2}$. So there are totally six distinct bipartite graphs in the set $K_{3,2}$. For

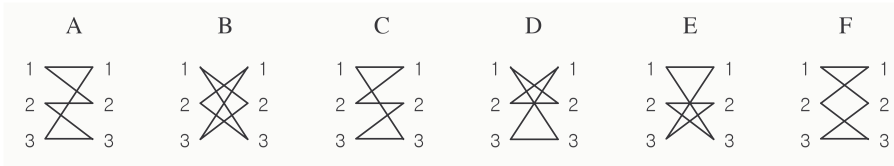

Figure 6: All the possible bipartite graphs in the set $K_{3,2}$: any graph can be obtained from any other by permutation and relabeling.

$K_{n,2}$ with arbitrary n, the cardinality is just the number of all the possible ways of making two edges from each left vertex to the right vertices under the following rule. The first edge from each left vertex must be connected with different right vertices, and second edge from the same left vertex must be connected with distinct right vertices which are not previously connected with the same left vertex. In other words, each left vertex can not be connected with the same right vertex twice.

Now we discuss a method of generation of different bipartite graphs in $K_{n,a}$ by permutation and relabeling any one of them. Let us consider bipartite graphs with $n = 3, a = 2$, and denote $K_{3,2}$={A, B, C, D, E, F} where A, B, C, D, E, and F are the bipartite graphs shown in Figure 6. In this case, all the elements in $K_{3,2}$ can be generated from any one element in the set by permutation and relabeling as shown in Figure 7. For example, given the bipartite graph A, first permute the right vertices $(1, 2, 3)$ into $(1', 3', 2')$ where the primes in the sequence $(1', 3', 2')$ indicate that they are permuted from the original sequence. Secondly, relabel $(1', 3', 2')$ as $(1, 3, 2)$, and move the right vertices together with their connected edges in the natural order $(1, 2, 3)$, then we get graph E in Figure 6. Similarly, we can get all other graphs in $K_{3,2}$ from A.

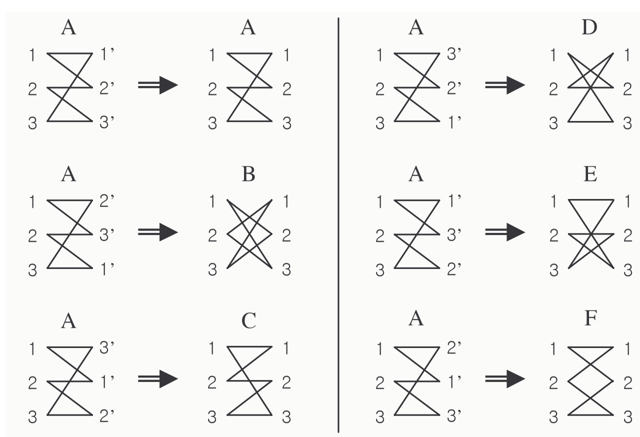

Figure 7: Generation of different bipartite graphs by permutation and relabeling in $K_{3,2}$.

However, in the case of $n = 4, a = 2$, we can not get all graphs in $K_{4,2}$ by just permutation and relabeling of any one graph in the set. There are a total of 90 distinct bipartite graphs in $K_{4,2}$, i.e., $|K_{4,2}| = 90$. These 90 graphs can be divided into mutually exclusive six subsets, denoted by **A**, **B**, **C**, **D**, **E** and **F**. Hence $K_{4,2} = \mathbf{A} \cup \mathbf{B} \cup \mathbf{C} \cup \mathbf{D} \cup \mathbf{E} \cup \mathbf{F}$ and $|K_{4,2}| = |\mathbf{A}| + |\mathbf{B}| + |\mathbf{C}| + |\mathbf{D}| + |\mathbf{E}| + |\mathbf{F}|$.

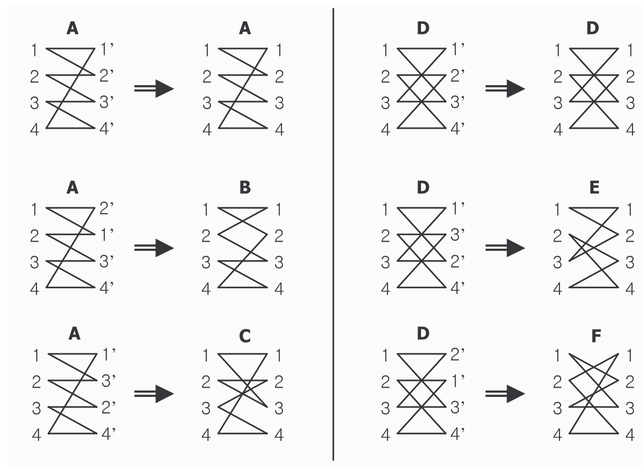

Figure 8: Generation of different bipartite graphs by permutation and relabeling in the subsets of $K_{4,2}$.

It can be verified that the set $K_{4,2}$ has six mutually exclusive small subsets, and they can be grouped into two bigger subsets, given by $\mathbf{A}_1$ and $\mathbf{A}_2$. All the graphs in the subset $\mathbf{A}_i$ (for $i = 1, 2$) can be obtained from any graph in that set by permutation and relabeling of the vertices. However, no graph in the subset $\mathbf{A}_1$ can be obtained by permutation and relabeling of a graph in $\mathbf{A}_2$ and vice versa. This is detailed in the Appendix.

In other words, the set $K_{4,2}$ can be partitioned into two *equivalence classes* where the cardinalities of these classes are 72 and 18 respectively, and the equivalence relation is characterized by the feasibility of obtaining one element in the class by permutation and relabeling of the vertices of the other.

It can be shown that a message-graph characterized by every graph in the set $\mathbf{D} \cup \mathbf{E} \cup \mathbf{F}$ has a common part. In other words, suppose the graph characterizing a message-graph belongs to $\mathbf{D} \cup \mathbf{E} \cup \mathbf{F}$, then we can represent this message-graph as a collection of 3 independent messages, each of length 2. This can be seen if we represent vertices $\{1, 2, 3, 4\}$ as binary numbers $\{00, 01, 10, 11\}$. Further, it can be shown that the message-graph characterized by every graph in the set $\mathbf{A} \cup \mathbf{B} \cup \mathbf{C}$ has no common part.

Further, it can be shown that similar partitions exist in $K_{n,a}$ for general n and a. At this point, we do not have a precise characterization of the number of equivalence classes in $K_{n,a}$.

Remark 2: We discussed this problem only for $n = 3, 4$ and $a = 2$, but from this result we learned about an interesting property of these bipartite graphs. We summarize the conclusions of this discussion:

- In $K_{n,a}$, graphs can be partitioned into equivalence classes, where one element in a class can be obtained from the other in the same class by permutation and relabeling. Thus if we have a code-graph with graph G_1 that does not cover a given message-graph with graph G_2, but if G_1 covers a graph G_3, where G_2 and G_3 belong to the same equivalence class, then we can use the given code-graph to transmit the message-graph.

- The elements of $K_{n,a}$ belonging to different equivalence classes may have different "correlation" structures.

These issues are further explored in [21]

4 Achievable Rate Region

In this section we characterize transmissibility of certain message-graphs over a multiple access channel. We want to emphasize that the symmetric system considered in this section can be easily generalized to non-symmetric cases. A word about the notation. For some finite sets $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$, if $\mathcal{C} \subset \mathcal{A} \times \mathcal{B}$, then for any $a \in \mathcal{A}$, let $h_1(a, \mathcal{C})$ denote the largest subset of $\mathcal{B}$, where $\forall b \in h_1(a, \mathcal{C})$, we have $(a, b) \in \mathcal{C}$. Similarly define $h_2(b, \mathcal{C})$ for $\forall b \in \mathcal{B}$ to be the largest subset of $\mathcal{A}$ such that $\forall a \in h_2(b, \mathcal{C})$ we have $(a, b) \in \mathcal{C}$. Define $h_3(\mathcal{C}) = \{a \in \mathcal{A} : h_1(a, \mathcal{C}) \neq \phi\}$ and $h_4(\mathcal{C}) = \{b \in \mathcal{B} : h_2(b, \mathcal{C}) \neq \phi\}$, where ϕ denotes the null set.

4.1 Summary of Results

We are given a symmetric stationary memoryless multiple access channel with symmetric conditional distribution $p(y|x_1, x_2)$, with input alphabets being the same and given by a finite set $\mathcal{X}$, and a finite output alphabet $\mathcal{Y}$.

A transmission system with parameters $(n, \Delta, \Delta', \tau)$ for the given multiple access channel with "correlated" messages would involve

- A set of code-graphs $\mathbb{C}^*$ where
  - $\mathbb{C}_i \subset \mathcal{X}^n$ and $|\mathbb{C}_i| = \Delta$ for $i = 1, 2$,
  - $\forall \mathbb{G} \in \mathbb{C}^*$, we have $\mathbb{G} \subset \mathbb{C}_1 \times \mathbb{C}_2$,
  - $|h_3(\mathbb{G})| = |h_4(\mathbb{G})| = \Delta$,
  - $|h_1(\mathbf{a}, \mathbb{G})| = |h_2(\mathbf{b}, \mathbb{G})| = \Delta', \quad \forall \mathbf{a} \in h_3(\mathbb{G})$ and $\forall \mathbf{b} \in h_4(\mathbb{G})$.
- A set of decoder mappings $\{d_{\mathbb{G}} : \mathbb{G} \in \mathbb{C}^*\}$, where $d_{\mathbb{G}} : \mathcal{Y}^n \to \mathbb{G}$.
- A performance measure given by the following minimum-average probability of error criterion: $\tau = \min_{\mathbb{G} \in \mathbb{C}^*} \sum_{(\mathbf{a},\mathbf{b}) \in \mathbb{G}} \frac{1}{|\mathbb{G}|} Pr\left[d_{\mathbb{G}}(\mathbf{Y}) \neq (\mathbf{a}, \mathbf{b}) | \mathbf{X_1} = \mathbf{a}, \mathbf{X}_2 = \mathbf{b}\right]$.

The goal is to find the closure $\mathcal{R}^*$ of the set of all achievable rate pairs (R, R'), where a rate pair (R, R') is said to be achievable for the given multiple access channel with "correlated" message sets if $\forall \epsilon > 0$, and for sufficiently large n, there exists a transmission system as defined above with parameters $(n, \Delta, \Delta', \tau)$ with $\frac{1}{n} \log \Delta > R - \epsilon$, $\frac{1}{n} \log \Delta' > R' - \epsilon$ and the corresponding minimum-average probability of error $\tau < \epsilon$.

An achievable rate region is given by the following theorem, which is the main result of this paper.

Theorem 1: An achievable rate region is given by the set of all (R, R') such that

$$R < \frac{1}{2}\left[I(X_1, X_2; Y) + I(X_1; X_2)\right], \text{ and } R' < R - I(X_1; X_2) \tag{3}$$

for some symmetric probability distribution $p(x_1, x_2)$ defined on $\mathcal{X}^2$.

4.2 Outline of Proof

In this section, we present the key ideas in the proof of the main result. The details are present in [21]. We use random coding and binning arguments and the notion of jointly typical sequences as given in [19].

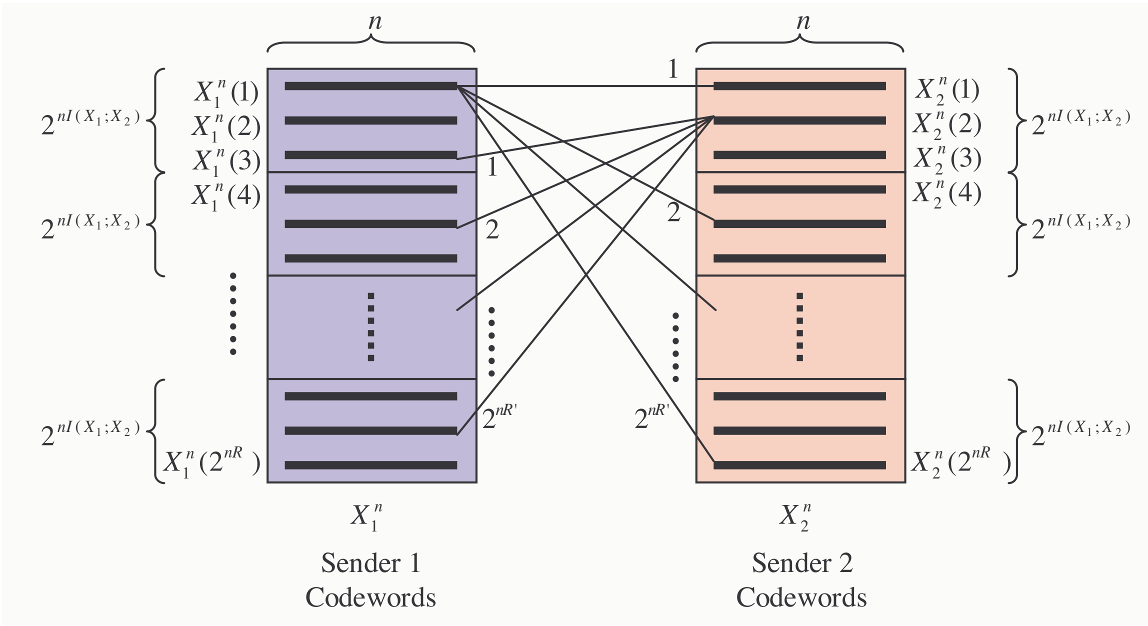

Figure 9: "Correlated" random codebook and jointly typical sequences: each codebook is randomly partitioned into $2^{nR'}$ bins, with each bin containing $\approx 2^{nI(X_1;X_2)}$ sequences.

Given the multiple access channel with distribution $p(y|x_1, x_2)$, fix a symmetric joint distribution $p(x_1, x_2)$ on $\mathcal{X}^2$.

Codebook generation: Generate 2^{nR} codewords $\mathbf{X}_1(i)$, $i \in \{1, 2, \ldots, 2^{nR}\}$, of length n, generating each element of every codeword independent identically distributed (i.i.d.) from the distribution $p(x_1)$, and call it $\mathbb{C}_1$. Similarly, generate 2^{nR} codewords $\mathbf{X}_2(i)$, $i \in \{1, 2, \ldots, 2^{nR}\}$, of length n, generating each element of every codeword i.i.d. from the distribution $p(x_2)$, and call it $\mathbb{C}_2$, where $p(x_1)$ and $p(x_2)$ are the marginals of $p(x_1, x_2)$.

Graph generation: Since $R - R' \simeq I(X_1; X_2)$ for $j = 1, 2$, each codebook (with size 2^{nR}) can be partitioned independently into $2^{nR'}$ random bins, with each bin having approximately $2^{nI(X_1;X_2)}$ codewords. For every codeword in $\mathbb{C}_2$, with high probability there is at least one codeword in every bin of $\mathbb{C}_1$ which are jointly typical, in the sense of $p(x_1, x_2)$. Thus we have $2^{nR'}$ codewords in $\mathbb{C}_2$ which are jointly typical with every codeword of $\mathbb{C}_1$, and similarly $2^{nR'}$ codewords in $\mathbb{C}_1$ which are jointly typical with every codeword of $\mathbb{C}_2$. This is illustrated in the Figure 9. This forms a code-graph with joint typicality as the associated desired property. In other words, with this codebook pair, we can associate a bipartite graph with parameters $(2^{nR}, 2^{nR}, 2^{nR'}, 2^{nR'})$, with one vertex associated with every codeword, and two vertices are connected by an edge if the corresponding codewords are jointly typical. Since our goal is to minimize the average probability of error over all message-graphs, for transmission, we assume that the message-graph and the code-graph belong to the same equivalence class. Note if $I(X_1; X_2) = 0$, the size of each bin becomes one. So every codeword in one codebook can be jointly typical with every codeword in the other. As $I(X_1; X_2)$ increases, the bins become larger, thus the total number of jointly typical sequence pairs in the codebook changes.

Encoding : Sender 1 sends the codeword $\mathbf{X}_1(i)$ to send message index i, similarly, sender 2 sends $\mathbf{X}_2(j)$ to send message index j.

Decoding : At the receiver, the index pair (i, j) is chosen as the transmitted message pair only if there exists a unique pair (i, j) such that $(\mathbf{X}_1(i), \mathbf{X}_2^n(j), \mathbf{Y}^n)$ are jointly typical in the sense of $p(x_1, x_2)p(y|x_1, x_2)$. Otherwise, an error is declared. It can be shown that if R and R' satisfies the condition as given Theorem 1, then the minimum-average probability of error can be made arbitrarily small. Hence this implies that there exists at least one good code-graph which has arbitrarily small error probability.

Remark 3: It is easy to find that Theorem 1 gives exactly the same rate region as given in Section 2.1 if there is no "correlation" between the messages, i.e., $R = R'$ which implies that $I(X_1; X_2) = 0$. In general, since mutual information is nonnegative, if the messages are "correlated", by using "correlated" codewords we can achieve higher rates which are beyond the capacity region which uses "independent" codewords.

5 Gaussian Multiple Access Channel

5.1 Achievable Rate Region

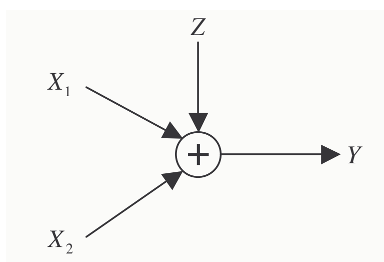

Figure 10: Gaussian multiple access channel.

The coding theorem given in the previous section can be extended to continuous-alphabet sources using standard techniques [14, 15]. Consider the Gaussian multiple access channel with the symmetric channel input distribution being jointly Gaussian as shown in Figure 10. There are two senders and one receiver. Each of the inputs has a power constraint, given by $E[X_i^2] \leq P$ for $i = 1, 2$. The received signal Y is given by

$$Y = X_1 + X_2 + Z \tag{4}$$

where Z is zero mean Gaussian random variable with variance N, denoted by $Z \sim \mathcal{N}(0, N)$, and X_1 and X_2 are zero mean jointly Gaussian random variables with covariance matrix K given by

$$K = \begin{bmatrix} P & \rho P \\ \rho P & P \end{bmatrix} \tag{5}$$

where ρ is the correlation coefficient.

The following information quantities associated with this Gaussian case can be easily computed. We analyze the behavior of (R, R') as a function of the correlation coefficient ρ.

$$I(X_1; X_2) = \frac{1}{2} \log \frac{1}{1-\rho^2}. \tag{6}$$

$$I(X_1; Y) = I(X_2; Y) = \frac{1}{2} \log \frac{2P + 2\rho P + N}{(1-\rho^2)P + N}. \tag{7}$$

$$I(X_1, X_2; Y) = \frac{1}{2} \log \left(1 + \frac{2P + 2\rho P}{N}\right). \tag{8}$$

$$I(X_1, X_2; Y) + I(X_1; X_2) = \frac{1}{2} \log \left[\frac{1}{1-\rho^2} \left(1 + \frac{2P + 2\rho P}{N}\right)\right]. \tag{9}$$

Hence, if we use "correlated" codewords which exploit the "correlation" between messages, an achievable rate region for the Gaussian multiple access channel with jointly Gaussian channel input is the set of rate pairs (R, R') satisfying

$$R < \frac{1}{4} \log \left[\frac{1}{1-\rho^2} \left(1 + \frac{2P + 2\rho P}{N}\right)\right], \qquad R' < R - \frac{1}{2} \log \left(\frac{1}{1-\rho^2}\right). \tag{10}$$

The boundary of the achievable rate region is thus given by

$$(R, R') = \left(\frac{1}{4} \log \left[\frac{1}{1-\rho^2} \left(1 + \frac{2P + 2\rho P}{N}\right)\right], \frac{1}{4} \log \left[(1-\rho^2) \left(1 + \frac{2P + 2\rho P}{N}\right)\right]\right). \tag{11}$$

If X_1 and X_2 are independent, i.e., $\rho = 0$, this gives the well known capacity region of the Gaussian multiple access channel with independent messages, which is the convex hull of the set of rate pairs (R, R') satisfying

$$R < \frac{1}{4} \log \left(1 + \frac{2P}{N}\right), \qquad R' = R. \tag{12}$$

As ρ varies from zero to one, mutual information $I(X_1; X_2)$ increases from zero to ∞. These values, which vary according to the correlation coefficient ρ, are shown in Figure 11, for $P = 10$ and $N = 1$.

Figure 11 shows the variation of R and R' as a function of correlation coefficient ρ. As ρ increases from 0 to almost equal to 1, R increases slowly when ρ is small, and then increases rapidly for large ρ, but R' becomes slightly bigger for small ρ and then decreases. For large ρ, R' decreases very rapidly and then becomes 0. This essentially is the upper bound on the values that ρ can take.

The achievable rate region is illustrated in Figure 12 for different signal to noise ratios as a function of the correlation coefficient. Using the boundary values for R and R' as given by (11), we get the bound for $R + R'$, which is given by

$$R + R' < \frac{1}{2} \log \left(1 + \frac{2P(1+\rho)}{N}\right). \tag{13}$$

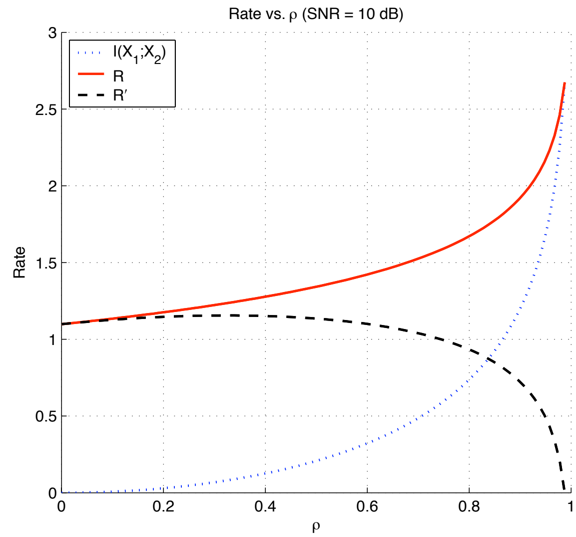

Figure 11: Variation of R and R' vs. correlation coefficient ρ, for $P = 10$ and $N = 1$.

5.2 Comparison with Separate Source and Channel Coding

We can use separate source and channel coding in order to send the same "correlated" messages over the Gaussian multiple access channel. In this case, we apply Slepian-Wolf source coding on the given "correlated" messages to transform them into new "independent" messages. This is done to minimize the amount of information to be sent over the channel. This is followed by conventional multiple access channel coding working on these new, transformed "independent" messages. Figure 13 illustrates a block diagram of a separate source and channel coding scheme for sending "correlated" messages over the Gaussian multiple access channel.

For the same Gaussian multiple access channel, we consider the case where P' is the power constraint on the inputs, and the rates of the two encoders are the same. According to the Slepian-Wolf theorem (see Section 2.2), we can encode the given "correlated" message-graph with parameters $(2^{nR}, 2^{nR}, 2^{nR'}, 2^{nR'})$ into two messages of length nR and nR'. Using time-sharing, we can now assume that each encoder of the channel coding module has access to an independent message of length $(nR+nR')/2$. If we now use the given multiple access channel n times, and use conventional multiple access channel coding, the transmission power required to sustain reliable communication is given by

$$P' = \frac{N}{2}\left[2^{2(R+R')} - 1\right]. \tag{14}$$

Now if we substitute for R and R', the values on the boundary of achievable rate region given by Theorem 1, then we get

$$P' = P(1+\rho). \tag{15}$$

Now we can compare the two different schemes for sending "correlated" messages over the Gaussian multiple access channel. One is a coding scheme with "correlated" codewords which exploits the existing message

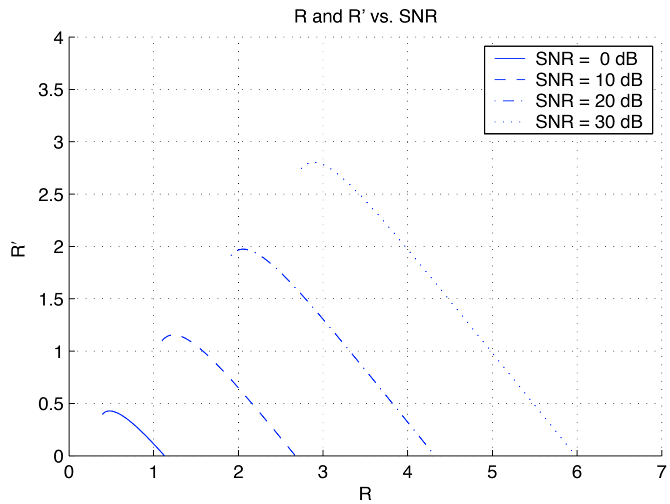

Figure 12: Variation of R and R' vs. correlation coefficient ρ, for different signal to noise ratios.

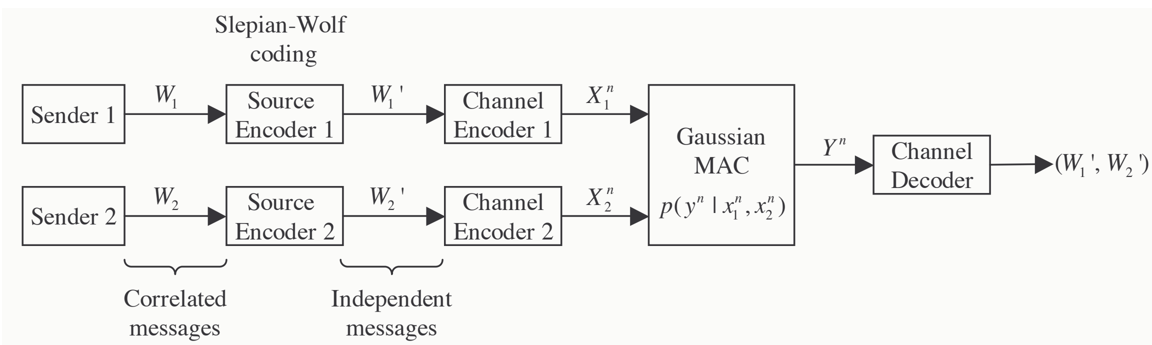

Figure 13: Separate source and channel coding involving Slepian-Wolf source coding and conventional multiple access channel coding for sending "correlated" messages over the Gaussian multiple access channel.

"correlation", the other is separate source and channel coding, working with "independent" codewords after applying Slepian-Wolf source coding. In order to have the same achievable rates in both schemes, we have the condition that $P' = (1+\rho)P > P$ if we choose a positive correlation coefficient. This means that if the given messages are not "correlated", i.e., $\rho = 0$, then the required power in both schemes are exactly the same, but if the messages are "correlated" with $\rho > 0$, we can send the same amount of information with less power by encoding with "correlated" codewords. In other words, we can send more information over the channel with the same given power.

5.3 Binary-Input Multiple Access Channel

Let us revisit the example considered in Section 2.3 that shows that error-free transmission of the given correlated sources over the given binary-input multiple access channel is possible with the special code $X_1 \equiv U$ and $X_2 \equiv V$. This can be considered as a match between the message-graph and the code-graph. So by applying the theorem, we can calculate the achievable rate region for this case.

$$I(X_1; X_2) = \log 3 - \frac{4}{3} \simeq 0.2516. \tag{16}$$

$$I(X_1, X_2; Y) = \log 3 \simeq 1.5850 \tag{17}$$

Thus, the achievable rate region is given by

$$R < \frac{1}{2}\left[I(X_1, X_2; Y) + I(X_1; X_2)\right] \simeq 0.9183 \quad \text{and} \quad R' < R - 0.2516. \tag{18}$$

Clearly $R = 0.9183$ and $R' = 0.6667$ is on the boundary of the given achievable rate region. It can be easily seen that $H(U) = H(V) = 0.9183$ and $H(U|V) = 0.6667$. Thus, while Sender 1 sends at a rate $R_1 = 0.9183$, Sender 2 also can send at a rate $R_2 = 0.9183$, along as their messages are "correlated".

So, in this case, achievable rates are the same as the entropies of two random variables, i.e., $R = H(U) = H(V)$. This means that all the typical sequences of U and V can be sent over the channel without any error.

6 Conclusion

We have considered multiple access channels with "correlated" messages. In the big picture, the vision is to build a multiterminal communication system with correlated information sources being transmitted reliably over a multiple access channel. In the proposed approach, we assume that the sources can be remotely mapped into message sets which preserve a predetermined "correlation". This "correlation" can be translated to the channel input using appropriate coding methods which will be more effective in combating the adversities of the channel. We emphasize that in our formulation, we do not associate any stochastic characterization with these messages. The problem is like distinguishing pairs of colors. If one knows that only certain pairs of colors are permitted, a better method of encoding can be accomplished which improves the performance. We have applied our analysis to two examples, one involving the Gaussian multiple access channel and the other involving binary-input multiple access channel to corroborate the claims made.

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A Appendix

Detail Explanation of Graphs in $K_{4,2}$

Let us consider one of the graphs in the subset **A** as shown in the top left corner of Figure 8. If we permute and relabel the left vertices of the graph in the same way as explained previously, then we get $4! = 24$ distinct graphs which are all the elements of **A**. Note that for all the graphs in **A**, all the left vertices can be connected with only one of the $\{(1, 2), (2, 3), (3, 4), (4, 1)\}$ where the numbers (i, j) are right vertices i and j connected with one left vertex in the graph. They, for example, can not have edges such as $1-1$ and $1-3$. Hence it can be shown that only other graphs in the same subset can be obtained by permuting and relabeling the left vertices. Now let us do this operation on the right vertices. By changing the right vertices 1 and 2 in the graph **A**, we can get a graph in the subset **B** in Figure 8. Similarly by permuting and relabeling the left vertices again, we can obtain all the distinct graphs in **B**. By changing the right vertices 2 and 3 in the graph **A**, we can get a graph in the subset **C** in Figure 8, and we can obtain all the graphs in **C** in a similar way.

Up to now, we could generate a total of 72 distinct graphs in the set $K_{4,2}$. Note that even after we permute the right and the left vertices of the original graph in **A**, we can not get the graph for which the edges belong to the set given by $\{(1, 3), (2, 4), (1, 3), (2, 4)\}$. This means that there are some graphs in $K_{4,2}$ which can not be obtained from a graph in the subset $\mathbf{A} \cup \mathbf{B} \cup \mathbf{C}$ by just permutation and relabeling.

Now consider the graph shown in the top right side of Figure 8. This is one of the graph in the subset **D**, for which the edges belong to the set given by $\{(1, 3), (2, 4), (1, 3), (2, 4)\}$. If we permute and relabel the left vertices, then we can obtain $4!/2! = 12$ distinct graphs in the subset **D**. Similarly, we can obtain all the remaining distinct 18 graphs in $K_{4,2}$ by changing the right vertices of the graph as shown in Figure 8. Hence we can obtain all the remaining 18 graphs in the subset $\mathbf{D} \cup \mathbf{E} \cup \mathbf{F}$ as well.

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