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# SIGNAL DETECTION WITH A PANORAMIC RECEIVER

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Department of Electrical Engineering

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ERRATA SHEET

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|---|---|
| Page 5 Line 5 | Change "one-tenth" to ".198". |
| Page 5 Line 7 | Change "100" to "50". |
| Page 5 Line 10 | Change "1/100" to "1/50". |
| Page 9 Last line | Add sentence –"Such a filter is referred to as inverse Gaussian". |
| Figure 3 thru 6 | Normalized signal-to-noise ratio is $(S/N)/(2EnN_0)^2$. |
| Table 1 | Change "2M" to "$\sqrt{2M}$" |
| Page 17 Line 8 | Change to read "$ad^2 = 160(Bdh)$ KMc/$s^2$, $bd = \sqrt{2}(Bdh)$ and wide open video". |

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ABSTRACT

Formulas for the signal-to-noise ratio at the output of a particular panoramic receiver are calculated. The receiver considered consists of a linearly sweeping local oscillator, an ideal mixer, an IF amplifier with a Gaussian passband, a square law detector, and a video amplifier with a Gaussian passband. For a given signal pulse duration the optimum IF bandwidth, video bandwidth, and sweep rate are determined. The results of this derivation are applied to compare a wideband non-scanning receiver with a narrow band receiver scanning at a rapid enough rate to make it effectively a wideband receiver over the same frequency range. If the optimum video filter is used, both receivers have very nearly the same output signal-to-noise ratio; but if the video filter is not matched, the fast-scanning narrow-band receiver is superior.

# SIGNAL DETECTION WITH A PANORAMIC RECEIVER

## I. INTRODUCTION

In this report formulas for signal-to-noise ratio are calculated for a model of panoramic receiver. The receiver model consists of a linearly sweeping local oscillator, an ideal mixer, an IF amplifier with Gaussian passband, a square law detector, and a video amplifier with a Gaussian passband. This model was chosen because an explicit formula for signal-to-noise ratio can be obtained for it— a rather remarkable fact in view of the complexity of the system.

Although the analysis is made only for the case of pulses with Gaussian shaped envelopes and filters with Gaussian passbands, the results should be indicative of what will occur in the more general case. Certainly the noise power will be little different. It was shown in Technical Report No. 3[1] that the Gaussian case is quantitatively consistent enough with the other cases studied to be used in many design problems involving peak amplitude, output pulse width, and apparent bandwidth. These are the features of the receiver output which are pertinent here.

Ideally, one would like to know probability of detection and probability of false alarm as a function of signal, noise, and receiver parameters. For a system as complicated as the panoramic receiver, this appears to be out of the question at this time. Signal-to-noise ratio has been used successfully for many years as a guide for designing linear systems for detection.

[1] H. W. Batten, R. A. Jorgensen, A. B. Macnee, and W. W. Peterson, "The Response of a Panoramic Receiver to CW and Pulse Signals" Technical Report No. 3, Electronic Defense Group, University of Michigan, June, 1952.

Signal-to-noise ratio is defined as the ratio of peak signal power in the absence of noise to the average noise power, at the receiver output. Since the noise and the signal are uncorrelated, the average voltage output with signal plus noise equals the sum of the output with signal only and the average output with noise alone.[1] Thus the peak signal output power measures the change in output voltage, on the average, caused by the signal. The noise power measures the average amplitude of noise fluctuations and hence signal-to-noise ratio is a measure of the amplitude of the change in voltage output caused by the signal, relative to the amplitude of noise fluctuations.

## II. A SIMPLE CASE

In this section, signal-to-noise ratio is calculated at the output of a Gaussian filter when white noise plus a Gaussian pulse are passed through the filter. This problem is relatively simple and serves as a good introduction to the more complicated panoramic receiver problem.

It is well known that the filter which maximizes signal-to-noise ratio has a transfer function proportional to the conjugate of the spectrum of the pulse.[2] It is the purpose of this section to consider the effect of matching the filter bandwidth improperly to the pulses.

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[1] This is strictly true only with a square law detector.

[2] See, for example, Part II, pp. 12 and 20 of Technical Report No. 13 by W. W. Peterson, and T. G. Birdsall, "The Theory of Signal Detectability"; Part I," The General Theory", June, 1953; Part II,"Applications with Gaussian noise," July, 1953. If the transfer function is proportional to the conjugate of the pulse spectrum, then the impulse response is proportional to the pulse reversed in time, and conversely.

Assume that the pulse is (the real part of)

$$f(t) = \left(\frac{E}{d}\sqrt{\frac{2}{\pi}}\right)^{1/2} \exp. \left(jat - \frac{t^2}{d^2}\right), \tag{2.1}$$

where a is the center frequency in radians per second, and d is the pulse duration in seconds between times when the amplitude is exp (-1/4). The constant factor was chosen to make the pulse have energy E,[1] i.e.,

$$\int_{-\infty}^{\infty} f(t)\ \overline{f(t)}\ dt = E. \tag{2.2}$$

The Fourier transform (or spectrum) of this pulse is[2]

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \exp(-j\omega t)dt$$

$$= \left(\frac{E}{d}\sqrt{\frac{2}{\pi}}\right)^{1/2} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(j(a-\omega)t - \frac{t^2}{d^2}\right) dt$$

$$= \left(\frac{E}{d}\sqrt{\frac{2}{\pi}}\right)^{1/2} \frac{d}{\sqrt{2}} \exp\left(-\frac{d^2}{4}(\omega-a)^2\right) \tag{2.3}$$

The pulse spectrum, and hence the optimum filter, has bandwidth $\frac{2}{d}$ radians per second between exp (-1/4) points. Consider a filter with pass band of the same shape and center frequency but of arbitrary bandwidth, i.e., a filter with transfer function

$$H(\omega) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(\omega-a)^2}{b^2}\right) \tag{2.4}$$

where b is the bandwidth in radians per second between exp(-1/4) points. The response of this filter to the pulse given by equation (2.1) is found by multiplying the pulse spectrum by the filter transfer function to get the spectrum of the output, and then transforming this back to the time domain, i.e., the

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1 This assumes a normalized load of one ohm, a convenient and non-restrictive assumption.

2 This integral, and a number of others appearing in this report, can be evaluated with the aid of equation B.4, page 69 of Technical Report No. 3: If U and V are complex numbers, and if the real part of U is positive, then

$$\int_{-\infty}^{\infty} \exp(-Ut^2 + Vt)dt = \sqrt{\pi/U}\ \exp(V^2/4U). \tag{2.5}$$

output pulse g(t) is

$$g(t) = \int_{-\infty}^{\infty} F(\omega)\ H(\omega)\ \exp\ (-j\omega t)\ dt$$

$$= \left(\frac{E}{d}\sqrt{\frac{2}{\pi}}\right)^{1/2} \frac{d}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{d^2}{4}(\omega-a)^2\right] \exp\left(\frac{-(\omega-a)^2}{b^2}\right) \exp(j\omega t) d\omega$$

$$= \left(\frac{E}{d}\sqrt{\frac{2}{\pi}}\right)^{1/2} \frac{d}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left\{\left[-\frac{d^2}{4}+\frac{1}{b^2}\right](\omega-a)^2 + j\omega t\right\} d\omega \qquad (2.6)$$

$$= \left(\frac{E}{d}\sqrt{\frac{2}{\pi}}\right)^{1/2} \cdot \frac{bd}{\sqrt{b^2d^2+4}} \cdot \exp\left(-\frac{t^2}{d^2+\frac{4}{b^2}}\right) \exp\ (jat)$$

The maximum power of the output pulse, i.e., the maximum of $g(t)\ \overline{g(t)}$, occurs when t = 0. It has the value

$$S = \left[g(t)\ \ \overline{g(t)}\right]_{max} = \frac{E}{d}\sqrt{\frac{2}{\pi}} \cdot \frac{b^2d^2}{b^2d^2+4} \qquad (2.7)$$

If the noise is assumed white with spectral power density of $N_o$ watts per cycle per second (or $\frac{N_o}{2\pi}$ watts per radian per second) at the input of the filter, then at the output the noise spectrum is, by equation (A.7) of Appendix A,

$$P_o(\omega) = \frac{1}{2} \cdot \frac{N_o}{2\pi} \cdot 2\pi H(\omega)\overline{H(\omega)}$$

and the total noise power is

$$N = \frac{N_o}{2}\int_{-\infty}^{\infty} H(\omega)\overline{H(\omega)}\ d\omega$$

$$= \frac{N_o}{4\pi}\int_{-\infty}^{\infty} \exp\left(-2\ \frac{(\omega-a)^2}{b^2}\right) d\omega \qquad (2.8)$$

$$= N_o\ b\ /4\sqrt{2\pi}$$

The signal-to-noise ratio is found from equations (2.7) and (2.8) to be

$$\frac{S}{N} = \frac{2E}{N_o} \cdot \frac{4bd}{b^2d^2+4} \qquad (2.9)$$

This function is graphed in Figure 1. The maximum signal-to-noise ratio is achieved when bd = 2, and it has the value $2E/N_o$.[1] The asymptotes are lines of slope one. A filter designed for ten microsecond pulses has one-tenth the optimum bandwidth for one microsecond pulses, and hence, from Fig. 1, the signal-to-noise ratio would be one-tenth of its maximum value if this filter rather than the optimum filter were used with one microsecond pulses. The signal-to-noise ratio would be reduced by a factor of 100 if a filter designed for ten microsecond pulses is used with one-tenth microsecond pulses. Likewise, using a filter designed for one-tenth microsecond pulses would result in a signal-to-noise ratio of 1/100 of the maximum value when used with ten microsecond pulses. Clearly, the filter bandwidth must be matched to the signal if the receiver sensitivity is an important consideration. This is true also in the case of a panoramic receiver.

This analysis assumed for simplicity that the pulse and the filter are Gaussian. The question arises as to whether it is safe to accept these results as approximately true for other types of signals and filters. A partial answer to this question is possible. In the first place it can be shown that the asymptotes are always parallel to $45^o$ lines. In the second place, if an optimum type filter is used, the maximum signal-to-noise ratio is $2E/N_o$.[2] If the curves always have the same maximum and parallel asymptotes, they are probably very similar. Curves for two other cases are shown on Fig. 1 for comparison. Note that in the case of a square pulse and a single-tuned circuit filter, the filter is not of the optimum type for the pulse.

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[1] It is true in general that the signal-to-noise ratio at the output of a filter which is optimum for a single signal is $2E/N_o$. See Technical Report No. 13, Part II, p 67. (In the first printing a factor 1/2 was omitted in equation (5.4), and therefore the output was found to be $E/N_o$ rather than the correct result $2E/N_o$).

[2] See footnote 1 above.

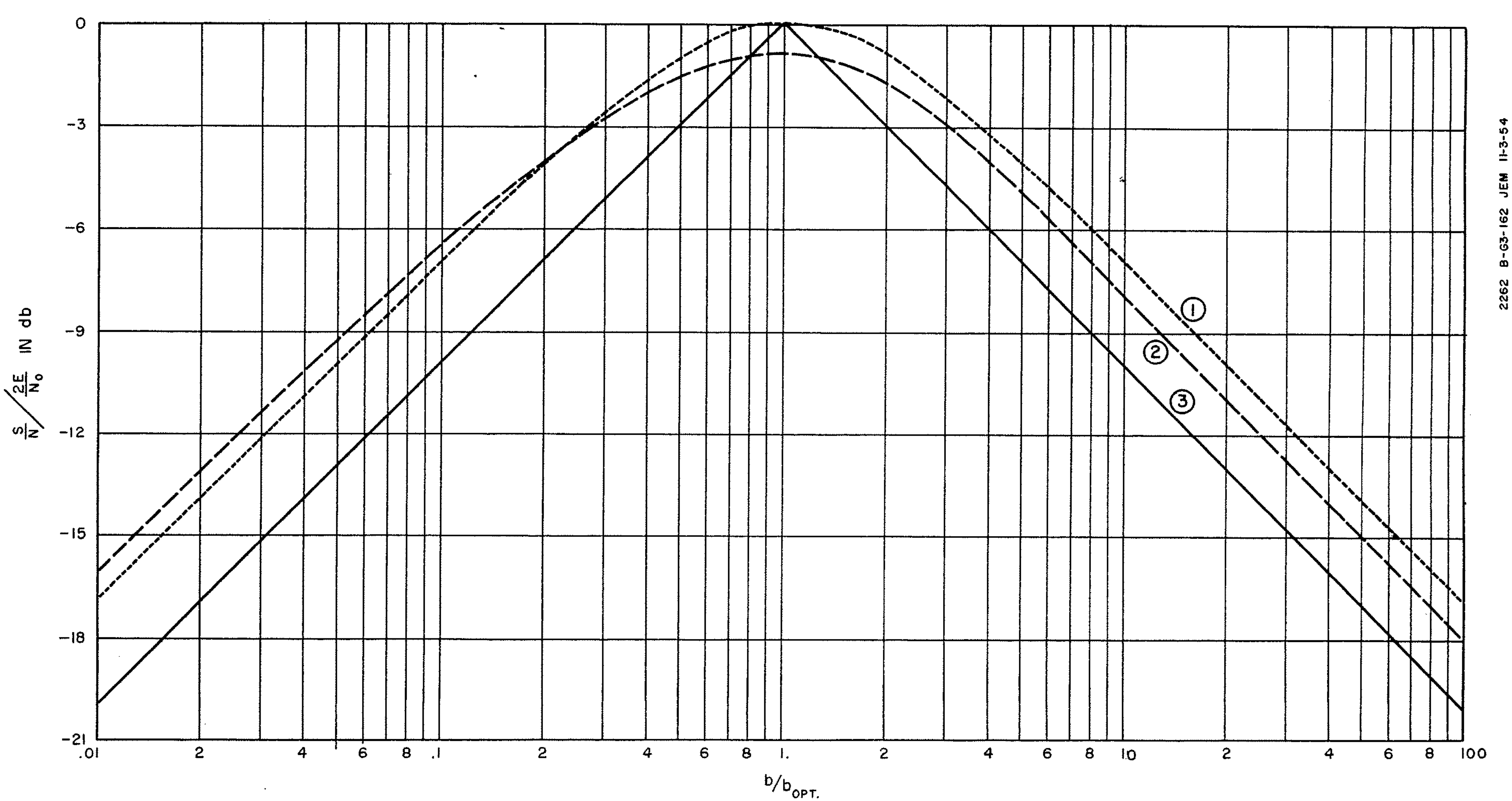


FIG. 1. NORMALIZED OUTPUT SIGNAL-TO-NOISE POWER RATIO VS. NORMALIZED FILTER BANDWIDTH.

| | | | |
|---|---|---|---|
| 1 | GAUSSIAN PULSE | GAUSSIAN BANDPASS | ( GAUSSIAN IMPULSE RESPONSE ) |
| 2 | SQUARE PULSE | R-C FILTER | ( EXPONENTIAL IMPULSE RESPONSE ) |
| 3 | SQUARE PULSE | "$\frac{\sin x}{x}$" BANDPASS | ( SQUARE IMPULSE RESPONSE ) |

## III. SIGNAL-TO-NOISE RATIO IN A PANORAMIC RECEIVER

In this section formulas and graphs are given for the signal-to-noise ratio at the output of a panoramic receiver. A block diagram of the receiver model is shown in Figure 2.

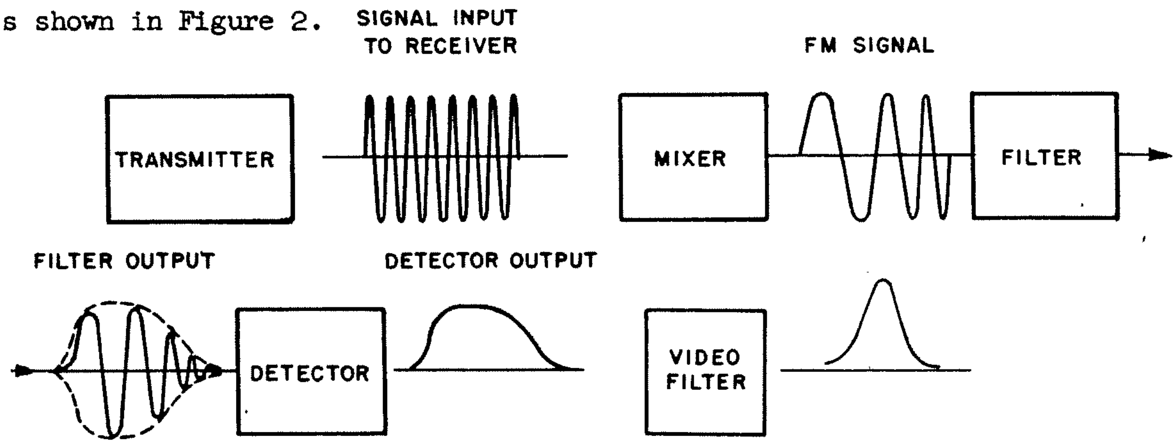


FIG 2. BLOCK DIAGRAM OF IDEALIZED PANORAMIC RECEIVER

CW signals and pulses with Gaussian shaped envelopes are considered. White Gaussian noise is assumed. The signal-to-noise ratio is calculated for this more complicated problem in much the same manner as in the previous section. Formulas in closed form are obtained for the signal-to-noise ratio as a function of pulse width, sweep rate, IF bandwidth and video bandwidth.

The derivation of the formula for signal-to-noise ratio is rather lengthy and not enlightening, and therefore it is placed in Appendix B. The signals are assumed to have the form

$$f(t) = \left(\frac{E}{d}\sqrt{\frac{2}{\pi}}\right)^{1/2} \exp\left(j(st^2 + at) - \frac{(t - c)^2}{d^2}\right) \qquad (3.1)$$

at the output of the ideal mixer, where the signal has a linearly varying frequency. The signal at the output of the IF filter is found with the aid of Technical Report No. 3. The square law detector is assumed to have at its output the square of the envelope of the input. The response of the video filter to this function is found also with the aid of Technical Report No. 3. The noise spectrum is assumed white at the input to the IF filter. At the output its spectrum is proportional to the power gain function of the IF filter. The spectrum after detection is found through the use of autocorrelation functions, and the power gain of the video filter is taken into account in finding the total noise power at the output of the receiver. The signal-to-noise ratio is found by dividing the peak signal power at the output by the average output noise power.[1]

The resulting expression for signal-to-noise ratio is

$$\frac{S}{N} = \frac{E^2}{N_o^2} \cdot \frac{16A_o^4 W^2 \beta \sqrt{\beta^2 + 2\beta^2}}{d^2(8s^2 + b^2\beta^2W^2)} \exp\left(-\frac{4}{B^2}\left(\frac{sc}{b}\right)^2\right) \tag{3.2}$$

where

- $A_o$ relative amplitude; the peak IF amplitude when the receiver is not sweeping and tuned to the pulse divided by the amplitude when the input is CW
- b bandwidth of the IF (radians/sec between $e^{-1/4}$ points)
- $\beta$ bandwidth of the video (radians/sec between $e^{-1/4}$ points)
- C the time difference between the occurance of the center of the pulse and the instant that the receiver is tuned to the pulse frequency (seconds)
- d input pulse duration (seconds)
- E energy of the pulse
- $N_o$ noise power per cycle per second
- S sweep rate of the receiver (radians/ sec)
- W normalized IF output pulse duration; the number of IF bandwidths swept through during the output pulse.

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[1] See Appendix B.

It will usually be possible to adjust the video bandwidth $\beta$, and therefore the optimum video bandwidth deserves consideration. By differentiating (3.2) with respect to $\beta$, it can be shown that the signal-to-noise ratio has its maximum for[1]

$$\beta = \frac{b}{\left(\frac{b^4W^2}{8s^2} - 1\right)^{1/2}} \tag{3.3}$$

If this value of $\beta$ is substituted in equation (3.2), the result simplifies to

$$\frac{S}{N} = \frac{E^2}{N_o^2} \frac{1}{B^2} \cdot \frac{2}{\left(\frac{b^4W^2}{4s^2} - 1\right)^{\frac{1}{2}}} \exp\left[-\frac{4}{B^2}(sc/b)^2\right] \tag{3.4}$$

By using the expressions for $A_o$, W, and B given in Technical Report No. 3, the following alternate forms can be derived from equations (3.3) and (3.4):

$$\beta_{opt.} = \frac{bB\sqrt{8}}{\sqrt{(4+b^2d^2)-4B^2}} \tag{3.5}$$

$$\left(\frac{S}{N}\right)_{\substack{\text{opt. video}\\ \text{filter}}} = \frac{E^2}{N_o^2} \frac{4}{B\sqrt{4+b^2d^2}} \exp\left(-\frac{4}{B^2}\left(\frac{sc}{b}\right)^2\right) \tag{3.6}$$

Equations (3.3) through (3.6) are derived in Appendix B.

The effective bandwidth, B, can be increased indefinitely by increasing the sweep rate. Hence for large enough sweep rates S, the expression

$$4 + b^2d^2 - 4B^2 \tag{3.7}$$

is negative, and the optimum video bandwidth is imaginary. If $\beta = j\gamma$, then the optimum video filter has a transfer function of the following type:

$$H(\omega) = \frac{1}{\sqrt{2\pi}} \exp\left(+\gamma^2\omega^2\right) \tag{3.8}$$

1. See Appendix B.

Such a filter is unrealizable, of course. It is not entirely unreasonable to consider such a filter; however, a situation very similar to this occurs in the problem of detecting signals in radar clutter, and Urkowitz[1] has shown that signal-to-noise ratio can be improved by using an approximation to a similar unrealizable filter. That case will not be considered in any detail in this report, however.

Since the ideal filter is unrealizable in certain ranges of the variables b, d, and s, signal-to-noise ratio with no video filter at all will be considered in these ranges. The expression for signal-to-noise ratio with no video filter can be found by letting $\beta$ approach infinity in equation (3.2). That equation then reduces to

$$\frac{S}{N} = \frac{E^2}{N_o^2} \frac{16A_o^4}{b^2d^2} \exp\left(-\frac{4}{B^2}\left(\frac{sc}{b}\right)^2\right) \tag{3.9}$$

The dividing line between real $\beta$ and imaginary $\beta$ occurs where $\beta$ becomes infinite and

$$4 + b^2d^2 - 4B^2 = 0 \tag{3.10}$$

If this equation is satisfied, the ideal video filter is no filter at all.

Signal-to-noise ratio is plotted in Figures 3, 4 and 5 to show its dependence upon sweep-rate and IF bandwidth. In these figures the pulse is always assumed centered on the passband, i.e., C is assumed to be zero.

Equation (3.6) shows the dependence of signal-to-noise ratio on IF bandwidth and effective bandwidth when an optimum video filter is used. The corresponding equation for the case of no video filter, obtained by eliminating $A_o$ in favor of B from equation (3.9) is

$$\left(\frac{S}{N}\right)_{\text{no video filter}} = \frac{E^2}{N_o^2} \frac{16}{4B^2+4+b^2d^2} \exp\left(-\frac{4}{B^2}\left(\frac{sc}{b}\right)^2\right) \tag{3.11}$$

[1] Harry Urkowitz, "Filters for Detection of Small Radar Signals in Clutter", Journal of Applied Physics, Vol. 24 No. 8, August 1953.

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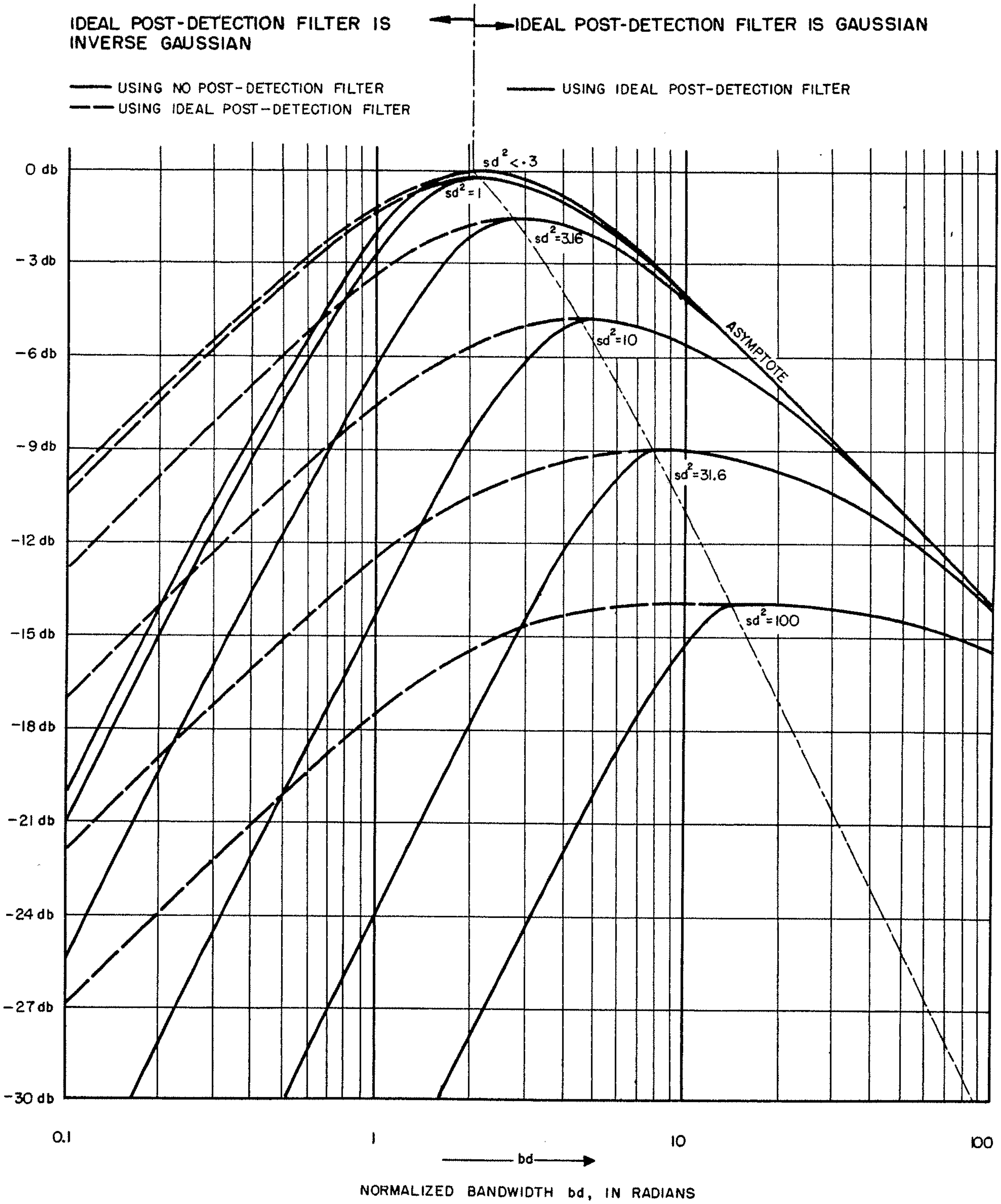
FIG. 3 . NORMALIZED SIGNAL-TO-NOISE RATIO VS. NORMALIZED BANDWIDTH, NORMALIZED SWEEP RATE AS A PARAMETER .



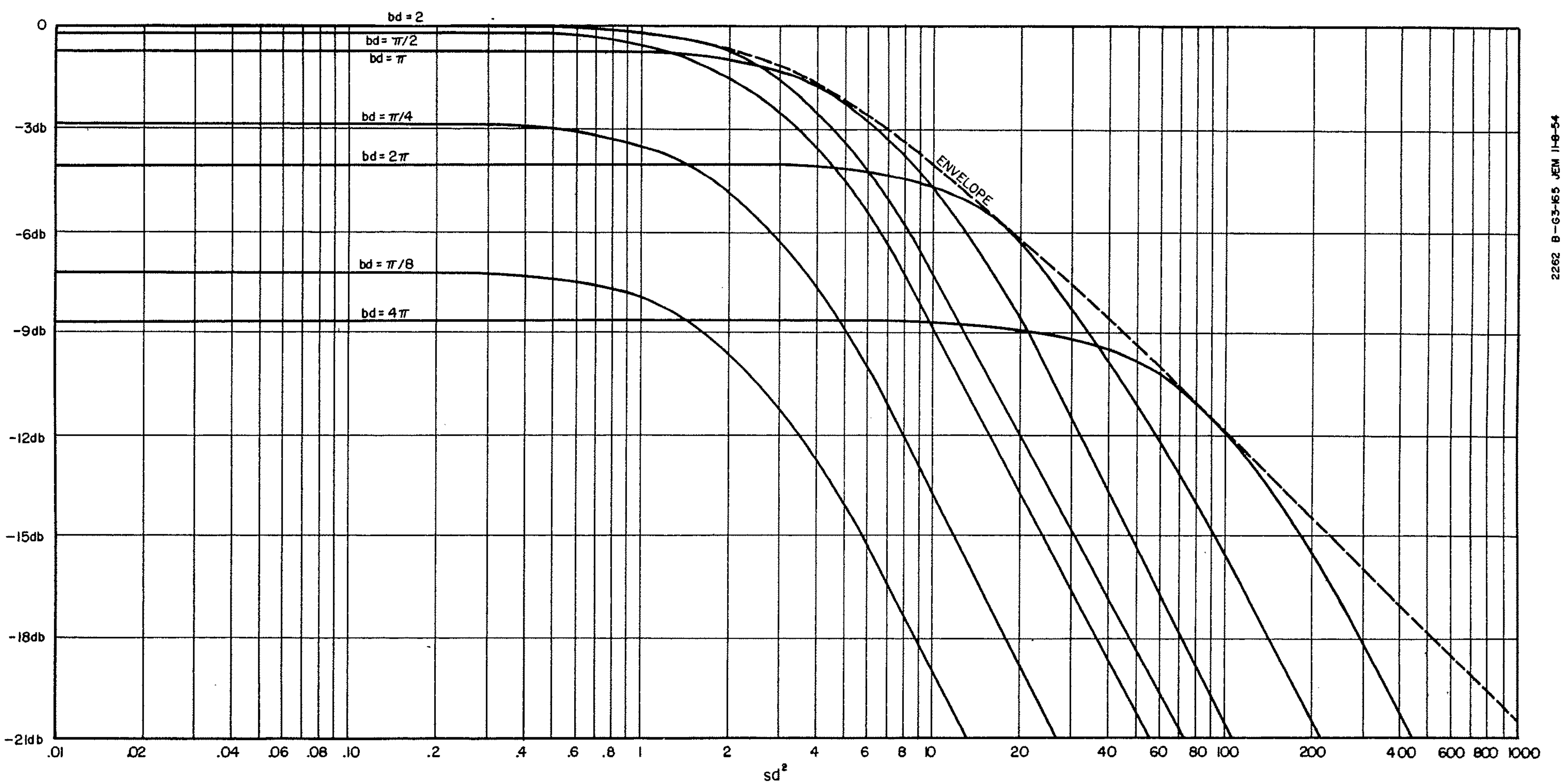



FIG. 4. NORMALIZED SIGNAL-TO-NOISE RATIO VS. NORMALIZED SWEEP RATE, NORMALIZED BANDWIDTH AS A PARAMETER, USING NO POST-DETECTION FILTER.

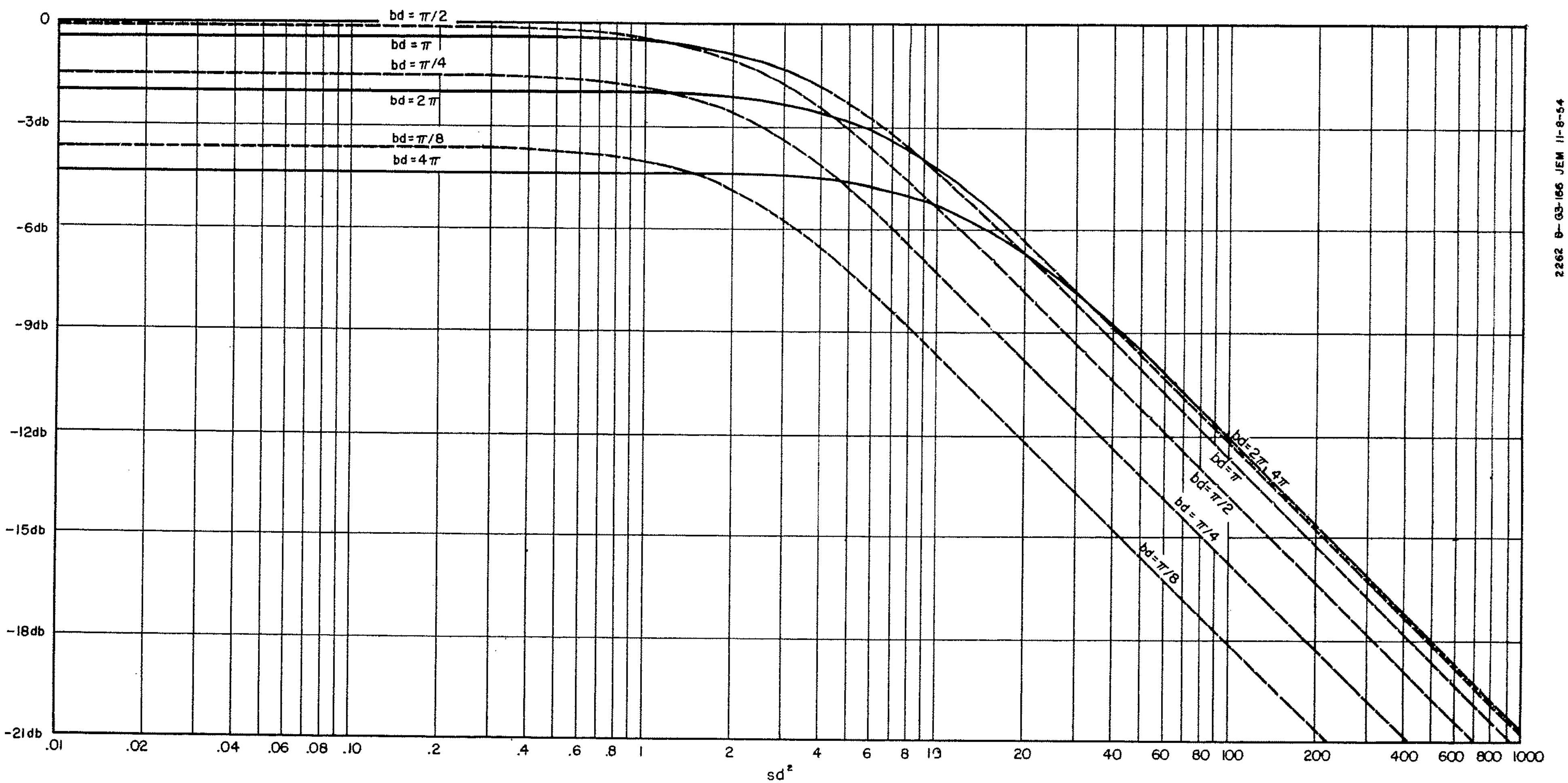



FIG. 5 . NORMALIZED SIGNAL-TO-NOISE RATIO VS. NORMALIZED SWEEP RATE, NORMALIZED BANDWIDTH AS A PARAMETER, USING IDEAL POST-DETECTION FILTER .

——— IDEAL POST-DETECTION FILTER IS GAUSSIAN

– – – IDEAL POST-DETECTION FILTER IS INVERSE GAUSSIAN

## IV. PANORAMIC RECEIVER DESIGNED TO COVER A FIXED BAND

Suppose that a receiver is required to be essentially wide open to pulses of duration d that may occur anywhere in a certain band. The effective bandwidth of a panoramic receiver is bB radians per second, and hence it is desired to adjust the receiver parameters so that bB always equals the band to be covered. Figure 6 is a plot of the signal-to-noise ratio as a function of IF bandwidth, bd, where the sweep rate has been adjusted along with the IF bandwidth in order to hold the effective bandwidth bdB constant.

Consider a receiver required to receive pulses that last ten microseconds and may be anywhere in a 1.6 megacycle band. This effective bandwidth could be accomplished either by having a 1.6 megacycle IF bandwidth, or by having a narrower IF bandwidth and by scanning rapidly. For this hypothetical case, bB is $2\pi \times 1.6 = 10$ megaradians per second, and dbB is therefore 100. It can be seen from Figure 6 that for bdB equal to 100, the maximum signal-to-noise ratio occurs for bd = 100 and s = 0. As the IF bandwidth is decreased to bd = 14, the point at which the optimum video bandwidth is infinite, the signal-to-noise ratio drops only .04 db.[1] Beyond that point, the signal-to-noise ratio begins to drop rapidly with the (unrealizable) ideal filter, and even more rapidly with no filter at all. The receiver could thus be operated with nearly maximum signal-to-noise ratio with IF bandwidth anywhere in the range from $b = \frac{100}{d}$ to $b = \frac{14.2}{d}$, 1.6 mc to 226 kc. At $b = \frac{100}{d}$, the sweep rate would be zero, while at $b = \frac{14.2}{d}$, the sweep rate would be $\frac{98.51}{d^2}$ radians per second per second, or 156 kilomegacycles per second per second.

[1] Table I summarizes this and several similar cases.

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FIG. 6 . NORMALIZED SIGNAL-TO-NOISE RATIO VS. NORMALIZED BANDWIDTH, NORMALIZED APPARENT BANDWIDTH AS A PARAMETER.

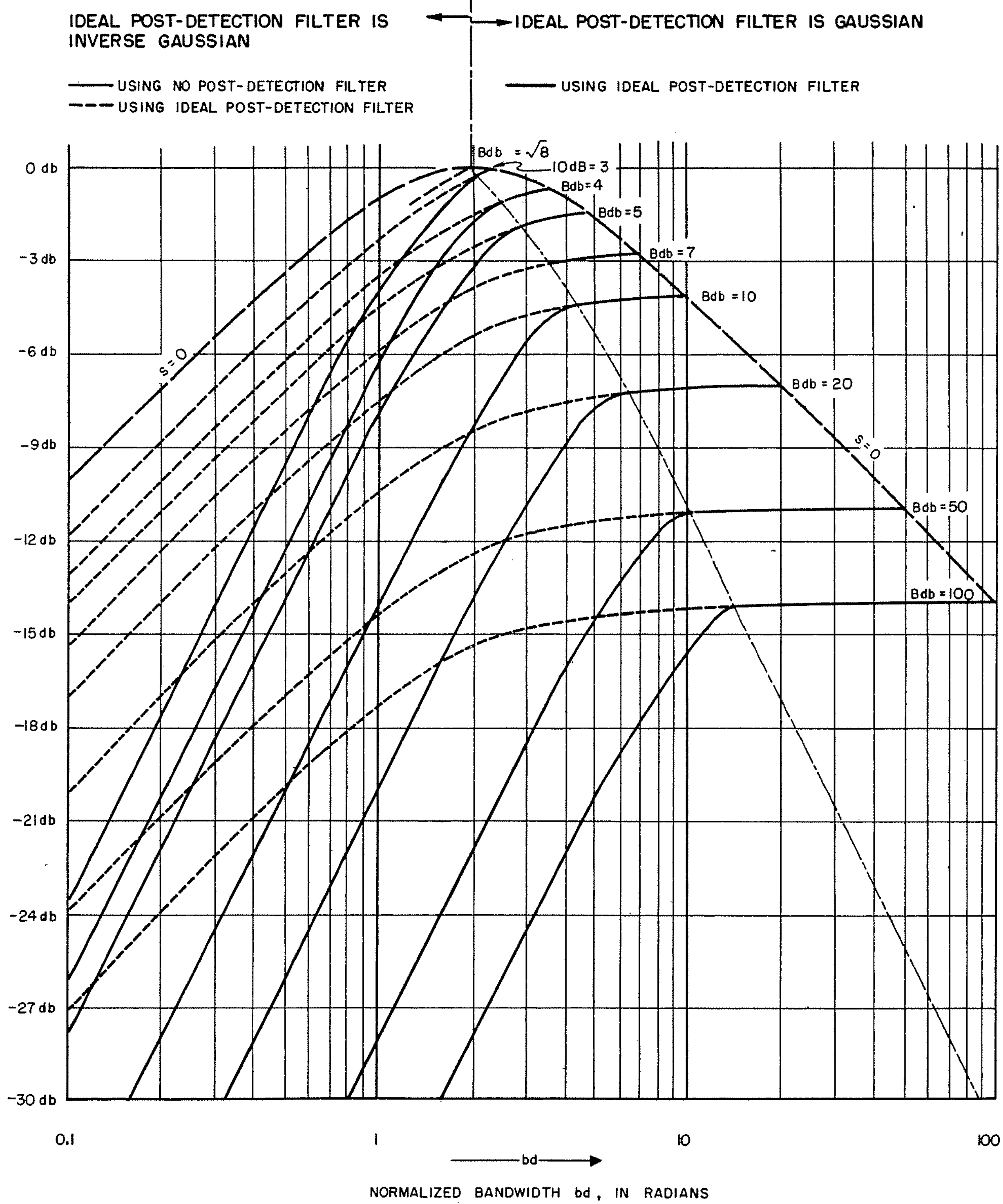

TABLE I

For 10 μsec Pulse

| Bdb | $sd^2$ | bd | $(\frac{S}{N})$ diff | Sweep Rate | IF Bandwidth | Video Bandwidth |
|---|---|---|---|---|---|---|
| 20 | 0 | 19.90 | .085 db | 0 | 318 Kc/s | 45.4 Kc/s |
| | 19.86 | 6.485 | | 31.6 $KMc/s^2$ | 103 Kc/s | --------- |
| 100 | 0 | 99.98 | .042 db | 0 | 1.6 Mc/s | 45.3 Kc/s |
| | 98.51 | 14.212 | | 156 $KMc/s^2$ | .226 Mc/s | --------- |
| M>100 | 0 | M | $\frac{4.343}{M}$ db | | | $\left(\frac{8}{b^2d^2-4}\right)^{1/2} b$ |
| | M | 2M | | | | |

The collected equations of interest for Bdb rad/sec are the following:

$$(bd)^2 = (Bdb)^2 - 4 - (sd^2)^2 \tag{4.1}$$

The video bandwidth, β, is found from (3.5) to be

$$\beta = \frac{\sqrt{2}\ Bdb}{\sqrt{.25\ b^4d^4 + b^2d^2 - (Bdb)^2}}\ b \tag{4.2}$$

For s = 0, the IF bandwidth, b, is determined by

$$(bd)^2 = (Bdb)^2 - 4 \tag{4.3}$$

and the video bandwidth β, is

$$\beta = \left(\frac{8}{(Bdb)^2 - 8}\right)^{1/2} b\ . \tag{4.4}$$

If the receiver is used for bdB >> 4, then

$$b \doteq \frac{(Bdb)}{d} \quad \text{radians/sec} \tag{4.6}$$

and

$$\beta \doteq \frac{\sqrt{8}}{Bdb}\, b \doteq \frac{\sqrt{8}}{d} \quad \text{radians/sec} \tag{4.7}$$

when the sweep rate is increased so that no video filter is necessary, equation (3.10) yields

$$bd = \left(2\sqrt{(Bdb)^2 + 1} \quad -2\right)^{1/2}, \tag{4.5}$$

and for $Bdb \gg 4$,

$$b = \frac{2(Bdb)}{d} \text{ radians/sec} \tag{4.8}$$

Summarizing, equations (4.1) through (4.5) apply whenever

$$(bd)^4 \geq 4(4 + s^2 d^4) \tag{4.9}$$

so that $\beta$ is real, and for large effective bandwidth, the same signal-to-noise can be obtained by adjustment of sweep rate, IF and video bandwidths from

$$\text{slow sweep} \quad bd = (Bdb) \quad \text{and } \beta d = \sqrt{8}$$

to

$$bd = 2(Bdb) \quad \text{and wide open video.}$$

Note that the fast-scanning narrow band and the slow-scanning or non-scanning broadband receivers have nearly equal signal-to-noise ratio only if the optimum video filter is used. <u>If the video filter is not matched, the fast-scanning narrow-band receiver is superior.</u>

Because the video filter is so important when a slow scanning broadband receiver is used to cover a fixed band, the equation for mismatched video bandwidth is determined from the general signal-to-noise ratio, Eq 3.2.

$$\left(\frac{S}{N}\right) = \left(\frac{2E}{N_o}\right)^2 \frac{A_o^2 w^2 b^2}{sd^2\sqrt{w^2 b^4 - 4s^2}} \; \frac{\frac{\beta}{\beta_{opt}}\left[\left(\frac{\beta}{\beta_{opt}}\right)^2 \left(\frac{\beta_{opt}}{b}\right)^2 + 2\right]^{1/2} \left[\frac{\beta_{opt}}{b}^2 + 2\right]^{1/2}}{\left(\frac{\beta}{\beta_{opt}}\right)^2 \left[\left(\frac{\beta_{opt}}{b}\right)^2 + 1\right] + 1} \tag{4.10}$$

The last factor in (4.10), the video mismatch factor, has a maximum value of one at $\beta = \beta_{opt}$, and depends on the parameter $(\beta_{opt}/b)$, the ratio of optimum video to IF bandwidth. In Fig. 7 the optimum video bandwidth, $\beta_{opt}d$, necessary to achieve an apparent bandwidth Bdb is plotted as a function of the IF bandwidth, bd. The video mismatch factor is plotted in Fig. 8. From Fig. 8 it is evident that it is better to have the video too wide than too narrow, and no video at all ($\beta = \infty$) is less than a decibel below optimum if the optimum video is as wide as

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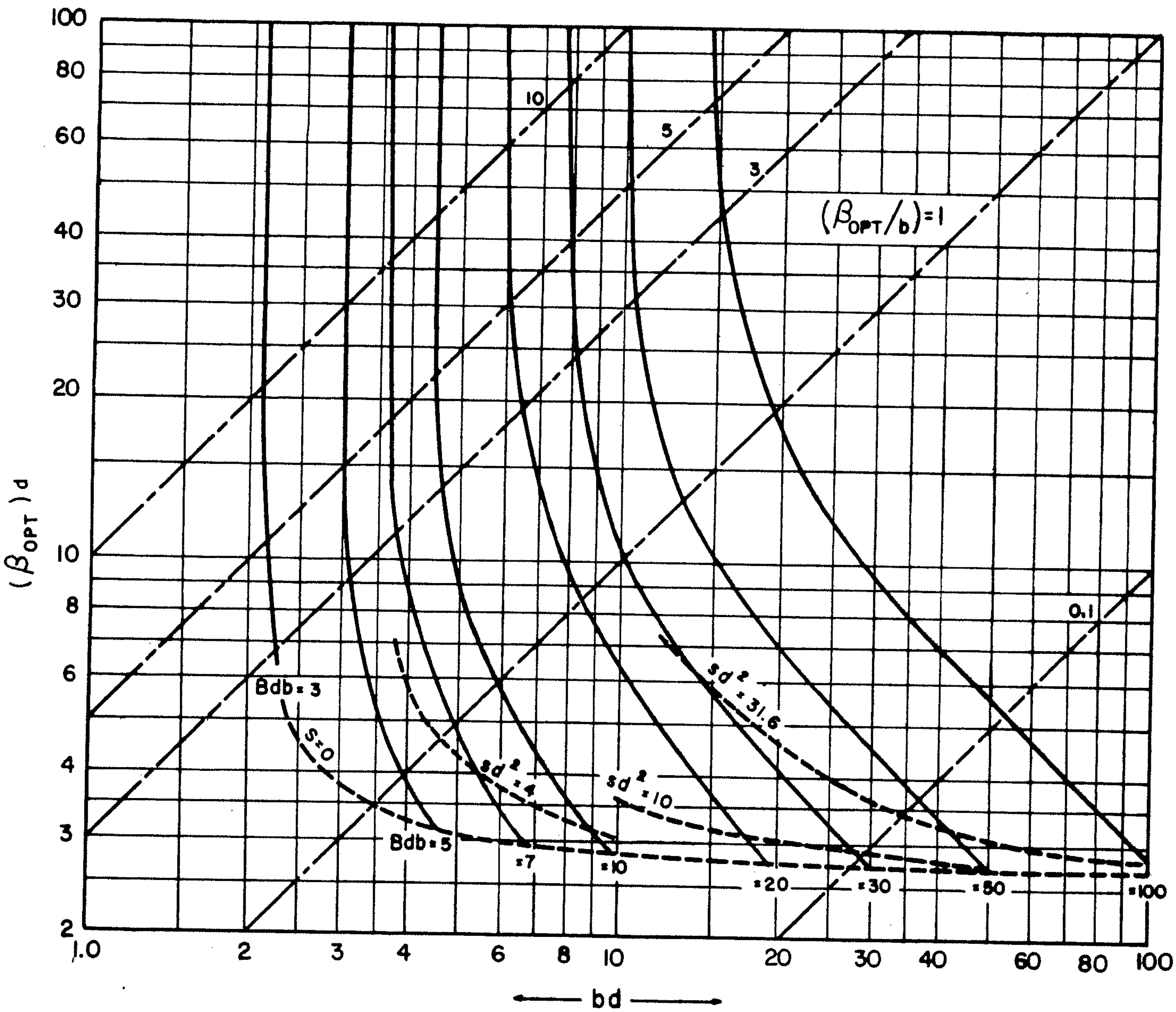


FIG. 7. OPTIMUM VIDEO BANDWIDTH VS. I.F. BANDWIDTH. APPARENT BANDWIDTH AS A PARAMETER.

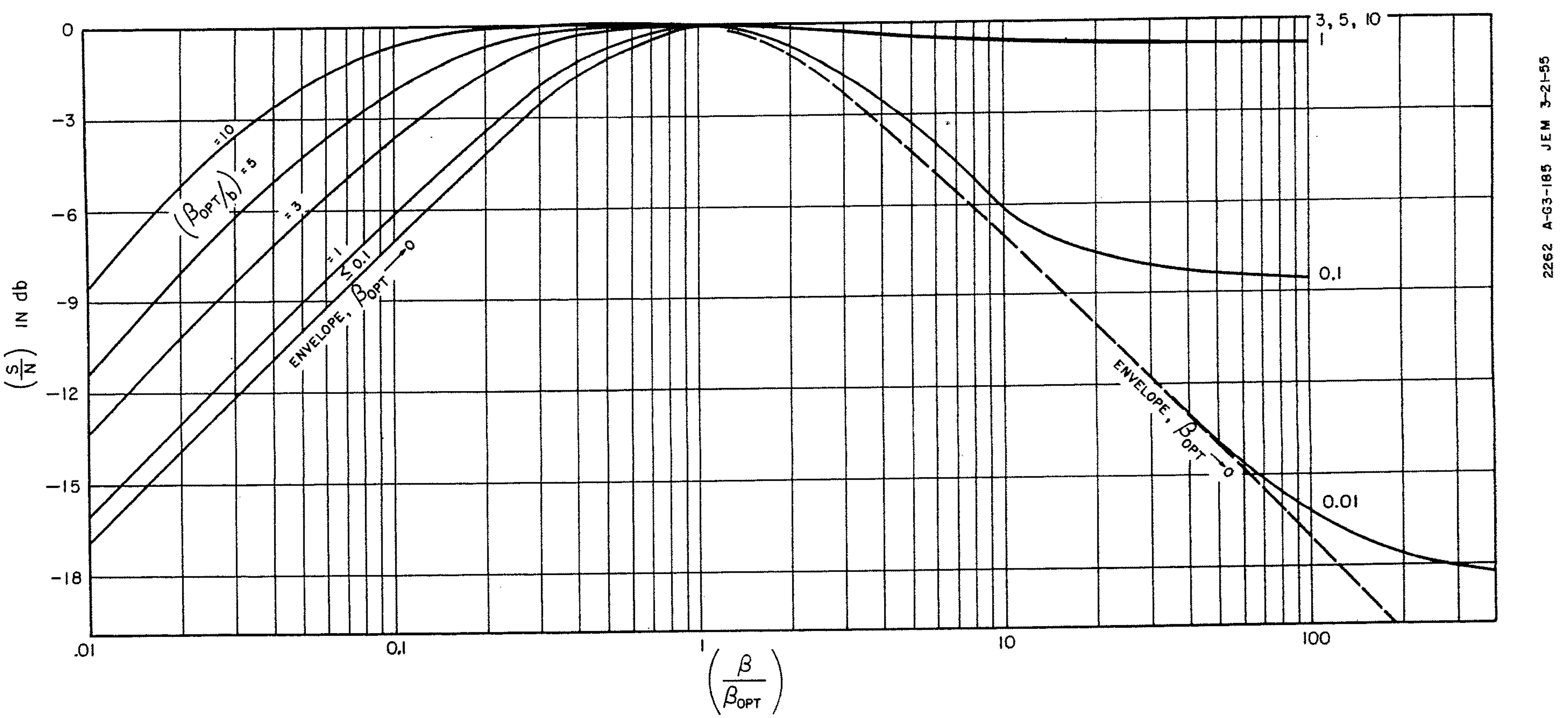


FIG. 8. SIGNAL-NOISE RATIO VS. VIDEO MISMATCH RATIO, OPTIMUM VIDEO TO I. F. BANDWIDTH AS A PARAMETER.

the IF. However, a computation of the sweep rate shows that $sd^2$ is only slightly less than Bdb for Bdb > 10.[1]

At the other extreme, if $sd^2$ is required to be less than one, then it can be shown with a little juggling that the optimum-video-to-IF ratio will be slightly greater than $\frac{\sqrt{8}}{Bdb}$ for Bdb > 10, and the video match is important.

Practical considerations may favor either the fast-scanning wide video or the non-scanning matched video type receiver. $N_o$, the noise power per unit bandwidth at the input of the receiver, was considered constant in the analysis made in this report. This quantity $N_o$ includes both the noise coming into the receiver and the noise produced in the receiver itself, and thus includes, for example, local oscillator noise and the noise produced in the IF amplifier. As bandwidth and sweep-rate are varied, the noise per unit bandwidth will not remain constant. Broadband IF amplifiers generally have higher noise figures than narrow-band amplifiers. It may not be possible to build a fast-scanning local oscillator with low noise level in its output. These factors must be taken into consideration, and thus the optimum sweep rate IF bandwidth and video bandwidth for a receiver is certainly a function of the state of the art, among other things.

---

1 For $\left(\frac{\beta_{opt}}{b}\right) = 1$

$$sd^2 = Bdb - \sqrt{3} - \frac{1.75}{Bdb} - \frac{2\sqrt{3}}{(Bdb)^2} \cdots$$

so that

$$Bdb - 2 < sd^2 < Bdb - \sqrt{3}$$

APPENDIX A

Fourier transforms are commonly used in several different forms, and with each form the constant multipliers are different. The form used in this report is

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\, dt \qquad (A.1)$$

where $F(\omega)$ is the Fourier transform of $f(t)$. The pertinent formulas are collected in this appendix for reference.

With this form of Fourier transform, the inverse transform is

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\, d\omega \quad . \qquad (A.2)$$

Parseval's theorem takes the form

$$\int_{-\infty}^{\infty} f(t)\, \overline{f(t)} dt = \int_{-\infty}^{\infty} F(\omega)\, \overline{F(\omega)}\, d\omega \quad , \qquad (A.3)$$

and it can be shown that if $H(\omega)$ is the transform of $h(t)$,

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H(\omega)F(\omega)e^{j\omega t}\, d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\lambda)h(t-\lambda)d\lambda \quad . \qquad (A.4)$$

Thus if a filter has impulse response $h(t)$, the response to a signal $f(t)$ is the transform of $H(\omega)F(\omega)$ multiplied by $\sqrt{2\pi}$ .

The energy spectrum of a signal $f(t)$ is

$$P(\omega) = F(\omega)\, \overline{F(\omega)} \quad , \qquad (A.5)$$

and the total energy is

$$\int_{-\infty}^{\infty} f(t)\, \overline{f(t)}\, dt = \int_{-\infty}^{\infty} F(\omega)\, \overline{F(\omega)}\, d\omega = \int_{-\infty}^{\infty} P(\omega)d\omega. \qquad (A.6)$$

If the signal $f(t)$ is passed through a filter with impulse response $h(t)$, or transfer function $H(\omega)$, the spectrum at the output is

$$P_o(\omega) = \left[\sqrt{2\pi}H(\omega)F(\omega)\right]\left[\overline{\sqrt{2\pi}H(\omega)F(\omega)}\right]$$

$$= 2\pi H(\omega)\overline{H(\omega)}F(\omega)\overline{F(\omega)}$$

$$P_o(\omega) = 2\pi H(\omega)\overline{H(\omega)}P_{in}(\omega) \qquad (A.7)$$

where $P_{in}(\omega)$ is the spectrum of the input signal. Equations (A.6) and (A.7) holds also if $P_o(\omega)$ and $P_{in}(\omega)$ are interpreted as the power spectrum of noise.

The autocorrelation function $p(\tau)$ for stationary noise is proportional to the transform of the spectrum. That is,

$$p(\tau) = K\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} P(\omega)e^{j\omega\tau}\,d\omega \quad , \qquad (A.8)$$

where $P(\omega)$ is the noise spectrum. The value of K depends upon the choice of the form of the Fourier transform. The constant can be determined from the fact that $p(0)$ must be the total noise power, and hence

$$p(0) = \int_{-\infty}^{\infty} P(\omega)d\omega \quad .$$

It follows that $K = \sqrt{2\pi}$ , and the auto correlation is the transform of the spectrum, multiplied by $\sqrt{2\pi}$ , i.e.,

$$p(\tau) = \int_{-\infty}^{\infty} P(\omega)e^{j\omega\tau}\,d\omega \quad . \qquad (A.9)$$

# APPENDIX B

In this appendix the expression for signal-to-noise ratio in a panoramic receiver is derived. The receiver model is shown in block diagram form in Figure B.1.

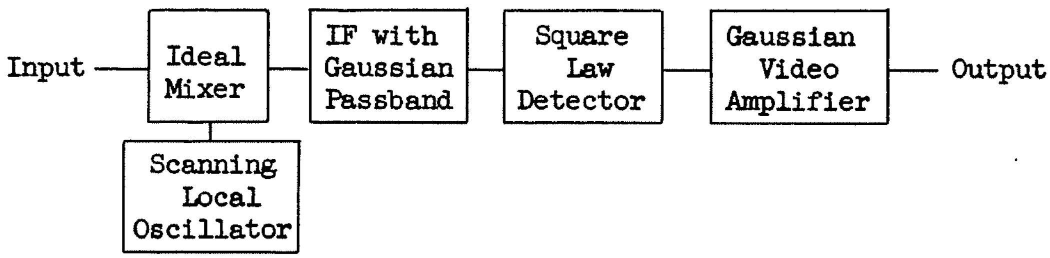


Fig. B.1 BLOCK DIAGRAM OF PANORAMIC RECEIVER

The signals assumed are pulses with Gaussian shaped envelopes. At the output of the mixer they have linearly varying frequency, and at this point are assumed to have the form

$$f(t) = \sqrt{\frac{E}{d}\sqrt{\frac{2}{\pi}}} \; \exp\left[ j\left(\frac{st^2}{2} + at\right) - \frac{(t-e)^2}{d^2} \right] \tag{B.1}$$

The constant factor was chosen to make the signal have energy E, i.e.,

$$E = \int_{-\infty}^{\infty} f(t)\,\overline{f(t)}\,dt \quad , \tag{B.2}$$

The noise is assumed to be white stationary Gaussian noise with a power of $N_o$ watts per cycle of bandwidth. The transfer function of the IF is assumed to be

$$H(\omega) = \frac{1}{\sqrt{2\pi}} \exp\left[ -\frac{(\omega-a)^2}{b^2} \right] , \tag{B.3}$$

and the transfer function of the video filter is

$$G(\omega) = \frac{1}{\sqrt{2\pi}} \exp\left[ -\frac{(\omega)^2}{\beta^2} \right] . \tag{B.4}$$

First the maximum signal power is found with the aid of <u>The Response of a Panoramic Receiver to CW and Pulse Signals</u>[1]. Then the noise power is calculated, and finally the expression is given for the ratio.

The signal function and the transfer function of the IF amplifier are the same except for a constant factor as those assumed in <u>The Response of a Panoramic Receiver to CW and Pulse Signals</u>[1], and hence the expression for the envelope of the signal at the output of the IF amplifier can be taken directly from that report.

$$\left|g(t)\right| = \sqrt{\frac{E}{d}\sqrt{\frac{2}{\pi}}}\; A_o \;\exp\left\{ -\frac{1}{W^2}\left[\frac{s(t-t_m)}{b}\right]^2 - \frac{1}{B^2}\left(\frac{sc}{b}\right)^2 \right\} \quad (B.5)$$

$$A_o = \frac{b}{\left[\left(\frac{4}{d^2} + b^2\right)^2 + 4s^2\right]^{1/4}}$$

$$B = \frac{1}{b}\sqrt{\frac{4}{d^2} + b^2 + s^2 \cdot d^2} \quad (B.7)$$

$$W = \frac{sd}{b^2}\sqrt{\frac{\left(\frac{4}{d^2} + b^2\right)^2 + 4s^2}{\frac{4}{d^2} + b^2 + s^2d^2}} = \frac{sd}{b}\cdot\frac{1}{A_o^2 B} \quad (B.8)$$

$$t_m = c\left[\frac{\frac{4}{d^2} + b^2}{\frac{4}{d^2} + b^2 + s^2d^2}\right] \quad (B.9)$$

The output of the square law detector is the square of the signal envelope:

$$\left|g(t)\right|^2 = \sqrt{\frac{2}{\pi}}\;\frac{E}{d}\;A_o^2\;\exp\left\{ -\frac{2}{W^2}\left[\frac{s(t-t_m)}{b}\right]^2 - \frac{2}{B^2}\;\frac{sc}{b}^2 \right\} (B.10)$$

---

[1] op. cit., page 1.

This is a Gaussian pulse, and the video filter is Gaussian also; therefore the formulas (B.5) through (B.9) can be applied again. This time the sweep rate, s, is zero, the center frequency, a, is zero, the filter bandwidth is $\beta$, the center time of the pulse, c, is $t_m$ , and the pulse width d is $\frac{bW}{s\sqrt{2}}$ . There is also the constant factor

$$\sqrt{\frac{2}{\pi}}\ \frac{E}{d}\ A_o^2\ \exp\left\{ -\frac{2}{B^2}\ \frac{sc}{b}\right\}^2 \tag{B.11}$$

to be carried along. Only the maximum response is required, and that is the factor, (B.11), times $A_o$ evaluated for these special values of sweep rate, filter bandwidth, and pulse width. From the expression (B.6) for $A_o$ we obtain

$$\sqrt{S} = \sqrt{\frac{2}{\pi}}\ \frac{E}{d}\ \ A_o^2\ \exp\left\{ -\frac{2}{B^2}\left(\frac{sc}{b}\right)^2\right\}\frac{\beta}{\sqrt{\frac{8s^2}{b^2W^2} + \beta^2}} \tag{B.12}$$

The peak signal power is the square of this voltage.

The noise spectrum at the input is assumed to be constant with a power of $N_o$ watts per cycle. If this is thought of as spread over all positive and negative frequencies (as is convenient in working with Fourier transforms), then one-half of the power should be associated with the negative frequencies and one-half with the positive frequencies. Also, since there are $2\pi$ radians per cycle, the density per cycle must be divided by $2\pi$ to give power density per radian. Hence the power density at the IF input is

$$P_{in}(\omega) = \frac{1}{2}\ \frac{1}{2\pi}\ N_o = \frac{N_o}{4\pi} \tag{B.13}$$

By equation (A.7), the spectrum at the output of the IF filter is

$$2\pi\ H(\omega)\ \overline{H(\omega)}\ P_{in}(\omega) = \frac{N_o}{4\pi} \exp\left[ - \frac{2(\omega - a)^2}{b^2} \right] \quad (B.14)$$

By equation (A.9), the autocorrelation function of the noise is $\sqrt{2\pi}$ times the Fourier transform of the noise spectrum, and hence the autocorrelation at the input to the detector is $\sqrt{2\pi}$ times the transform of (B.14), or

$$r_{in}(\tau) = \frac{N_o b}{4\sqrt{2\pi}} \exp\left[ ja\tau - \frac{b^2 \tau^2}{8} \right] \quad (B.15)$$

A formula for the autocorrelation at the output of a square law detector is presented in Threshold Signals[1]. In that book, the noise is represented in the form

$$n(t) = x(t) \cos \omega t + y(t) \sin \omega t \quad (B.16)$$

where $n(t)$ is the noise voltage, $\omega$ the center frequency of the noise spectrum, and $x(t)$ and $y(t)$ are assumed to be uncorrelated and to have the same autocorrelation function $\rho(\tau)$. The autocorrelation function of this noise $n(t)$ is

$$\begin{aligned} r(\tau) &= \text{average} \left\{ n(t)\ n(t+\tau) \right\} \\ &= \text{average} \left\{ \left[ x(t) \cos \omega T + y(t) \sin \omega T \right] \left[ x(t+\tau) \cos \omega(t+\tau) + \right.\right. \\ &\qquad \left.\left. y(t+\tau) \sin \omega(t+\tau) \right] \right\} \\ &= \text{average} \left\{ x(t)\, x(t+\tau) \cos \omega t \cos \omega (t+\tau) + x(t) y(t+\tau) \right. \\ &\qquad \cos \omega t \sin \omega(t+\tau) + y(t) x(t+\tau) \sin \omega t \cos \omega(t+\tau) \\ &\qquad \left. + y(t) y(t+\tau) \sin \omega t \sin \omega(t+\tau) \right\} \qquad (B.17) \\ &= \left[ \text{average} \left\{ x(t) x(t ++\tau) \right\} \right] \cos \omega t \cos \omega(t+\tau) + \\ &\qquad + \left[ \text{average} \left\{ y(t) y(t+\tau) \right\} \right] \sin \omega t \sin \omega(t+\tau) \end{aligned}$$

[1] J. A. Lawson, G. E. Uhlenbeck, "Threshold Signals," McGraw-Hill, 1950.

At this point the other terms drop out because x(t) and y(t) are uncorrelated. The expressions in brackets in (B.16) are the autocorrelation of x(t) and of y(t), i.e., $p(\tau)$. Thus

$$\begin{aligned} r(\tau) &= p(\tau)\left[\cos \omega t \cos\omega(t+\tau) + \sin \omega t \sin \omega (t+\tau)\right] \\ &= p(\tau) \cos \omega \tau \end{aligned} \tag{B.18}$$

Thus $p(\tau)$ is the envelope of $r(\tau)$. Since $r(\tau)$ is given in complex form, the envelope is the absolute value, $|r(\tau)|$.

The autocorrelation function at the output of the detector as given by Lawson and Uhlenbeck is

$$r_{out}(\tau) = 4N^2 + 4\left[\rho(\tau)\right]^2 . \tag{B.19}$$

or

$$r_{out}(\tau) = 4N^2 + 4\left[r_{in}(\tau)\right]^2 .$$

The first term in this expression is the dc component of the output, and hence may be dropped. Thus by equation (A.9), the noise spectrum is the Fourier transform of the second term divided by $\sqrt{2\pi}$ , or

$$\begin{aligned} N(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} 4 \; r_{in}(\tau)^{\;2} \exp\left[-j\omega\tau\right] d\tau \\ &= \frac{N_o^2 b^2}{16\pi^2}\int_{-\infty}^{\infty}\left[\exp - \frac{b^2\tau^2}{4} - j\omega\tau\right] d\tau \\ &= \frac{N_o^2 b^2}{16\pi^2}\sqrt{\frac{4\pi}{b^2}} \exp\left[-\frac{\omega^2}{b^2}\right] \\ &= \frac{N_o^2 b}{8\pi\sqrt{\pi}} \exp\left[-\frac{\omega^2}{b^2}\right] \end{aligned} \tag{B.20}$$

By (A.7), the spectrum at the output of the video filter is obtained by multiplying (B.20) by $2\pi\, G(\omega)\overline{G(\omega)}$, where $G(\omega)$ is the transfer function of the video filter. The noise power output of the receiver is the integral of the noise spectrum:

$$N = \int_{-\infty}^{\infty} \frac{N_o^2 b}{4\sqrt{\pi}} \exp\left[-\frac{\omega^2}{b^2}\right] G(\omega)\,\overline{G(\omega)}\; d\omega$$

$$= \frac{N_o^2 b}{8\pi\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{\omega^2}{b^2} - \frac{2\omega^2}{\beta^2}\right] d\omega$$

$$= \frac{N_o^2 b}{8\pi}\left(\frac{1}{b^2} + \frac{2}{\beta^2}\right)^{-1/2} \tag{B.21}$$

Then the signal-to-noise ratio is obtained from (B.12) and (B.21):

$$\frac{S}{N} = \frac{E^2}{N_o^2} \cdot \frac{16A_o^{\,4}\, W^2\beta\sqrt{\beta^2 + 2b^2}}{d^2(8s^2 + b^2\beta^2W^2)} \exp\left\{-\frac{4}{B^2}\left(\frac{sc}{b}\right)^2\right\} \tag{B.22}$$

The signal-to-noise ratio with no video filter can be found from (B.22) by taking the limit as the video bandwidth $\beta$ becomes infinite. If the numerator and denominator of (B.22) are divided by $\beta^2$, the equation takes the form

$$\frac{S}{N} = \frac{E^2}{N_o^2}\; \frac{16A_o^{\,4}W^2\sqrt{1 + \dfrac{2b^2}{\beta^2}}}{d^2\left(8\,\dfrac{s^2}{\beta^2} + b^2W^2\right)} \exp\left\{-\frac{4}{\beta^2}\left(\frac{sc}{b}\right)^2\right\} \tag{B.23}$$

and as $\beta$ approaches infinity, the limiting signal-to-noise ratio is

$$\left(\frac{S}{N}\right)_{\text{no video filter}} = \frac{E^2}{N_o^2}\;\frac{16A_o^{\,4}}{b^2d^2} \exp\left\{-\frac{4}{B^2}\left(\frac{sc}{b}\right)^2\right\} \tag{B.24}$$

The optimum video filter bandwidth can be found by differentiating (B.22) with respect to $\beta$ and solving for the value of $\beta$ which makes the derivative zero.

$$\frac{\partial}{\partial \beta}\left(\frac{S}{N}\right) = \frac{E^2}{N_o^2} \cdot \frac{16A_o{}^4W^2}{d^2} \cdot \exp\left\{- \frac{4}{B^2}\left(\frac{sc}{b}\right)^2\right\} \frac{\partial}{\partial \beta}\left[\sqrt{\frac{\beta^4 + 2b^2\beta^2}{(8s^2 + b^2\beta^2W^2)^2}}\right]$$

$$0 = 1/2\left[\frac{\beta^4 + 2b^2\beta^2}{(8s^2 + b^2\beta^2W^2)^2}\right]^{-1/2} \left[ (4\beta^3 + 4b^2\beta)(8s^2 + b^2\beta^2W^2)^2 - \right.$$

$$\left. - (\beta^4 + 2b^2\beta^2)2(8s^2 + b^2\beta^2W^2)(2b^2\beta W^2)\right]$$

or

$$0 = (\beta^2 + b^2)(8s^2 + b^2\beta^2W^2) - (\beta^4 + 2b^2\beta^2)(b^2W^2)$$

$$0 = 8s^2\beta^2 + b^2\beta^4W^2 + 8b^2s^2 + b^4\beta^2W^2 - \beta^4b^2W^2 - 2b^4\beta^2W^2$$

$$0 = 8b^2s^2 - (b^4W^2 - 8s^2)\beta^2$$

or

$$\frac{b^2}{\beta^2} = \frac{b^4\,W^2}{8s^2} - 1 \tag{B.25}$$

This can also be written in the following form:

$$\beta = \frac{b}{\sqrt{\frac{b^4W^2}{8s^2} - 1}} \tag{B.26}$$

The signal-to-noise ratio with the optimum filter can be found by substituting the expression (B.26) for $\beta$ in equation (B.22). It is somewhat simpler to substitute for $1/\beta^2$ from (B.25) into (B.23).

$$\frac{S}{N} = \frac{E^2}{N_o^2} \; \frac{16A_o^4}{d^2} \; \frac{\sqrt{\dfrac{b^4W^2}{4s^2} - 1}}{\dfrac{8s^2}{b^2}\left(\dfrac{b^4W^2}{8s^2} - 1\right) + b^2W^2} \; \exp\left\{ - \frac{4}{B^2}\left(\frac{sc}{b}\right)^2 \right\}$$

$$= \frac{E^2}{N_o^2} \; \frac{16A_o^4W^2}{d^2} \; \frac{\sqrt{\dfrac{b^4W^2}{4s^2} - 1}}{\dfrac{8s^2}{b^2}\left(\dfrac{b^4W^2}{4s^2} - 1\right)} \; \exp\left\{ - \frac{4}{B^2}\left(\frac{sc}{b}\right)^2 \right\}$$

$$= \frac{E^2}{N_o^2} \; \frac{2A_o^4 W^2}{\dfrac{s^2d^2}{b^2}\sqrt{\dfrac{b^4W^2}{4s^2} - 1}} \; \exp\left\{ - \frac{4}{B^2}\left(\frac{sc}{b}\right)^2 \right\} \qquad (B.27)$$

By equation (3.11) of Technical Report No. 3,[1]

$$W = \frac{sd}{b} \; \frac{1}{A_o^2 B} \; . \qquad (B.28)$$

Substituting for W in (B.27) and (B.26) yields

$$\left(\frac{S}{N}\right)_{\substack{\text{optimum}\\ \text{video}\\ \text{filter}}} = \frac{E^2}{N_o^2} \; \frac{2}{B^2\sqrt{\dfrac{b^2d^2}{4A_o^4B^2} - 1}} \; \exp\left\{ - \frac{4}{B^2}\left(\frac{sc}{b}\right)^2 \right\} \qquad (B.29)$$

and

$$\beta_{opt} = b\sqrt{\frac{b^2d^2}{8A_o^4B^2} - 1} \qquad (B.30)$$

[1] op. cit., p.l.

From equations (3.9) and (3.10) of Technical Report No. 3,[1]

$$\frac{b^2d^2}{A_o^4} = \left(\frac{4}{b^2d^2} + 1\right)(4 + b^2d^2) + \frac{4s^2d^2}{b^2} \quad , \tag{B.31}$$

and

$$B^2 = \frac{4}{b^2d^2} + 1 + \frac{s^2d^2}{b^2} \quad , \tag{B.32}$$

and therefore

$$\frac{b^2d^2}{A_o^4} - \left(\frac{4}{b^2d^2} + 1\right)(4 + b^2d^2) = 4\left[B^2 - \frac{4}{b^2d^2} - 1\right] \quad ,$$

or

$$\frac{b^2d^2}{A_o^4} = 4B^2 + 4 + b^2d^2 \tag{B.33}$$

This expression can be used to eliminate $A_o$ from (B.24), (B.29) and (B.30).

Thus

$$\left(\frac{S}{N}\right)_{\substack{\text{no video}\\ \text{filter}}} = \frac{E^2}{N_o^2} \frac{16}{4B^2 + 4 + b^2d^2} \exp\left\{-\frac{4}{B^2}\left(\frac{sc}{b}\right)^2\right\} \tag{B.34}$$

and

$$\left(\frac{S}{N}\right)_{\substack{\text{opt video}\\ \text{filter}}} = \frac{E^2}{N_o^2} \frac{4}{B\sqrt{4 + b^2d^2}} \exp\left\{-\frac{4}{B^2}\left(\frac{sc}{b}\right)^2\right\} \tag{B.35}$$

and

$$\beta_{opt} = \frac{dB\sqrt{8}}{\sqrt{4 + b^2d^2 - 4B^2}} \tag{B.36}$$

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[1] Op. Cit. P. 1

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