Guaranteed uncertainty management (GUM) for sensor provisioning in missile defense

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Abstract

Sensor provisioning is the problem of determining the number of sensors required to accomplish a complicated system level task, e.g. tracking or discriminating between N targets. It is a central problem in missile defense radar systems where the number of targets can easily exceed the number of radars. Here we present a method for conservative sensor provisioning that guarantees a prescribed level of system performance, e.g., multiple target detection and position uncertainty levels, regardless of the scenario.

1 Introduction

This report describes a general approach to sensor provisioning for multiple sensor systems that uses the guaranteed uncertainty management (GUM) philosophy. By sensor provisioning we mean using radar models for target detection, estimation and classification to specify fundamental limits on performance (system stability, track entropy, discrimination error probability, radar occupancy rates) for a given provisioning of the radar system (number of sensors, pulse repetitions, revisit rates) as a function of signal processing parameters (target SNR, state error covariance, desired P_f and P_d or classification error P_e).

While these methods apply to more general multiple radar multiple target detection, tracking, and discrimination problems, our motivation was sensor management for engagement planning in missile defense. The guaranteed uncertainty management approach is more conservative than standard stochastic scheduling approaches to radar system provisioning. We feel it is better suited to the missile defense application since it carries strict and absolute guarantees on the probability of loss of track of the system. This is as contrasted to average performance guarantees or approximations that have been adopted in previous approaches to sensor management [4] for similar applications. By adopting this strict performance assurance approach the sensor management problem becomes non-stochastic and we can obtain strong results that could not be easily obtained in the less stringent stochastic scheduling context.

The report is organized as follows. In Sec. 2 we summarize the selection of radar parameters. In Sec 3 the main statistical models for radar tracking and discrimination are reviewed and a compact asymptotic approximation to the classification error probability is given for Swerling targets using Chernoff and union bounds. In Sec 4 we develop the theory for guaranteed uncertainty management for tracking radar including stability conditions and radar provisioning for multiple targets. In Sec. 5 this theory is extended to multi-purpose radar that engages in tracking and other activities such as discrimination and search. A step-by-step illustration of how to apply these results is given in this section. Finally in Sec. 6 a numerical example is shown.

In general we denote time parameters by τ and t, probabilities by P, positive integers by upper and lower case i, j, k, l, m, n, p and sets by caligraphic capital letters $\mathcal{C}$.

2 Radar and target parameters

The radar scan parameters in the missile defense application are:

- Pulse repetition interval τ_{PRI}. Determined by the maximum range r_{max} acquired by the radar. A rule of thumb is to set $\tau_{PRI} = 2r_{max}/c$, where c is speed of light, but if there are multipath radar returns the factor of 2 may not be sufficiently large.
- The number N_p of radar pulses used to probe a particular cell. More pulses translate into better detection and discrimination performance by reduction of statistical sampling error variance.
- Gain G. This is not important unless we wish to account for smart targets that deploy countermeasures upon detection of sufficient amounts of incident radar illumination energy.
- Mode M of radar. In FTI mode the RCS is different from that of MTI mode. MTI mode contains information about target position, velocity and RCS, while FTI only carries information about position and RCS. This information can be quantified by the inverse Fisher information on these parameters, see e.g., the FASM book [4] chapter 10.
- The field of view (FOV) of each radar and, in particular, the overlap and coverage diversity. The FOV is important in all phases of radar operation. It determines the subset of radars that are capable of tracking a target as it crosses from the FOV of one radar into another's FOV and thus determines the handoff protocol. Cooperative tracking and identification by multiple (usually 2) radars can greatly improve performance for targets lying in overlapping regions once detected.
- Radar service time required for revisiting a target and performing a particular task, which is also called the radar service load. This ”busy time” will depend on target type, radar mode, and desired performance level (operating point) in detection, tracking, or classification.
- The occupancy of a radar is the percent of the time that it is busy servicing targets. For maximal utilization efficiency and minimal idle time, the sensor manager should strive to maintain a few sensors at 100% occupancy rather than many sensors at less than 100% occupancy.

The target parameters are

1. Number N of targets in the FOVs of the R radars.
2. Target radar cross section(RCS) denoted by σ_s.
3. Noise power σ_o.
4. Confuser type. This is determined by the presence or absence of chaff or other countermeasures.
5. Target class S. This is determined by the condition of the target, e.g. boost phase, ballistic phase, transition phase, etc.

3 Statistical models for radar return signal

3.1 Target return models

Different target classes induce different joint distributions $f(Y|\alpha,\beta)$ of the pulse-averaged radar return $Y = \sum_{i=1}^{N_p} Y_i$ resulting from a scan of a given cell. Swerling's models are common used to model the distribution of the magnitude of the radar returns and determine the detection and classification performance [11]. These models are special cases of the (scaled) Chi-square distribution

$$\mathcal{X}(Y;\nu,\sigma_s) = \Gamma(Y;\nu/2, 2\sigma_s), \tag{1}$$

where Γ is the (2 parameter) Gamma distribution

$$\Gamma(Y;\alpha,\beta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} Y^{\alpha-1} e^{-Y/\beta}, Y > 0 \tag{2}$$

where $\alpha > 0, \beta > 0$ $\Gamma(\alpha)$ and is the complete Gamma function [5]. In particular, for $\alpha = 1$ the Gamma distribution is the exponential distribution

$$\Gamma(Y;1,\beta) = \frac{1}{\beta} e^{-Y/\beta}, Y > 0.$$

Consider a multipulse radar probing a target cell with N_p pulses detector. Assume that the radar receiver performs phase incoherent envelope detection for each detected pulse and outputs a voltage Y equal to the sum of these pulse returns. Then, if the radar receiver noise is dominated by the target return signal we have the following density models for Y:

1. **Cell with single slowly fluctuating target (Swerling I)**: the density $f(Y|\sigma_s, N_p)$ is Gamma with parameters $\alpha = 1, \beta = 2N_p\sigma_s$.
2. **Cell with single rapidly fluctuating target (Swerling II)**: the density $f(Y|\sigma_s, N_p)$ is Gamma with parameters $\alpha = N_p, \beta = 2\sigma_s$.
3. **Cell with slowly fluctuating target and clutter (Swerling III)**: the density $f(Y|\sigma_s, N_p)$ is Gamma with parameters $\alpha = 2, \beta = 2N_p\sigma_s$.
4. **Cell with rapidly fluctuating target and clutter (Swerling IV)**: the density $f(Y|\sigma_s, N_p)$ is Gamma with parameters $\alpha = 2N_p, \beta = 2\sigma_s$.

Other models have been proposed for complex targets, e.g., the Rician, Weibul, and log-normal densities [11, Ch. 5],[Ch. 2][13]. These models can also be handled but, unlike the Swerling models, there is no closed form for the Kullack-Liebler divergence (4) between them.

3.2 Detection, tracking, and classification performance

Assume that the radar probes a particular range/cross-range cell centered at location $\overline{z}$ for presence of target, estimation of target trajectory, or target classification.

The parameters of importance are

- Desired detection operating point P_f, P_d on ROC curve. This determines the number of radar pulses required to probe each cell for a particular radar sensitivity and RCS of target.

- If there is a target present, the standard errors σ_z, σ_ϕ, σ_v in the estimates of target position z, direction of motion ϕ, and speed of motion v of the target within the cell. The inverse Fisher information, the Kalman error covariance estimate, or other signal processing provides these parameters, e.g., see [4] chapter 10. These standard errors can be specified as desired operating points for the radar.
- Desired operating point for the confusion matrix of probabilities $p(i|j)$ of deciding target condition i when j is true:

$$P_e = ((P(i|j)))_{i,j=1,\ldots,C}$$

  where C is the number of target conditions of interest, e.g., lethality of live target, disintegration of intercepted target, target deployment of chaff or other countermeasures. This desired operating point will determine radar requirements.

The tasks (modes) that we consider are detection, tracking, and classification. In each of these cases the required radar scan time to detect, estimate, or classify within a given cell can be expressed as

$$T = N_p \tau_{PRI}.$$

The parameter N_p is the number of pulses required to achieve a specific per-cell performance level. The computation of N_p is summarized in the following:

- Detection of a target in a radar cell. Here $N_p = N_p(S, \sigma, M, P_F, P_D)$ is determined from the equation

$$N_p = \min\{n : \beta(P_f; n) \geq P_d\}$$

  and $\beta(\alpha, n)$, $\alpha \in [0, 1]$, is the ROC curve associated with the number of pulses, the mode M of the radar, the class of target return, and the RCS σ of the target.
- Estimation of target position, direction, and velocity in a cell. Here $N_p = N_p(S, \sigma, M, \epsilon)$ is now defined relative to a desired level ϵ of worst case standard error on these three parameters.

$$N_p = \min\{n : \max(\sigma_z, \sigma_\phi, \sigma_v\} \leq \epsilon\}.$$

- Classification of target type within a cell. Here $N_p = N_p(M, \delta)$ can be specified to attain a desired level of misclassification error probability. In keeping with the need to maintain the most stringent performance guarantees, we propose to select N_p in order to ensure the worst case misclassification probability

$$N_p = \min\{n : \max_j (P_e(n|i)\} \leq \delta\},$$

  where $P_e(n|i)$ is the probability of misclassification of target type i when n radar pulses are used to scan the cell. This error probability will depend on the decision rule used for classification, the radar mode used, the target RCS, and on the distribution of the radar returns under the different classes of target.

3.2.1 Classification performance prediction via Chernoff bound

Unless there are only two target classes, the misclassification probability is a complicated function of the density functions $P(Y|i)$, $i = 1, \ldots, M$. We propose to use a upper bound on this probability that applied to the optimal maximum likelihood (ML) classifier. In keeping with our conservative approach an upper bound will enable us to specify absolute classification performance guarantees as contrasted with average performance guarantees or approximations.

The misclassification error probability can be upper bounded using a combination of the union bound and the Chernoff bound (see for example [2] or [3, Sec. 5.6]). This results in

$$P_e(n|i) = P(\cup_{j\neq i}\{f(Y|j) > f(Y|i)\}|i) \leq \sum_{j\neq i} P(f(Y|j) > f(Y|i)|i) \leq \sum_{j\neq i} \exp(-n\mathrm{KL}(i\|j)), \tag{3}$$

where

$$\mathrm{KL}(i\|j) = \int_Y f(Y|i) \ln \frac{f(Y|i)}{f(Y|j)} dY \tag{4}$$

is the Kullback-Liebler divergence between the densities $f(Y|i)$ and $f(Y|j)$ of the radar return signal due to a single pulse when the target class is i and j, respectively.

To apply this to target classification between different possible Swerling models, in (3) we can use the expressions for the KL divergence between two Gamma distributions $f(Y|i) = \Gamma(Y;\alpha_i,\beta_i)$ and $f(Y|j) = \Gamma(Y;\alpha_j,\beta_j)$ [12, 9]

$$\begin{aligned}\mathrm{KL}(i\|j) &= (\alpha_i - 1)\Psi(\alpha_i) - \log(\beta_i) - \alpha_i - \log\left(\Gamma(\alpha_i)/\Gamma(\alpha_j)\right) \\ &\quad + \alpha_j \log\beta_j - (\alpha_j - 1)\left(\Psi(\alpha_i) + \log\beta_i\right) + \frac{\alpha_i\beta_i}{\beta_j}\end{aligned} \tag{5}$$

where $\Psi(z) = \frac{d}{dz}\ln\Gamma(z)$ is the "psi function" (also called the digamma function and available as Matlab function `psi(0,z)`).

4 Stable information-driven sensor management strategies

The sensor management problem is to utilize the available R radars in an optimal fashion to detect, classify and track targets most accurately.

4.1 Information-optimal sensor management

The information criterion is often used to manage the sensors by selecting the sensor or sensor action that maximizes the drop in entropy of the posterior distribution of the target detections, positions, and/or class. This problem has been studied extensively in our previous work, see e.g., [7],[6],[8].

Assume that when a radar works on a task that it finishes it, in the sense that it works long enough on the task to reduce the entropy to a specified (small) value. Assume also that the entropy $H(n)$ associated with each task is known and increases at a rate that is independent of the task. Under these assumptions, a standard exchange argument establishes that the optimal allocation of R radars to N different tasks is as follows. First rank order the tasks in terms of decreasing entropy of the posteriors associated with each task. Initially assign the R radars to the R tasks on the top of the list. As radars finish their initial tasks they are assigned to the most highly ranked task that have not yet been started. This results in the fastest overall decrease in the average entropy, $\frac{1}{N}\sum_{n=1}^{N} H(n)$.

4.2 Stable sensor management under resource constraints

Since the radars cannot revisit and update target tracks instantaneously and the uncertainty (entropy) grows over time it is obvious that too few radars will quickly be overwhelmed as the number of targets increases. The stability question is: what is the minimum number of radars required to maintain bounded uncertainty on the positions of all targets?

Entropy minimization does not account for resource constraints such as service load, slew rate, handoff, or others. Such resource constraints must be combined with information-driven sensor management to ensure stable and efficient system operation. To ensure stable information-drive sensor management we find conditions that must be satisfied for the steady state radar load to be bounded.

To do this we give tight bounds on the service load (in seconds) on a radar to revisit a target track and reduce the target positional uncertainty to the volume of a single range/cross-range cell. These bounds are then used to determined the total number of radars required to maintain a given number of tracks, the associated steady state revisit rates and track entropies. We use these results to establish stability regions for the multiple track maintenance problem.

4.3 Guaranteed uncertainty management: single target

Assume that at time 0 a target is detected in a cubical radar cell

$$\mathcal{C}_0 = \{z = (z_1, z_2, z_3) : -\sigma_z \le z_1 - \overline{z}_1 \le \sigma_z,\ -\sigma_z \le z_2 - \overline{z}_2 \le \sigma_z,\ -\sigma_z \le z_3 - \overline{z}_3 \le \sigma_z\}$$

where $\overline{z} = [\overline{z}_1, \overline{z}_2, \overline{z}_3,]$ is the center position of the cell. From the integrated radar returns from this cell, the radar signal processing algorithms extract an estimate $(\hat{z}, \hat{\phi}, \hat{\theta}, \hat{v})$ of target position, direction angles, and speed along with a set of standard errors $\sigma_z, \sigma_\phi, \sigma_\theta, \sigma_v$. This could be the output of a Kalman filter, sigma tracker, particle filter or other common tracking algorithm.

From these estimates and standard errors a confidence region for z, ϕ, θ, v having coverage probability of at least $1 - \epsilon_T$ can be specified. In particular, assume that

$$[\hat{z} - \sigma_z, \hat{z} + \sigma_z] \times [\hat{\phi} - \sigma_\phi, \hat{\phi} + \sigma_\phi] \times [\hat{\theta} - \sigma_\theta, \hat{\theta} + \sigma_\theta] \times [\hat{v} - \sigma_v, \hat{v} + \sigma_v]$$

is such a confidence region.

Assume for simplicity that $\sigma_\phi = \sigma_\theta$. Then with probability no less than $1 - \epsilon_T$, after an elapsed time of τ seconds from the last revisit of the target, the above $1 - \epsilon_T$ confidence region will map to the union of an uncountable number of conical segments, each with conical apex at some point within the radar cell $\mathcal{C}_0$. This union is a complicated set and we approximate it with with the circumscribing conical segment

$$\mathcal{C}_\tau = \{z = (r, \phi, \theta) : -\sigma_z - \tau\sigma_v \le r + \Delta - \tau\hat{v} \le \sigma_z + \tau\sigma_v,\ -\sigma_\phi - \sigma_z/(\tau\hat{v}) \le \phi - \hat{\phi} \le \sigma_\phi + \sigma_z/(\tau\hat{v}),\ \theta \in [-\pi, \pi]\},$$

where

$$\Delta = \frac{\sigma_z/\sqrt{2}}{\tan(\sigma_\phi)}$$

is the distance between the apex of the cone and the center of the cell $\mathcal{C}_0$. See Figure 1 and 2 for illustration. The volume of this cork-shaped region is

$$|\mathcal{C}_\tau| = \frac{2\pi}{3}\left[(\tau(\hat{v} + \sigma_v) + \Delta + \sigma_z)^3 - (\tau(\hat{v} - \sigma_v) + \Delta)^3\right](1 - \cos(\sigma_\phi + \sigma_z/(\tau\hat{v})). \tag{6}$$

Consider a radar or sensor with scan cell size $|\mathcal{C}_0|$ that revisits this target after an elapsed time of τ seconds from the previous revisit. This radar will be occupied scanning subcells in $\mathcal{C}_\tau$ in order to reduce uncertainty on the target parameters (position/speed/direction) back down to a $1 - \epsilon_T$ confidence region of size $|\mathcal{C}_0| = \sigma_z^3$. The load on this radar would be the time to scan the region $\mathcal{C}_\tau$ which is equal to:

$$q(\tau) = \gamma(\tau) N_p \tau_{PRI}, \tag{7}$$

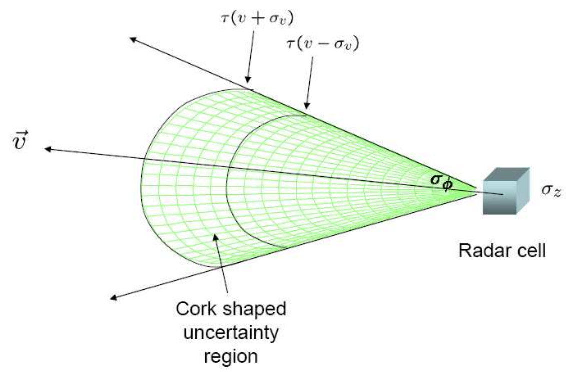

Figure 1: The small radar cell at right contains a target with high certainty immediately after revisit. If the target speed is v with confidence $\pm\sigma_v$ and the target direction is $\vec{v}$ with confidence σ_ϕ then we can be confident that a target at the center of the radar cell will lie in a cork shaped region after an elapsed time of τ secs. When the the target can lie anywhere in the radar cell then we can only be confident that the target will lie in the union of all induced cork-shaped regions by cones with vertices somewhere in the radar cell.

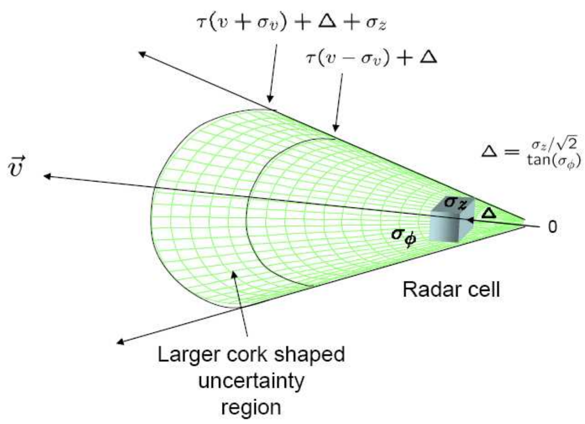

Figure 2: The confidence region after an elapsed time of τ seconds can be bounded by a larger cork shaped region induced by the cone that inscribes the radar cell.

where

$$\gamma(\tau) = |\mathcal{C}_\tau|/|\mathcal{C}_0| - 1$$

is the growth of the confidence region and N_p is the number of radar pulses per cell required to achieve the P_f, P_d and prescribed $1 - \epsilon_T$ standard error on target parameters.

For R radars and N targets let $q_{r,n}(\tau)$ denote the load (in seconds) on the r-th radar to revisit and update the n-th target after an elapsed time of τ.

$$q_{r,n}(\tau) = \gamma_{r,n}(\tau) N_p(r,n) \tau_{PRI}, \tag{8}$$

where $\gamma_{r,n}$ and $N_p(r,n)$ are analogously defined as possibly radar and target dependent quantities that ensure that the confidence region $\mathcal{C}_\tau = \mathcal{C}_\tau(r,n)$ for the n-th target has confidence $1-\epsilon_T$ and that the number of pulses $N_p = N_p(r,n)$ is sufficient to guarantee the P_f, P_d criteria.

4.4 Guaranteed uncertainty management: multiple targets

The problem of scheduling of multiple radars to maintain multiple target tracks is a resource allocation problem analogous to the problem of optimal processor allocation to different types of jobs in a parallel processing system. The problem falls in the framework of dynamic scheduling of multiple heterogeneous servers (radars) to multiple heterogeneous queues (targets) [1, 14].

4.4.1 The PLQ sensor manager for tracking

The sensor manager must assign radar servers to queues of target-revisit jobs in queues that grow as time elapses. The target revisit jobs mat have different service requirements. In its full generality, solving for the optimal allocation of servers to queues is a difficult, if not intractable, problem. However, several sub-optimal strategies have been proposed. A suboptimal ”prioritized longest queue” (PLQ) strategy is to assign free servers to the longest queues, where each queue is processed by the server that is best matched to the queues' service requirements, e.g., an EO sensor can best service the tracks of hot targets. The following implementation of this strategy is the ”largest weighted queue length” policy proposed in Wasserman *etal* [14] for heterogeneous multiqueueing systems. Let $\mathcal{N} \subset \{1, \ldots, N\}$ be the number of target tracks not in the process of being revisited.

Prioritized longest queue (PLQ) sensor scheduling policy

When a radar sensor r is unoccupied and available for assignment to updating a target track then either

1. *idle* the sensor if all target tracks are in process of being revisited ($\mathcal{N}$ is empty).
2. *deploy* the radar sensor on the target tracks $n \in \mathcal{N}$ that maximizes the weighted service time $\max_{n\in\mathcal{N}} q_{r_n}(\tau_n)$ where τ_n is the elapsed time since the last revisit of target n.

Note that PLQ policy can be related to a penalized entropy minimization sensor manager since:

$$\ln(q_{r,n}(\tau) + N_p \tau_{PRI}) = \Delta H_\tau(r,n) + c_{r,n}$$

where $\Delta H_\tau(r,n) = \ln|\mathcal{C}_\tau(r,n)| - \ln|\mathcal{C}_0(r,n)|$ is the decrease in the track entropy attained by updating the n-th target with the r-th radar and c_n is a penalty that is independent of growth in the uncertainty of target location.

In the context of queuing systems, Wasserman *etal* [14] give conditions on the number of servers (R), the number of queues (N), the rates of service ($\gamma(r,n)$) and arrival rates ($d\mathcal{C}_\tau/d\tau$) that guarantee that the mean queue size ($E[\mathcal{C}_\tau]$) is bounded, i.e. the scheduling policy does not lose track. These results are applicable to the missile defense scenario but would only provide guarantees on mean tracking performance, which is too weak for our purposes. In the following we give more useful conditions that guarantee that with probability $1-\epsilon_T$ the policy does not lose track.

The results are simpler to develop for a single radar tracking N targets so we treat this case first.

4.4.2 Balance equations guaranteeing system stability: single radar

Balance equations for stable operation of the radar are equations that guarantee that at the time of revisit of a target its service load has not grown larger than it was at the previous revisit. As there is only one radar we drop the index r from $q_{r,n}(\tau)$. We also assume that these functions have been indexed such that $q_1(\tau) \geq q_2(\tau) \geq \ldots \geq q_N(\tau)$, i.e. the targets have been ranked in decreasing order of service load, and that the radar revisits the targets in this order. This is the PLQ policy and it will give the best and least stringent stability conditions.

Define the functions $q^{(i)}$ as follows

$$\begin{aligned} q^{(1)}(\tau) &= q_1(\tau) \\ q^{(2)}(\tau) &= q_2(q^{(1)}(\tau)+\tau) \\ &\vdots \\ q^{(N)}(\tau) &= q_N(q^{(N-1)}(\tau)+\tau). \end{aligned} \tag{9}$$

The function $q^{(i)}(\tau)$ is the service load (in seconds) of the i-th target at the time of revisit. Next define the radar system loading function

$$Q^{(N)}(\tau) = \sum_{i=2}^{N} q^{(i)}(\tau). \tag{10}$$

When $Q^{(N)}(\tau) < \tau$ the service loads will remain bounded and the radar system is stable. When $Q^{(N)}(\tau) > \tau$ the system is unstable and when $Q^{(N)}(\tau) = \tau$ is critically stable. If a solution exists, let $\tau = \tau^*$ be the solution to the following *balance equation*

$$Q^{(N)}(\tau) = \tau. \tag{11}$$

We can state the following result

Proposition 1 *For a single radar tracking N targets the PLQ policy (service targets in decreasing order of load) is stable, in the sense that the system maintains bounded tracking errors, if the following conditions hold:*

1. *a solution to (11) exists;*
2. *the revisit rate is at least $1/\tau^*$;*
3. *The maximum target speed v is such that $\tau^*(v+\sigma) \ll \sigma_z\sqrt{3}$.*

The value τ^* can be interpreted as the steady state total time required for the radar to cycle through a complete sequence of target revisits. $q^{(k)}(\tau^*)$ is the time required to service the target k, $k = 1,\ldots,N$. The stability result of Proposition 1 is tight in the sense that the radar system becomes unstable if Conditions 1 and 2 are not satisfied.

When stability of the PLQ policy is guaranteed, we have a tight bound on the associated tracking error

elapsed time(s)	job	job duration(s)	T1 load	T2 load
τ	initialize	τ	$q_1(\tau)$	$q_2(\tau)$
$q_1(\tau)+\tau$	T1-update	$q_1(\tau)$	0	$q_2(q_1(\tau)+\tau)$
$q_2(q_1(\tau)+\tau)+q_1(\tau)+\tau$	T2 update	$q_2(q_1(\tau)+\tau)$	$q_1(q_2(q_1(\tau)+\tau))$	0
$q_2(q_1(\tau)+\tau)+q_1(\tau)+\tau+\ldots$	T1-update	$q_1(q_2(q_1(\tau)+\tau))$	0	$q_2(q_1(q_2(q_1(\tau)+\tau)))$
$\vdots$	$\vdots$	$\vdots$		

Table 1: Table for a single radar tasked to track $N = 2$ targets. For stable system we require that the load per update of any given target satisfy a stable growth condition, i.e., T1 load in 3-th row be equal to T1-load in 1st row: $q_1(q_2(q_1(\tau)+\tau)) = q_1(\tau)$. Since q_1 is monotonic increasing function this is equivalent to $q_2(q_1(\tau)+\tau) = \tau$.

Corollary 1 *If the radar system is stable in the sense of Proposition 1 then the entropy of the tracking error of the k-th target will never exceed $H^*(k) = \ln \mathcal{C}_{\tau^*}(k)$.*

The proof of the above proposition is straightforward but we do not provide details here. Table 1 provides an illustration of the stability condition for $N = 2$. The full proof relies on the fact that $q_k(\tau)$ is monotonic increasing in τ. We then use mathematical induction to obtain equations (9) as the time required to service the targets, and apply standard load balancing condition of optimal scheduling theory to obtain (11).

4.4.3 A simple slope criterion for stability

The system load function $Q^{(N)}(\tau)$ defined in (10) is zero at $\tau = 0$ and is smooth, differentiable, and monotonic increasing. Thus a necessary condition for the balance equation (11) to have a solution is that its derivative be less than or equal to one at the point $\tau = 0$. By induction the derivative $[Q^{(N)}]'(0) = dQ^{(N)}(\tau)/dt|_{\tau=0}$ can be shown to be of the form:

$$[Q^{(N)}]'(0) \quad = \quad \sum_{j=2}^{N}\sum_{k=2}^{j}\prod_{i=1}^{k} q_i'(0) \leq \sum_{j=2}^{N}\sum_{k=2}^{j}[q'(0)]^k, \tag{12}$$

where we have defined $q'(0) = \max_i q_i'(0)$.

If $\min_i q_i'(0) > 1$ then necessarily $[Q^{(N)}]' > 1$ so that $Q^{(N)}(\tau) > \tau$ and the system is unstable. If $\max_i q_i'(0) < 1$ then the system may be stable.

To obtain closed form results we will derive sufficient conditions on N that guarantee stability by using the upper bound on the right of (12) instead of the exact expression in the middle of (12). This upper bound is attained when all service load functions are identical $q_i(0) = q_j(0)$ in which the conditions derived below will also be necessary. Therefore, the conditions will be tight for a worst case scenario but will be more stringent than might be required for a typical scenario.

As $q'(0) \geq 0$, the geometric series summation formula applied to the right hand side of (12) gives the simple formula :

$$[Q^{(N)}]'(0) = \frac{q'(0)}{1-q'(0)}\left(N - \frac{q'(0)}{1-q'(0)}(1-[q'(0)]^N)\right) - q'(0). \tag{13}$$

Furthermore after differentiating (6) and plugging into (7)

$$q'(0) = 2\pi(1-\cos\sigma_\phi))\left((\Delta+\sigma_z)^2(v+\sigma_v) - \Delta^2(v-\sigma_v)\right)N_p\tau_{PRI}. \tag{14}$$

Proposition 2 *A solution τ^* to the balance equations (11) exists if and only if,*

$$[Q^{(N)}]^{'}(0) = \frac{q^{'}(0)}{1-[q^{'}(0)]^2}\left(N - \frac{q^{'}(0)}{1-q^{'}(0)}(1-[q^{'}(0)]^N)\right) < 1.$$

Define N_{max} as the maximum value of N such the inequality in Proposition 2 is satisfied. When the radar is tasked to track N_{max} targets then the system will be stable (however, we must still verify that the associated τ^* is such that condition 3 of Proposition 1 is satisfied). In the case $N = N_{max}$ the radar is fully utilized and operating at maximum efficiency. When $q^{'}(0)$ is small N_{max} can be found approximately as

$$N_{max} = \frac{q^{'}(0)}{1-[q^{'}(0)]^2} + \frac{1-q^{'}(0)}{q^{'}(0)}. \tag{15}$$

Furthermore since $0 \leq 1-[q^{'}(0)]^N \leq 1$ we can assert that if the number of targets N exceeds N_{max} in (15) then no solution to the balance equations exists and the radar tracker system diverges.

4.4.4 Balance equations guaranteeing system stability: multiple radars

When there are $R > 1$ radars to manage we can obtain stability conditions in a similar manner to the previous section. Define the ratio of targets per radar $b = \text{ceil}(N/R)$ as the smallest integer greater than N/R. Define $q(\tau) = \max_{n,r} q_{r,n}(\tau)$ and defined the functions $q^{(i)}$ as

$$\begin{aligned}
q^{(1)}(\tau) &= q(\tau) \\
q^{(2)}(\tau) &= q(q^{(1)}(\tau)+\tau) \\
\vdots &\;\vdots\; \vdots \\
q^{(b)}(\tau) &= q(q^{(b-1)}(\tau)+\tau)
\end{aligned} \tag{16}$$

In analogy to the previous section, define the radar system loading function $Q^{(b)}((\tau) = \sum_{i=1}^{b} q^{(i)}(\tau)$. Again, when $Q^{(b)}((\tau) < \tau$ the system is stable. The multiple radar *balance equation* is

$$Q^{(b)}(\tau) = \sum_{i=1}^{b} q^{(i)}(\tau) = \tau. \tag{17}$$

Proposition 3 *A system of R tracking radars is stable under the PLQ policy if the following conditions hold:*

1. *a solution to (17) exists;*
2. *the revisit rate is at least $1/\tau^*$;*
3. *The maximum target speed v is such that $\tau^*(v+\sigma) \ll \sigma_z\sqrt{3}$.*

The stability result of Proposition 3 is tight in the sense that the radar system becomes unstable if Conditions 1 and 2 are not satisfied for at least one scenario. This scenario is when all targets have the same revisit service requirements, i.e., $q_{n,r}(\tau)$ is independent of n, r.

Again, when stability of the PLQ policy is guaranteed, we have a tight bound on the associated tracking error

Corollary 2 *If the R radar tracking system is stable in the sense of Proposition 3 then the entropy of the tracking error of any of the N targets is upper bounded by $H^* = \ln \mathcal{C}_{\tau^*}$. This bound is achieved when all targets have the same revisit service requirements.*

The proof of the above proposition is similar to that of Proposition 1 except that we group the targets into b groups of at most R targets that can be revisited at any point in time and proceed as previously. The details are omitted.

A similar slope condition to (18) can easily be derived for the case of multiple radars.

Proposition 4 *For a system of R radars a solution τ^* to the balance equations (17) exists if and only if,*

$$[Q^{(b)}]^{'}(0) = \frac{q^{'}(0)}{1-[q^{'}(0)]^2}\left(b - \frac{q^{'}(0)}{1-q^{'}(0)}(1-[q^{'}(0)]^{b+1})\right) < 1,$$

where $b = \mathrm{ceil}(N/R)$.

4.5 Determining track-only radar occupancy

We can use the Propositions to determine the efficiency of a radar system in terms of its occupancy rates, defined as one minus the proportion of time a radar in the system is idle. We assume that the radars are scheduled under the PLQ policy. In steady state a stable system of R radars will be at maximum utilization when the the system is critically stable. This occurs when there are approximately $b^* = N/R$ targets per radar where b^* is the solution to the equation

$$\frac{q^{'}(0)}{1-[q^{'}(0)]^2}\left(b - \frac{q^{'}(0)}{1-q^{'}(0)}(1-[q^{'}(0)]^{b+1})\right) = 1. \tag{18}$$

Define $N_{max} = \mathrm{floor}(b^* R)$. At this critically stable operating point of N_{max} targets, the radars are fully occupied performing *just-in-time* revisits of the targets. In this case the the maximum service load that each target places on the system is $Q^{(N_{max}/R)}(\tau^*)$ where τ^* is the solution of $Q^{(N_{max}/R)}(\tau) = \tau$. When the same system is assigned to track a fewer number $N < N_{max}$ of targets there will be idle time. We define the occupancy of the track-only system as

$$\rho = \tau^*/\tau_e$$

where τ_e is the value of τ that satisfies

$$Q^{(N/R)}(\tau) = Q^{(N_{max}/R)}(\tau^*). \tag{19}$$

The interpretation is that τ_e is operating point of the radar system that results in the same loading for the underloaded system tracking N target as the fully loaded system tracking N_{max} targets. The definition of ρ is illustrated in the next section.

5 Multi-purpose radar for tracking and other tasks

Finally we turn to scenarios when the radar may be engaged in other tasks in addition to tracking. For example, the provisioning results of the previous section are extensible to a system that combines wide area search, discrimination and tracking. This is handled by building in headroom into the track update stability equations.

When there is an additional load on the radar of Δ secs after updating a target track, Table 1 needs to be modified to Table 2. Here Δ is the time spent after each revisit on other tasks than tracking. In general if there are N targets

elapsed time(s)	job	job duration(s)	T1 load	T2 load
τ	initialize	τ	$q_1(\tau)$	$q_2(\tau)$
$q_1(\tau)+\tau$	T1-update	$q_1(\tau)$	0	$q_2(q_1(\tau)+\tau)$
$q_1(\tau)+\tau+\Delta$	Δ-update	Δ	$q_1(\Delta)$	$q_2(q_1(\tau)+\tau+\Delta)$
$q_2(q_1(\tau)+\tau+\Delta)+q_1(\tau)+\tau+\Delta$	T2 update	$q_2(q_1(\tau)+\tau+\Delta)$	$q_1(q_2(q_1(\tau)+\tau+\Delta)+\Delta)$	0
$q_2(q_1(\tau)+\tau+\Delta)+q_1(\tau)+\tau+2\Delta$	Δ-update	Δ	$q_1(q_2(q_1(\tau)+\tau+\Delta)+2\Delta)$	$q_2(\Delta)$
$q_2(q_1(\tau)+\tau+\Delta)+q_1(\tau)+\tau+2\Delta+\ldots$	T1-update	$q_1(q_2(q_1(\tau)+\tau+\Delta)+2\Delta)$	0	$q_2(q_1(q_2(q_1(\tau)+\tau+\Delta)+2\Delta)+\Delta)$
$\vdots$	$\vdots$	$\vdots$		

Table 2: Table for a single radar tasked to track $N=2$ targets in addition to spend Δ seconds after each revisit on other tasks. For stable system we require that the load per update of any given target satisfy a stable growth condition, i.e., T1 load in 5-th row be equal to T1-load in 1st row: $q_1(q_2(q_1(\tau)+\tau+\Delta)+2\Delta)=q_1(\tau)$. Since q_1 is monotonic increasing function this is equivalent to $q_2(q_1(\tau)+\tau+\Delta)+2\Delta=\tau$.

and R radars then each radar can be assigned to b of the N targets according to the PLQ strategy. Without loss of generality, let there be only a single radar and N targets. For given Δ the stability condition is that there must exist asolution $\tau=\tau^*$ such that (refer to Table 2 for the case $N=2$)

$$Q^{(N)}(\tau,\Delta)+N\Delta=\tau$$

where

$$Q^{(N)}(\tau,\Delta)=\sum_{i=2}^{N}q^{(i)}(\tau,\Delta)$$

and $q^{(i)}$ is defined recursively as

$$\begin{array}{rcl} q^{(1)}(\tau,\Delta) & = & q_1(\tau) \\ q^{(2)}(\tau,\Delta) & = & q_2(q^{(1)}(\tau)+\tau+\Delta) \\ \vdots & \vdots & \vdots \\ q^{(N)}(\tau,\Delta) & = & q_N(q^{(N-1)}(\tau)+\tau+\Delta) \end{array} \tag{20}$$

As the q_i's are monotonic increasing we have the bound

$$Q^{(N)}(\tau,\Delta)\le Q^{(N)}(\tau+\Delta)$$

where $Q^{(N)}(\tau)$ is the simpler univariate function defined in (11). Therefore, for specified Δ, a sufficient condition for stability is that there exists a $\tau=\tau^*$ such that:

$$Q^{(N)}(\tau+\Delta)+N\Delta=\tau.$$

Rexpressing this in terms of the variable $u=\tau+\Delta$, we have the equivalent condition that there exist a solution $u=u^*$ to

$$Q^{(N)}(u)=u-(N+1)\Delta.. \tag{21}$$

5.1 Multi-purpose radar load margin, excess capacity, and occupancy

The load margin represents the maximum additional load that can be accommodated by a radar tracking system that must perform joint operations such as tracking, detection, etc. The load margin Δ_{max} is defined as the maximum value Δ for which a solution u to (21) exists. As the slope of $Q^{(N)}(u)$ is increasing in u, the load margin Δ_{max} is easily determined graphically (see Fig. 8). If for given Δ the load curve $Q^{(N)}(u)$, $u>0$, intersects the line $u-(N+1)\Delta$ then a solution $u=u^*$ exists. As one increases Δ beyond Δ_{max} the line moves below the curve and there is no intersection. Thus, if it exists, Δ_{max} must be the y-intercept of the line of slope one that is tangent to the load curve. The point of tangency specifies a pair $(u^*, Q^{(N)}(u^*))$ and the quantity $Q^{(N)}(u^*)$ is the time it takes to revisit a given target when the radar system is operating at maximum capacity (engaged in servicing N targets and doing other tasks within the load margin Δ_{max}).

Correspondingly, when there are N targets and the multi-purpose radar spends Δ seconds per update performing other tasks we define the excess capacity

$$c_{excess}(\Delta) = 1 - \frac{\Delta}{\Delta_{max}}. \tag{22}$$

c_{excess} specifies the headroom available for yet other tasks. Likewise we define the multi-purpose radar occupancy as

$$\rho(\Delta) = \frac{Q^{(N)}(u^*) - (\Delta_{max} - \Delta)N}{Q^{(N)}(u^*)}. \tag{23}$$

Note that this differs from our definition of radar occupancy (19) for tracking-only radar.

5.2 Illustration of multi-purpose radar occupancy computation

As concrete example, consider the following radar requirements.

1. **Discrimination Mode**: Probability of correct classification of a target is constrained to be greater than or equal to 0.95.
2. **Search Mode**: Probability of missing a new target is less than or equal to 0.05 and the amount of time that a new target remains on the search fence is τ_{fence}.
3. **Track mode**: For targets already in track the desired track accuracy is σ_z.

The procedure to achieve these benchmarks is:

1. **Fix the track dwell time**: N_p^{track} and T_{PRI}^{track} are selected to achieve the detection, false alarm, and resolution requirements of the radar within a single radar track cell.
2. **Fix the search dwell time**: N_p^{search} and T_{PRI}^{search} are selected to achieve the detection, false alarm, and resolution requirements of the radar within a single radar search cell.
3. **Fix the discrimination dwell time**: N_p^{disc} and T_{PRI}^{disc} are selected to achieve the classification probability of error requirements of the radar within a single radar discrimination cell.
4. **Establish track-only feasibility**: For N targets compute the minimum number of radars R required to maintain track by finding b^* in (18). The minimum number of required radars is then $R = N/\text{floor}(b^*)$. This only guarantees track-only feasibility. If the radar load margin (below) is exceeded then more radars will be required to jointly track, search and discriminate.
5. **Compute load margin of tracking radar**: For N targets and R radars the available load margin Δ_{max} (secs per target update) is computed by exploring the existence of a solution $\tau = \tau^*$ to (21). If a solution exists then the radar system has an available load margin of Δ for accommodating other tasks besides tracking. If a solution does not exist then the number of radars must be increased or the tracking performance requirements must be relaxed.

6. **Compute search load**: The search load will be $N_p^{search}T_{PRI}^{search}$ per cell. The total search load will be $\tau_{search} = N_{fence}N_p^{search}T_{PRI}^{search}$ where N_{fence} is the number of cells on the search fence to be searched after each revisit cycle. There is an additional constraint that the every radar cell on the search fence be probed at least once every τ_{fence} seconds.

7. **Compute discrimination load**: The discrimination load will be $N_p^{disc}T_{PRI}^{disc}$ per target. The total discrimination load will be $\tau_{disc} = N_{targets}N_p^{disc}T_{PRI}^{disc}$ where $N_{targets}$ is the number of targets to be classified after each revisit cycle.

8. **Compute radar occupancy and excess capacity**: If the sum of search and discrimination loads τ_{search}/N and τ_{disc}/N does not exceed the load margin Δ, computed above, and if the revisit period (time for the radar to revisit all targets) is less than τ_{fence} then the radar can perform all joint operations without additional provisioning. In this case, with τ^* and Δ_{max} given by the solution to (21), the radar occupancy is given by

$$\rho = \frac{Q^{(N)}(\tau^*) - \Delta_{max}N + \tau_{search} + \tau_{disc}}{Q^{(N)}(\tau^*)}$$

and the excess capacity of the radar is the fraction of load margin that is available for other tasks

$$\eta = 1 - \frac{\tau_{search} + \tau_{disc}}{\Delta_{max}}.$$

6 Example application

Here we illustrate the results presented in the last two sections.

6.1 Track-only Provisioning

For specified standard errors on radar tracking accuracy, e.g., available from Kalman tracking covariance estimates, these results can be used to generate tables and curves on the required number of radar sensors, their revisit rates, and their occupancy, for tracking N targets with prescribed track error (entropy).

The scenario we have in mind for these examples is tracking N ballistic missile trajectories with R radars that have the following characteristics (these parameters were selected to correspond to typical C-band multipulse monostatic surveillance radars (see [11]).

1. The radars use a pulse repetition interval $\tau_{PRI} = 1$ms for detecting targets at a maximum range of 150 km.
2. The radar pulse duration corresponds to a radar range resolution of 150 meters.
3. The transformed cartesian coordinate cubical radar cell has volume $\sigma_z^3 = (150)^3$.
4. The radar probes each cell with 10 pulses. This corresponds to performance $P_f = 10^{-6}, P_d > 0.999$ for detecting a Swerling II target at a SNR=10dB [10, Fig. 12.17].
5. The speed of each of the targets is estimated to be 300m/sec with standard error of 30m (10%).
6. The estimated direction of travel of each of the targets is accurate to within 18 degrees (10%).

We first illustrate the application of the balance equations to study the loading of the radar system for different numbers of targets and radars. Then we use the slope criterion to study optimal radar allocation strategies for this example.

6.1.1 Loading of track-only radar system

In Figs. 3-5 we consider the case where the number R of radars and the number N of targets are such that $N/R = 5, 15, 23$, respectively.

Define $T_{cell} = \sigma_z\sqrt{3}/(v+\sigma_v)$ as an upper bound on the time required for a target to move out of the neighborhood of a single radar cell. T_{cell} defines an upper limit on the revisit service time for each target. The solid curves in these figures give $q^{(N/R)}(\tau)$, the time for the radar to work through a complete cycle of target updates, which we call the radar system load curve and is plotted against the variable τ. The diagonal line, called the stability boundary, separates two regions of operation. When the load curve is below the diagonal track is maintained on all targets (as long as the revisit service time is less than T_{cell}). Above the diagonal line the system is unstable: the radars are overloaded and cannot keep track of all targets.

When the system load curve intersects the stability boundary, as in Fig. 3, the system is stable and the radars are fully occupied revisiting targets. In this regime the entropies of the tracking error distributions are allowed to grow to the very edge of the stability region. Because of this the radars have no time to spare for other tasks. For the radar parameters chosen, a maximum of 23 targets per radar can be tracked stably. The system is thus fully provisioned with radars and is at full occupancy ($\rho = 100\%$) for maintaining tracks with standard error of 202.8m.

When the load curve is always below the stability boundary, as in Fig. 4, the R radars easily maintain the target tracks but are not fully utilized, i.e., they are idle some of the time. For this case there are only 5 targets so the system is severely overprovisioned in number of radars. The sensor manager can correct this inefficiency by assigning some of the R radars to other tasks and operating those remaining at the full utilization point intersecting the stability line.

In Fig. 5 $N/R = 15$ and the radars can allow the uncertainty in target tracks ($\sigma = 290$m) to grow beyond the uncertainty guaranteed by the fully provisioned case and still maintain stability. However, the system would operate more efficiently if the number of radars were reduced and more fully utilized at the lower standard error ($\sigma = 202$m) of the fully provisioned case.

When the load curve is always above the stability boundary, as in Fig. 6, the R radars are overwhelmed and the entropies of some of the tracks will grow without bound. This can only be corrected by the sensor manager by increasing the number of radars so that the system is not underprovisioned

6.1.2 Just-in-time track-only radar provisioning

The results of Sec. 4.4.3 allow us to study the minimal stability-insuring radar provisioning requirements, i.e., number of radars, for different numbers of targets. To illustrate, for a radar with parameters given at the beginning of this section, the value of $q^{'}(0)$ is 0.042 and the solution b to (15) is 23.8. We know from Proposition 2 that this value of b would result in unstable operation. The fully provisioned system would assign a radar to every 23 targets that need be tracked.

For example for 2 radars and 46 targets the revisit rate for each target is 18.8 times per sec. and the occupancy of the system is 100%. Furthermore, at this operating point, in steady state the entropy of the tracking errors will grow no larger than $\ln|\mathcal{C}_\tau| = 15.9$ which corresponds to a maximal positional uncertainty on the order of $\sigma_z = |\mathcal{C}_\tau|^{-1/3} =$ 202m. This can be compared to the 150m positional resolution of the radar, the positional uncertainty attained after a revisit. Thus, the radar system allows the positional uncertainty to grow by only 33.3% between revisits.

Figure 7 shows the *provisioning matrix* whose (i, j) entry is equal to 1 if i radars can track j targets stably and equal to 0 otherwise. The matrix is represented using Matlab's `spy` command and the dark region of the matrix corresponds to stable operation.

6.2 Multipurpose radar provisioning

Figure 8 illustrates a computation of the excess capacity, occupancy, and load margin for the same radar as in the previous section but when it is tracking only 12 targets and can devote resources to other tasks. Unlike the case of 23 targets, for which the load curve is the upper curve that only intersects the diagonal line $y(u) = u - \Delta$ when $\Delta = 0$, there is a substantial load margin for the case of 12 targets, Δ_{max} of $0.176/N$ secs. At this full utilization operating point the radar devotes approximately 0.58 of its time to tracking and the rest of its time to other tasks. The distance between the upper and lower diagonal lines $y(u) = u$ and $y(u) = u - \Delta_{max}N$ is 0.176 secs. If the actual load for other tasks was set to only $\Delta = 0.05/N$ secs., giving an excess capacity $c_{excess} = 0.72$ and an occupancy of $\rho(\Delta) = 0.70$, the radar would be idle 30% of the time.

7 Conclusions

The results presented here were applied to sensor management for multiple target radar tracking subject to typical radar resource constraints. These results were based on finding solutions to load balance equations (Propositions 1 and 4) that guarantee system stability. These solutions yield the required system provisioning of radars, along with their associated steady state revisit rates, track entropy and occupancy, to guarantee stable tracking with prescribed level of statistical confidence.

The provisioning results given here are conservative and specify the system requirements, steady state occupancy, revisit times, and track entropy in terms of the PQL sensor scheduling policy. The PQL policy will always perform at least as well as the performance predictions we provide. One can expect considerably better performance of the system than these predictions for typical scenarios, although there exists a scenario (namely, all targets are equally difficult to track) where the predictions are exact. Less stringent provisioning requirements might be explored using a stochastic optimization, e.g., Wasserman's multiqueue analysis.

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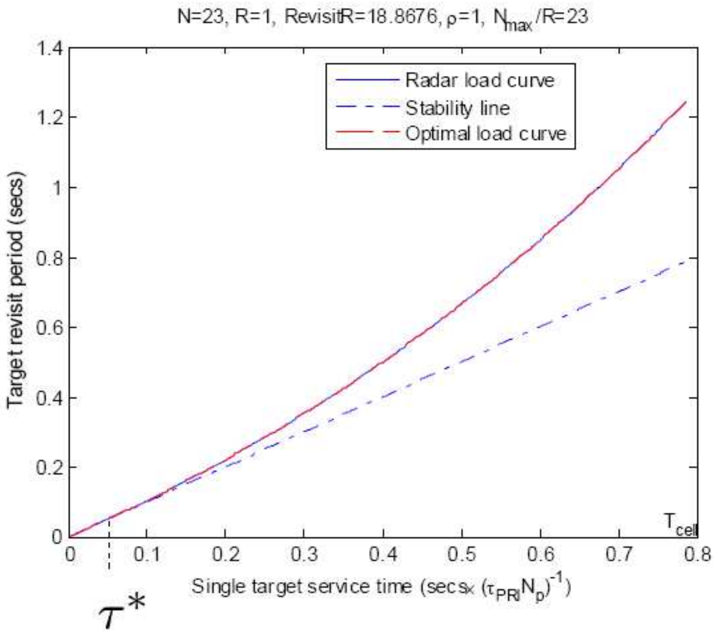

Figure 3: The system loading curve for N/R=23 for track-only radar, the maximum possible value for which the system is stable. The system is critically provisioned at the operating point $\tau = \tau^*$. At this operating point it maintains positional uncertainty of all targets at less than 202m at a revisit rate of 18.9 targets/s.

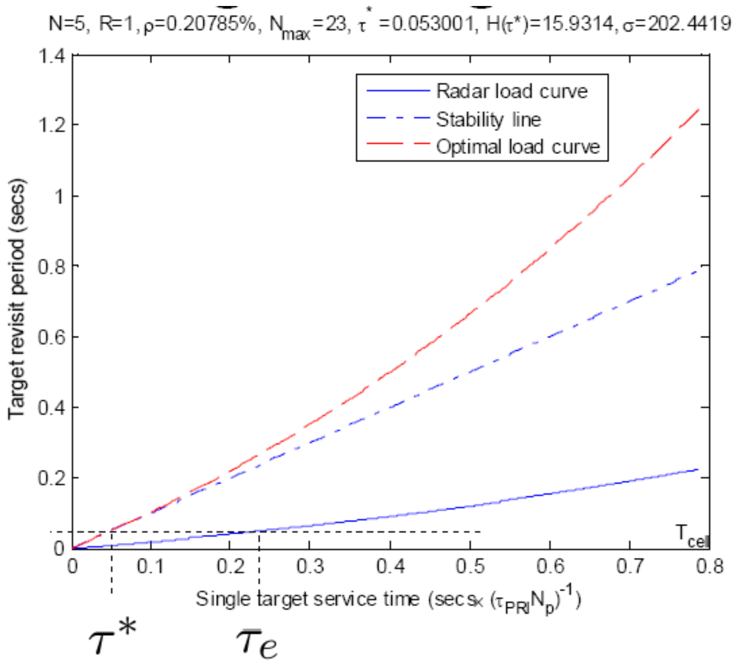

Figure 4: The system loading curve for the case of $N/R = 5$ for track-only radar. The system is stable but severely overprovisioned as compared to the critically stable case $N/R = 23$ shown in Fig. 3. Shown is the operating point τ_e where the overprovisioned system has same loading (revisit rate) as the critically provisioned system in the previous figure. The occupancy of the system is the ratio $\rho = \tau/\tau_e$.

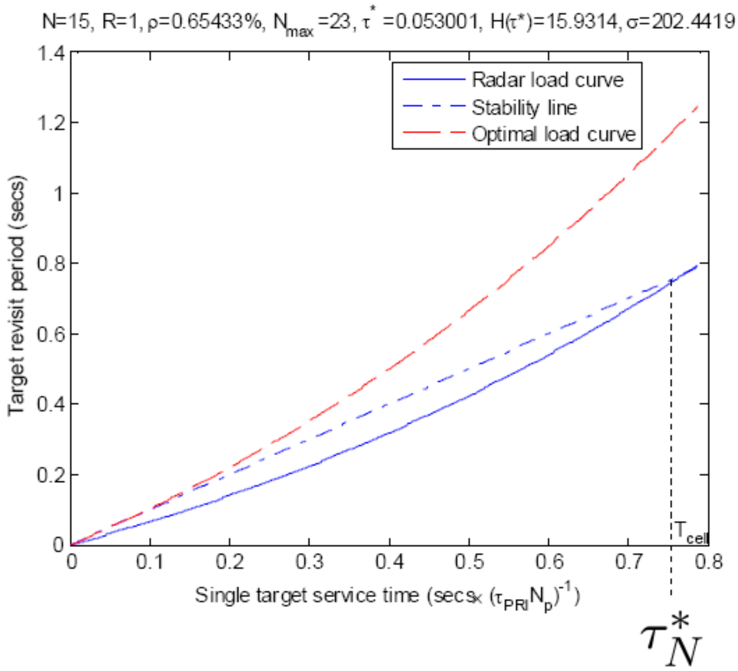

Figure 5: The system loading curve for the case of $N/R = 15$ for track-only radar. The system is stable, but is operating dangerously close to the exit time T_{cell} of the target in a cell. The system is overprovisioned for achieving the standard error $\sigma = 202$m of the fully provisioned system in Fig. 3. It reaches its critical stability limit for $\tau = \tau_N^*$ where target track uncertainties are allowed to grow to $\sigma = 290$m and track revisit rate is 1.3 targets/s.

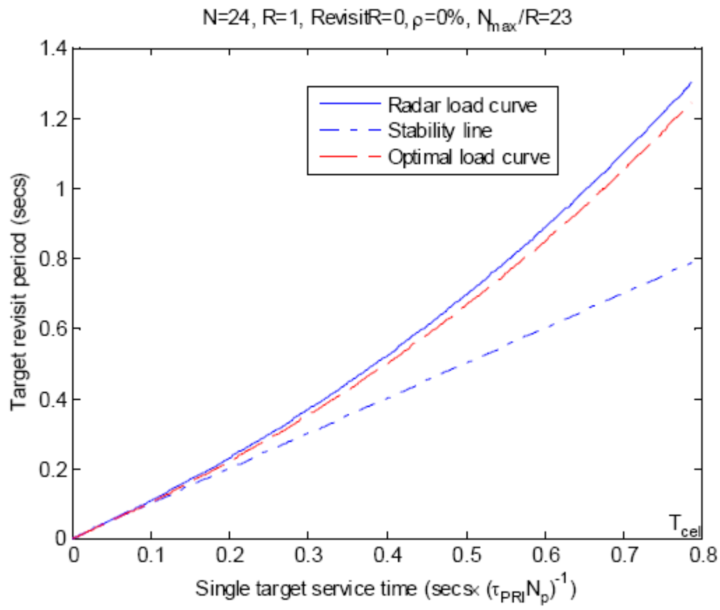

Figure 6: The system loading curve for $N/R = 24$ for track-only radar. The system is underprovisioned, overloaded and unstable and the number of radars is insufficient to keep track of all the targets for any revisit rates.

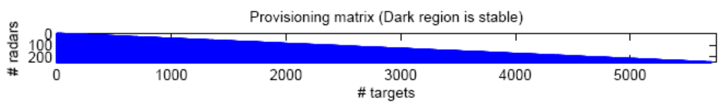

Figure 7: The system provisioning matrix specifies stability region (dark) as a function of the numbers of radars and the number targets for track-only radar.

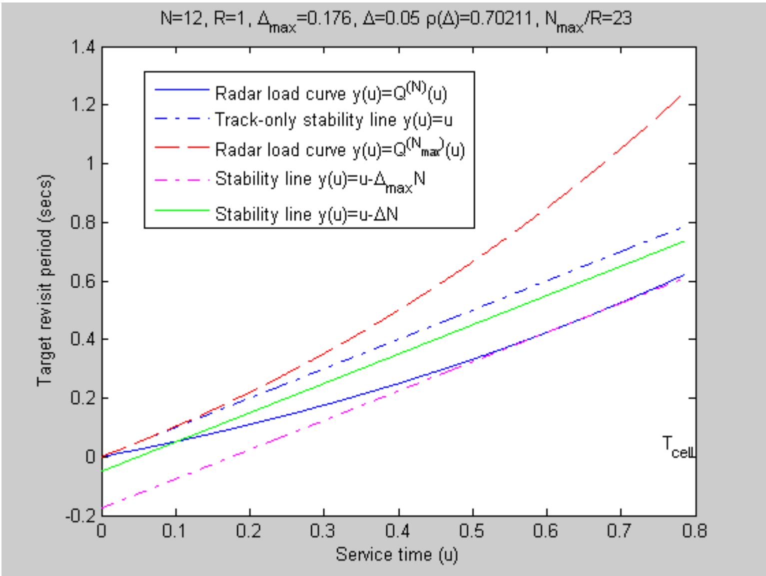

Figure 8: System loading curves for computing occupancy and excess capacity for the multipurpose radar tracking example. Fully loaded track-only radar load curve at top is shown for comparison.