Suboptimal Receivers in CDMA

Aditya Mahajan Ravi Agarwal

EECS 598-3: Digital Signal Processing and Analysis
Term Project

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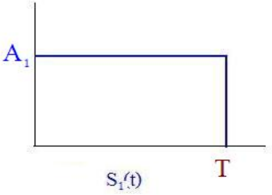
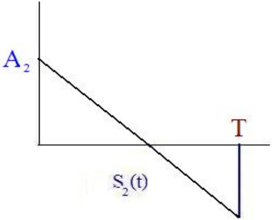
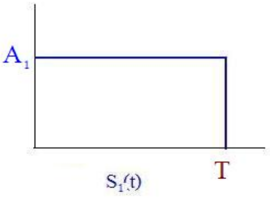


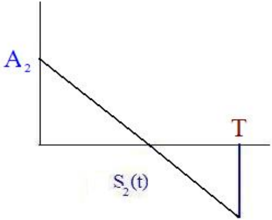


$$r(t) = b_1 s_1(t) + b_2 s_2(t)$$

Key Idea: Signature waveforms are orthogonal over $[0, T]$

$$\int_0^T r(t)s_1(t)dt = \int_0^T b_1 s_1(t) \times s_1(t)dt + \int_0^T b_2 s_2(t) \times s_1(t)dt$$
$$= b_1$$





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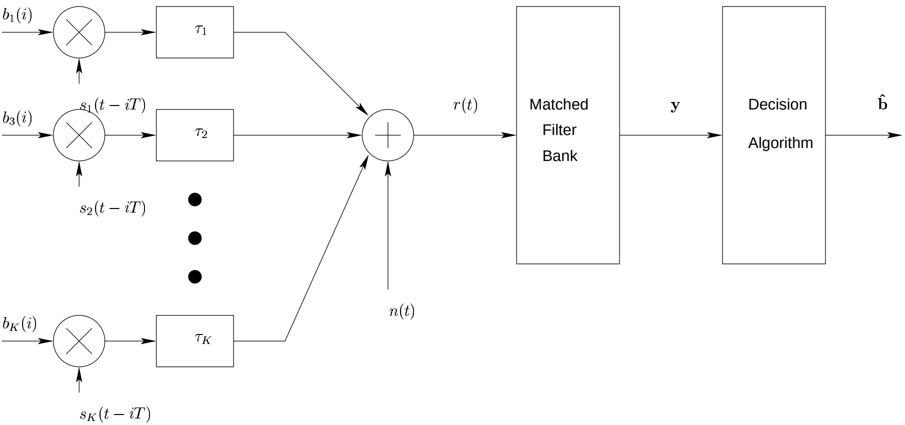
Typical CDMA System



Bit Waveform Matched Filter

Near Far Problem

- Same Received Signal Strength
- Different Transmitted Power
- Users near the base station face large MUI

Near Far Resistance

The ability to reject structured interference.

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Notation

- K Number of users
- M Number of bits to be transmitted
- N Number of chips per bit (processing gain)

Signal Space Representation

(Received Signal) $$r(t) = \sum_{k=1}^{K} A_k \sum_{i=1}^{M} b_k(i) s_k(t - iT - \tau_k) + n(t)$$

(Sig Waveform) $$s_k(t) = \sum_{j=0}^{N-1} \beta_j^k \psi(t - jT)$$

(MF O/P) $$\boldsymbol{y} = \boldsymbol{\beta_M} \boldsymbol{b} + \boldsymbol{n}$$

(Estimate) $$\hat{\boldsymbol{b}} = \boldsymbol{T}\boldsymbol{y}$$

MMSE Receiver

MMSE Receiver

A mapping $\boldsymbol{T} : \mathbb{R}^{MK} \times \mathbb{R}^{MK}$ so as to minimize $\boldsymbol{E}[(\boldsymbol{b} - \hat{\boldsymbol{b}})^{(\boldsymbol{T})}(\boldsymbol{b} - \hat{\boldsymbol{b}})]$

Solution

$$\boldsymbol{T} = \left(\beta_M + \frac{N_0}{2}\boldsymbol{I}\right)^{-1}$$

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Computation Complexity

- To detect MK transmitted bits requires $3MK^2$ multiplications.
- Thus complexity of detector is $3K$ multiplications/bit
- Complexity Linear in K and independent of M

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Limitation

- The receiver operates on the entire sequence at once.
- Large M implies unacceptable detection delay.
- A suboptimal scheme needed to ensure finite detection delay

Linear Equalizer

Constraint

MMSE detection problem, with a constraint that the detection delay, J is finite. $J \ll M$

(Edge Effects) $\boldsymbol{y} = \beta_{2J+1}\boldsymbol{b} + \boldsymbol{n} + \boldsymbol{Pz}$

Solution

$$\boldsymbol{T} = \boldsymbol{A} \cdot \left(\beta_{2J+1}^2 + \frac{N_0}{2}\beta_{2J+1} + \boldsymbol{PP}^T\right)^{-1}$$

$$\boldsymbol{A} = [0, \cdots, 0, \boldsymbol{H}(1), \boldsymbol{H}(0), \boldsymbol{H}(1)^T, 0, \cdots, 0]$$

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WLS Detector

Constraint

Minimizing the likelihood function $f(\boldsymbol{y}|\boldsymbol{b})$ by a linear operator. This is equivalent to minimizing

$$\Omega = (\boldsymbol{y} - \boldsymbol{\beta_M}\hat{\boldsymbol{b}})^T \beta_M^{-1}(\boldsymbol{y} - \boldsymbol{\beta_M}\hat{\boldsymbol{b}})$$

Solution

The vector that minimizes the above equation is given by

$$\hat{\boldsymbol{b}} = \beta_M^{-1}\boldsymbol{y}$$

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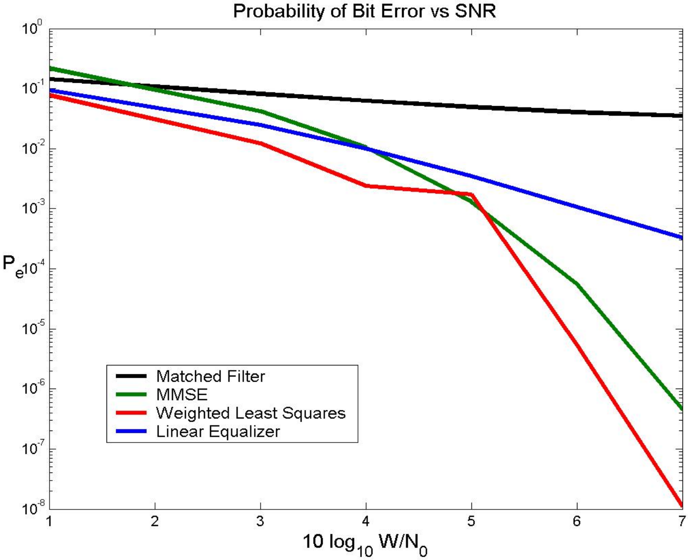
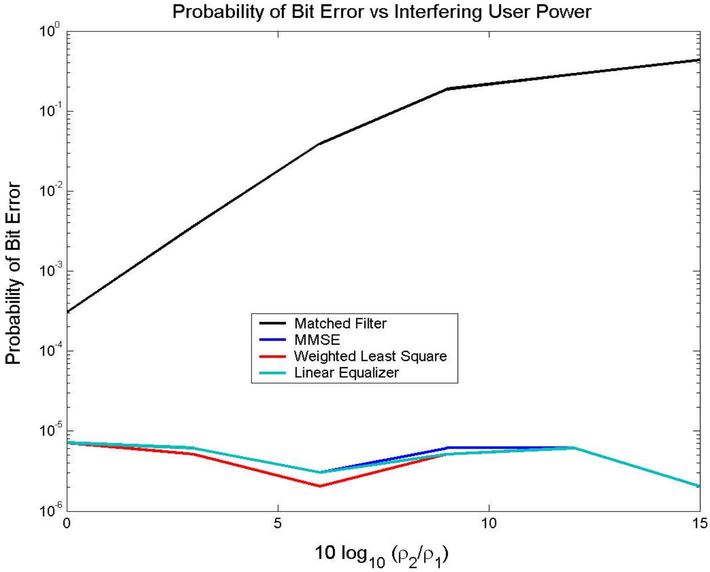

Probability of Bit Error vs SNR
P_e
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
1
2
3
4
5
6
7
$10 \log_{10} W/N_0$
Matched Filter
MMSE
Weighted Least Squares
Linear Equalizer


Probability of Bit Error vs Interfering User Power
Probability of Bit Error
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
0
5
10
15
10 log10 (ρ2/ρ1)
Matched Filter
MMSE
Weighted Least Square
Linear Equalizer

Conclusion

- Linear Suboptimal Receivers eliminate the Near-Far Problem

Limitations

- Assumes that the signature waveform and bit timing of all the other users is known
- Finding this information from training sequences not feasible due to the temporal nature of the channel
- Blind Adaptive Equalization needed

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Subspace Concept

(Chip MF O/P) $\boldsymbol{r} = \sum_{k=1}^{K} A_k b_k \boldsymbol{s_k} + \sigma \boldsymbol{n}$

Let, $\boldsymbol{S} = [\boldsymbol{s_1}, \boldsymbol{s_2}, \cdots, \boldsymbol{s_K}]$

$\boldsymbol{A} = \text{diag}(A_1^2, \cdots, A_K^2)$

(autocorrelation) $\boldsymbol{C} \triangleq E\{\boldsymbol{r}\boldsymbol{r}^T\} = \boldsymbol{S}\boldsymbol{A}\boldsymbol{S}^T + \sigma^2 \boldsymbol{I_N}$

(Eigen Value Decomposition)

$$\boldsymbol{C} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^T = \begin{bmatrix} \boldsymbol{U_s} & \boldsymbol{U_n} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Lambda_s} & \\ & \boldsymbol{\Lambda_n} \end{bmatrix} \begin{bmatrix} \boldsymbol{U_s}^T \\ \boldsymbol{U_n}^T \end{bmatrix}$$

$$\boldsymbol{S}\boldsymbol{A}\boldsymbol{S}^T = \boldsymbol{U_s}(\boldsymbol{\Lambda_s} - \sigma^2 \boldsymbol{I_K})\boldsymbol{U_s}^T$$

Subspace Based Detectors

WLS Receiver

$$\boldsymbol{T} = \beta_M^{-1}\boldsymbol{r}$$

Decorrelating Receiver

$$\boldsymbol{T} = \sum_{k=1}^{K} [\boldsymbol{S}^T\boldsymbol{S}]_{1k}^{-1}\boldsymbol{s}_k$$

$$\boldsymbol{T} = \frac{\boldsymbol{U}_s(\boldsymbol{\Lambda}_s - \sigma^2\boldsymbol{I}_K)^{-1}\boldsymbol{U}_s^T\boldsymbol{s}_1}{[\boldsymbol{s}_1^T\boldsymbol{U}_s(\boldsymbol{\Lambda}_s - \sigma^2\boldsymbol{I}_K)^{-1}\boldsymbol{U}_s^T\boldsymbol{s}_1]}$$

MMSE Receiver

$$\boldsymbol{T} = \left(\beta_M + \frac{N_0}{2}\boldsymbol{I}\right)^{-1}$$

Linear MMSE Receiver

$$\arg\min_{\boldsymbol{T}^T\boldsymbol{s}_1=1}\left[\mathbb{E}\{(A_1b_1 - \boldsymbol{T}^T\boldsymbol{r})^2\}\right]$$

$$\boldsymbol{T} = \frac{\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\boldsymbol{s}_1}{[\boldsymbol{s}_1^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\boldsymbol{s}_1]}$$

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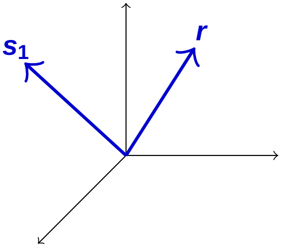
Explaination by Eigen Vectors

$$\hat{b} = \text{sgn}\left(\boldsymbol{s_1}^T \boldsymbol{r}\right)$$



$$\boldsymbol{C} = \boldsymbol{U\Lambda U}^T = \begin{bmatrix} \boldsymbol{U_s} & \boldsymbol{U_n} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Lambda_s} & \\ & \boldsymbol{\Lambda_n} \end{bmatrix} \begin{bmatrix} \boldsymbol{U_s}^T \\ \boldsymbol{U_n}^T \end{bmatrix}$$

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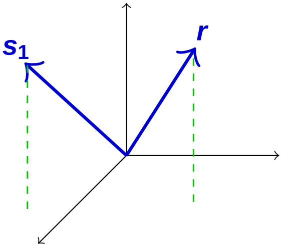
$$p_1 = {U_s}^T s_1$$



$$y = {U_s}^T s_2$$

$$C = U \Lambda U^T = \begin{bmatrix} U_s & U_n \end{bmatrix} \begin{bmatrix} \Lambda_s & \\ & \Lambda_n \end{bmatrix} \begin{bmatrix} {U_s}^T \\ {U_n}^T \end{bmatrix}$$

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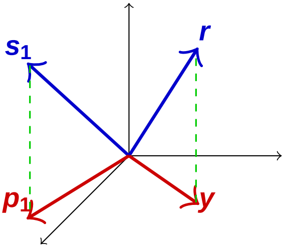

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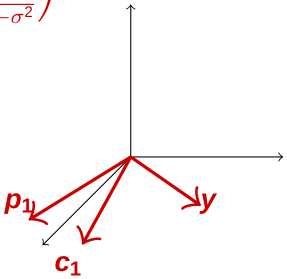
Explaination by Eigen Vectors

$$\mathbf{c}_1^d = \begin{pmatrix} \frac{1}{\lambda_1 - \sigma^2} & & \\ & \ddots & \\ & & \frac{1}{\lambda_K - \sigma^2} \end{pmatrix} \mathbf{p}_1 \qquad \mathbf{c}_1^m = \begin{pmatrix} \frac{1}{\lambda_1} & & \\ & \ddots & \\ & & \frac{1}{\lambda_K} \end{pmatrix} \mathbf{p}_1$$

$$\mathbf{p}_1 = \mathbf{U_s}^T \mathbf{s}_1 \qquad \mathbf{y} = \mathbf{U_s}^T \mathbf{s}_2$$

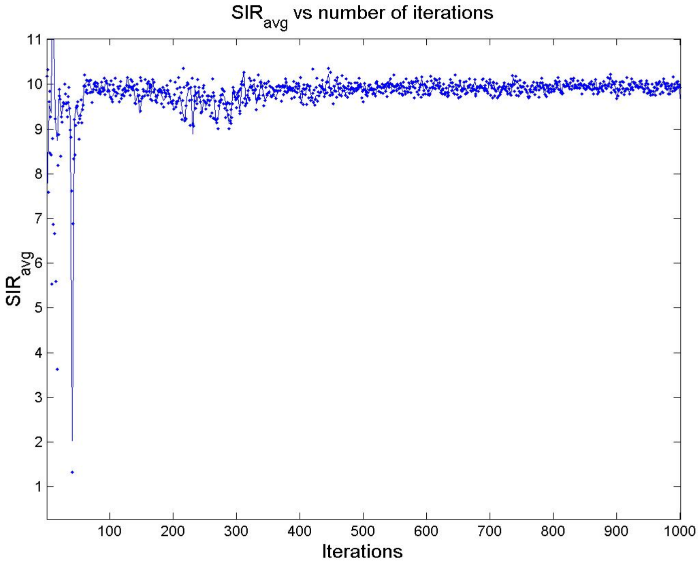


$$\hat{\mathbf{b}} = \operatorname{sgn}\left(\mathbf{c}_1^T \mathbf{y}\right)$$

Blind Estimation

Autocorrelation Matrix

- Need to estimate C to find its eigenvalues.
- Can be estimated blindly
  - batch eigenvalue decomposition
  - recursive adaptive schemes
- Almost sure convergence
- Low computational complexity
- Rank tracking capabilities


SIR$_{avg}$ vs number of iterations
11
10
9
8
7
6
5
4
3
2
1
SIR$_{avg}$
100
200
300
400
500
600
700
800
900
1000
Iterations

References

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