Multiuser Detection in DS-CDMA

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Contents

1 Introduction

Code Division Multiple Access (CDMA), as a spread spectrum technique has become widely accepted as the future of cellular systems. In an orthogonal DS-CDMA cellular system, multiple users are allowed to transmit simultaneously by assigning to them orthogonal signature waveforms. The orthogonality of these signature waveforms ensures that each user's signal can be extracted from its superposition with all the other signals at the receiver end. However, theoretical limits on the number of orthogonal sequences places an upper bound on the number of users that can be supported by such a system. Also practically, the inherent nature of the wireless channel introduces multi-path fading and delays, which destroy the orthogonality of these signature waveforms. In such a scenario, it is often desirable to use signature waveforms with low cross correlation's (almost orthogonal) to ensure minimum degradation in performance.

The role of multi-user detection becomes increasingly crucial in non-orthogonal CDMA systems. A communication system can achieve significant capacity gains if the negative effect of Multiple-User Interference (MUI) can be removed. In multi-user detection we exploit the correlation properties of the signature waveforms of different users to extract the desired signal(s). Then instead of acting as interference, the other users' signals are used for mutual benefit by implementing joint detection at the receiver. The optimal ML receiver for multi-user detection was found by Verdu in [1]. However, this receiver has exponential complexity per bit and thus is not feasible for implementation in a real time system.

Since then, a wide range of multi-user receivers have been proposed as a trade-off between complexity and optimality. In this pursuit, the criteria of optimality have varied from ML sequence estimation to MMSE to marginally ML optimal. An important measure of any multi-user detector is its near-far resistance. In a situation when there is a large variation in the energies of the signals received from different users, the detection of relatively lower energy signals becomes increasingly erroneous even in high SNR regions due to multiple-user interference. Near-far resistance is defined as the ability of a receiver to reject the structured interference offered by the other users. A conventional multi-user detector does not account for the structure of the MUI, considering it as just background noise and does not have good near far resistance. Improved detectors have been suggested in literature, see [2], [3], [8], having higher near far resistance and whose performance is comparable to that of the optimal ML detector. A popular class of linear suboptimal detectors was introduced by Lupas [2] and independently by Xie et. al. [3]. These linear detectors offer high near-far resistance and their complexity is also linear in the number of users. We recommend the reader to [4] for a comprehensive reading on the subject.

For multi-user detection, a common assumption is that the receiver has the complete knowledge of

1) The signature waveform of the desired user.

2) The signature waveform of the interfering users.

3) The timing(bit epochs) of the desired user.

4) The timing(bit epoch) of each of the interfering user.

5) The received amplitudes of the interfering users(relative to that of the desired user).

In linear detectors it is assumed that the receiver knows all of these. This limits the user of linear receiver to the reverse link, where the base station has complete knowledge about the signalling characteristics of all its users. However, the situation is quite different of the forward link. The mobile receiver has no information about the signature waveforms of the other users, so it must acquire them. Such detectors are called adaptive detectors. Initially adaptive detectors substitute the need to know 2), 4) and 5) by the need to have

6) Training sequence data for each user.

The training sequence is known apriori and is used by the receiver for the initial adaptation process. However, any change in the channel conditions during transmission implies that the adaptation would have to be made in a decision directed mode by the receiver, which becomes highly unreliable. This motivated the need for a blind adaptive multiuser detection scheme for CDMA systems, which requires no more than 2) and 4). Blind-adaptive estimation was first introduced in [**?**]. In this report, we focus on a particular class of sub-optimal receivers namely, Linear Multi-user detectors. Our aim is to summarize the vast literature available on this subject. We will also explain a signal subspace based method for blind adaptive estimation proposed by [7]. Finally, through simulation, we present a comparative study of the various receivers within this class of linear suboptimal detectors and their blind implementation.

bibliographic notes: It was first shown by Verdu [1] that significant gains can be achieved by implementing joint multi-user detection instead of conventional single-user matched filter at the receiver. Since then the field of multi-user detection has been at the forefront of research related to multiple access communications. This is a survey paper and our original contribution here is modest- throughout the paper, the form of address 'we' is meant conversationally to suggest us and the reader.

2 CDMA System Model

The typical CDMA system is shown in Fig. 1. There are K users in the system, transmitting digitally modulated signatures waveforms with anti-podal signaling {-1,+1}. The bit duration of each of the users is T, the chip duration is T_c. Thus $N = T/T_c$ is the processing gain of each user. Basic differences in the nature of interference in the case of Synchronous and Asynchronous CDMA systems motivate the use of different representation in each case, although the basic underlying model and notation remain the same.

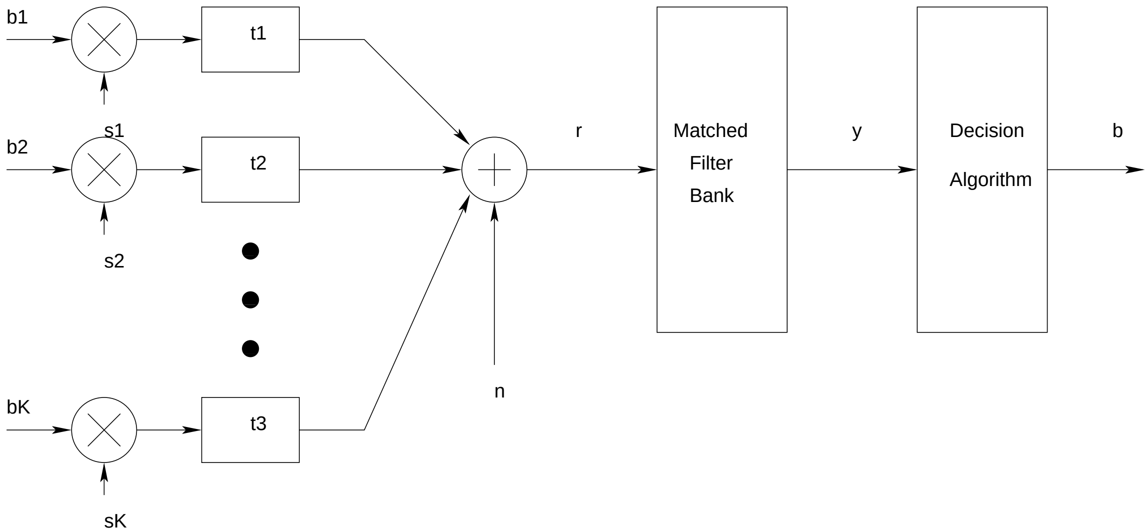

Figure 1: A typical CDMA system

2.1 Asynchronous CDMA model

The baseband received signal $r(t)$ can be written as

$$r(t) = \sum_{k=1}^{K} A_k \sum_{i=1}^{M} b_k(i) s_k(t - iT - \tau_k) + n(t) \tag{1}$$

where,

$$s_k(t) = \sum_{j=0}^{N-1} \beta_j^k \psi(t - jT_c) \qquad t \in [0, T] \tag{2}$$

is the signature waveform of the k^{t} user with β_j^k as the corresponding chip sequences, $n(t)$ is gaussian noise with power spectral density $N_0/2$, M is the number of bits transmitted by each user, $b_k(i) \in \{-1, +1\}$ is the i^{th} transmitted bit of the k^{th} user and τ_k is the delay of the k^{th} user. Without loss of generality the users are assumed to be ordered such that $0 \leq \tau_1 \leq \tau_2 \leq \cdots \tau_K < \tau$.

The received signal is passed through a matched filter bank, and the output can be represented in vector form as

$$\boldsymbol{y} = \beta_M \boldsymbol{b} + \boldsymbol{n} \tag{3}$$

where $\boldsymbol{y}$, $\boldsymbol{b}$, $\boldsymbol{n}$ is the row stacked version of the all the received bits, transitted bits and the noise respectively, i.e.

$$\boldsymbol{y} = [y_1(1), y_2(1) \cdots y_K(1), y_1(2), y_2(2) \cdots y_K(2) \cdots]^T \tag{4}$$

and so on. β_M is the block correlation matrix i.e

$$\beta_M = \begin{bmatrix} \boldsymbol{H}(0) & \boldsymbol{H}(1)^T & \mathbf{0} & \cdots & \mathbf{0} & \\ \boldsymbol{H}(1) & \boldsymbol{H}(0) & \boldsymbol{H}(1)^T & \mathbf{0} & \cdots & \mathbf{0} \\ & & \vdots & & & \\ \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{H}(1) & \boldsymbol{H}(0) & \boldsymbol{H}(1)^T \\ \mathbf{0} & \cdots & & \mathbf{0} & \boldsymbol{H}(1) & \boldsymbol{H}(0) \end{bmatrix} \tag{5}$$

where,

$$H_{i,j}(m) = \int_{-\infty}^{\infty} s_i(t-\tau_i)s_j(t+mT-\tau_j)dt \tag{6}$$

2.2 Synchronous CDMA Model

Setting $\tau_1 = \tau_2 = \cdots = \tau_K = 0$, we get a model for Synchronous CDMA system. Here, the symbol boundaries of each of the user's is aligned, hence it is sufficient to consider just one symbol interval and the received signal model becomes

$$r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + n(t) \qquad t \in [0,T] \tag{7}$$

Notice here that we have removed the outer summation in (1) and restricted the interval of observation to $[0,T]$ corresponding to just one bit duration. Other than this everything remains the same to Section 2.1. In the analysis for blind adaptive detectors, we will be using chip waveform matched filtering, whose output is given by

$$\boldsymbol{r} = \sum_{k=1}^{\infty} A_k b_k \boldsymbol{s}_k + \boldsymbol{n} \tag{8}$$

3 Family of Linear Suboptimal Multi-user Detectors

In this section we concentrate on of linear multi-user detectors for asynchronous CDMA, whose system equation is characterized by (3). Any linear detector can be characterized by two operations. First, a linear estimate $\tilde{\boldsymbol{b}} = T\boldsymbol{y}$ of the transmitted bits is obtained from the matched filter outputs. Then, the components of the estimate $\tilde{\boldsymbol{b}}$ are compared to a threshold of zero and a bit decision is taken to be the corresponding element of $\hat{\boldsymbol{b}} = \operatorname{sgn} \tilde{\boldsymbol{b}}$. The linear transformation $T : R^{MK} \to R^{MK}$, is chosen so as optimize a performance criteria such as MSE $= \mathbf{E}[(\hat{\boldsymbol{b}} - \tilde{\boldsymbol{b}})^T(\hat{\boldsymbol{b}} - \tilde{\boldsymbol{b}})]$ or the likelihood function $f(\boldsymbol{y}|\boldsymbol{b})$.

ML Optimal Matched Filter Detector

The objective of maximum-likelihood sequence estimation is to find the input sequence which maximizes the conditional probability, or the likelihood of the given output sequence [5]. It was shown by Verdú [1], that the K-user Maximum-Likelihood (ML) sequence detector consists of bank of single-user matched filters followed by a Viterbi Algorithm. The vector $\boldsymbol{y}$ obtained from the matched filter bank output (see (3)) has dimension MK and optimal detection requires determining each of the MK components at once, incurring in a complexity of $\mathrm{O}(2^K)$. Although the optimal detector has excellent performance, it is too complex for practical implementation.

3.1 Conventional Detector or Matched Filter Receiver

The conventional multi-user detector uses the same approach as the optimal detector for a single user case. It detects the bit streams by passing the received signal through a matched filter bank. The result is then hard limited $\{+1, -1\}$ to find the corresponding bit of the users. The bit estimate can be written as, $\hat{\boldsymbol{b}} = \mathrm{sgn}(\boldsymbol{y})$. This is the simplest possible receiver where the linear mapping $T = I$, the identity map. The conventional matched filter receiver treats multiple user interference as background noise and does not take into account the structure of the MUI to enhance the performance.

3.2 Linear Minimum Mean Squared Error(MMSE) Detector

As the name suggests, the MMSE detector is the linear mapping which minimizes the mean squared error criteria. Formally, the problem can be stated as follows:

$$\min_{\boldsymbol{T}} \mathbf{E}(\hat{\boldsymbol{b}} - \boldsymbol{b})^T(\hat{\boldsymbol{b}} - \boldsymbol{b}) \tag{9}$$

where $\boldsymbol{T} : R^{MK} \to R^{MK}$, $\hat{\boldsymbol{b}} = \boldsymbol{T}\boldsymbol{y}$. The mapping that achieves this minimum is

$$\boldsymbol{T} = \left(\beta_M + \frac{N_0}{2}\boldsymbol{I}\right)^{-1} \tag{10}$$

The above is equivalent to solving the set of linear equations given by

$$\left(\beta_M + \frac{N_0}{2}\boldsymbol{I}\right)\hat{\boldsymbol{b}} = \boldsymbol{y} \tag{11}$$

a process which can be carried out efficiently using LU decomposition [10, pp. 111]. It can be shown that to detect MK bits, this method requires $3MK^2$ multiplications incurring a complexity of $3K$ multiplications per bit, which is linear in K and independent of the transmission length M. The linear MMSE receiver is computationally much more viable as compared to the optimal ML detector, but detection of a bit stream requires waiting for the entire length of the transmission before the first decision can be made. In case of long transmissions, the large latency of MMSE detector makes it practically unacceptable.

3.3 Linear Equalizer

The linear equalizer is a suboptimal implementation of the MMSE detector with the constraint that the detector delay be finite. Here, the sequence of M data bits is divided into subsequences of length $J \ll M$. Each of the subsequences can detected separately incurring a delay of only J symbols periods. Mathematically, the problem can be stated as follows:

$$\min_{\boldsymbol{T}} \mathbf{E}(\hat{\boldsymbol{b}} - \boldsymbol{b})^T(\hat{\boldsymbol{b}} - \boldsymbol{b}) \tag{12}$$

where $\boldsymbol{T} : \boldsymbol{R}^{(2J+1)K} \rightarrow \boldsymbol{R}^K$ and $\hat{\boldsymbol{b}} = \boldsymbol{T}\boldsymbol{y}$.

The $(2J+1)K$-dimensional matched filter output vector $\boldsymbol{y}$ is defined as

$$\boldsymbol{y} = [\boldsymbol{y}(i-J)^T, \boldsymbol{y}(i-J+1)^T, \cdots, \boldsymbol{y}(i+j)^T]^T \tag{13}$$

For $i + J < M$ and $i - J > 0$, $\boldsymbol{y}$ can be expressed as

$$\boldsymbol{y} = \beta_{2J+1}\boldsymbol{b} + \boldsymbol{n} + \boldsymbol{P}\boldsymbol{z} \tag{14}$$

where,

$$\boldsymbol{b} = [\boldsymbol{b}(i-J)^T, \boldsymbol{b}(i-J+1)^T, \cdots, \boldsymbol{b}(i+J)^T]^T \tag{15}$$

$$\boldsymbol{z} = [\boldsymbol{b}(i-J-1)^T, 0, \cdots, 0, \boldsymbol{b}(i+J+1)^T] \tag{16}$$

β_{2J+1} is as defined in (5) with proper dimensions, and

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{H}(1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{H}(1)^T \end{bmatrix} \tag{17}$$

The term $\boldsymbol{P}\boldsymbol{z}$ in (14) is a consequence of the edge effects associated with previous and future bits outside the observation window. The mapping that achieves the minimum in (12) is

$$\boldsymbol{T} = \boldsymbol{A} \cdot \left(\boldsymbol{\beta^2_{2J+1}} + \frac{\boldsymbol{N_0}}{\mathbf{2}} \boldsymbol{\beta_{2J+1}} + \boldsymbol{P}\boldsymbol{P^T} \right)^{-\mathbf{1}} \tag{18}$$

$$\boldsymbol{A} = [\mathbf{0}, \cdots, \mathbf{0}, \boldsymbol{H}(\mathbf{1}), \boldsymbol{H}(\mathbf{0}), \boldsymbol{H}(\mathbf{1})^{\boldsymbol{T}}, \mathbf{0}, \cdots, \mathbf{0}] \tag{19}$$

which is well defined since the matrix in parentheses is positive definite. The linear equalizer can be seen as moving window implementation of the MMSE detector where the window operates on $(2J+1)$ blocks of matched filter outputs rather than the entire sequence of M blocks. It is clear that the performance of the linear equalizer improves as J increases. For the case of $J = 1$, the linear equalizer is also referred to as *Least Mean Square Error detector*.

3.4 Weighted Least Square Detector(WLS)

Till now we have focused on the MMSE as the performance criteria. One consequence of this is that the detectors obtained were biased estimates of the transmitted bits. An alternative performance criteria to be maximized can be the likelihood function $f(\boldsymbol{y}|\boldsymbol{b})$. As can be seen from (3), the received vector $\boldsymbol{y}$ is Gaussian with mean $\beta_M \boldsymbol{b}$ and variance equal to $(N_0/2)\beta_M$. Thus maximizing the likelihood function is equivalent to minimizing the exponent in the joint Gaussian density of the vector $\boldsymbol{y}$. Formally, the problem can be stated as follows

$$\min_{\boldsymbol{T}} \left\{ \left(\boldsymbol{y} - \beta_M \hat{\boldsymbol{b}} \right)^T \beta_M^{-1} \left(\boldsymbol{y} - \beta_M \hat{\boldsymbol{b}} \right) \right\} \tag{20}$$

where $\hat{\boldsymbol{b}} = \boldsymbol{T}\boldsymbol{y}$. The minimum is acieved by taking

$$\hat{\boldsymbol{b}} = \beta_M^{-1} \boldsymbol{y} \tag{21}$$

Once again the problem involves solving the set of linear equations $\beta_M \hat{\boldsymbol{b}} = \boldsymbol{y}$. Substituting for $\boldsymbol{y}$ from (3), we see that

$$\hat{\boldsymbol{b}} = \boldsymbol{b} + \beta_M^{-1} \boldsymbol{n} \tag{22}$$

Notice that the preliminary estimate $\hat{\boldsymbol{b}}$ of the transmitted bits is unbiased, hence independent of the interfering signal powers. Thus multi-user interference has been completely removed as the expense of enhancing the noise power. For this reason, the WLS detector is also referred to as the *decorrelating detector*. In this sense, the WLS detector is asymptotically optimum as the noise power is made zero and provides near far resistance similar to the ML optimal detector. However, the delay associated with the WLS detector is unacceptably high since it operates on the entire transmitted sequence at once.

4 Blind Adaptive Multi-user Detectors

As explained in the introduction of the report, the linear detectors illustrated in Section 3 assume the complete knowledge about all the users in the system, which is in general not satisfied. For the downlink situation, we need to consider blind adaptive receivers. In this section we will first develop the signal subspace model and then explain how the linear detectors can be modified to get corresponding detectors with blind adaptation. In this section we consider synchronous CDMA model of Section 2.2. This is not a limitation as it has been shown in [8] that an asynchronous K user system can be viewed equivalent to a synchronous system with $2K-1$ users.

4.1 A subspace approach

The N-dimensional vector of chip waveform matched filter output is given by (7). Without loss of generality we can assume that the signature waveforms $\{\boldsymbol{s}_k\}_{k=1}^K$ of the K users are linearly independent. Let,

$$\boldsymbol{S} = [\boldsymbol{s}_1, \boldsymbol{s}_2, \cdots, \boldsymbol{s}_K] \tag{23}$$

$$\boldsymbol{A} = \mathrm{diag}(A_1^2, A_2^2, \cdots, A_K^2) \tag{24}$$

The autocorrelation matrix od the received signal $\boldsymbol{r}$ is then given by

$$\boldsymbol{C} = \mathbf{E}\{\boldsymbol{r}^T \boldsymbol{r}\} \tag{25}$$

$$= \sum_{k=1}^{K} A_k^2 \boldsymbol{s}_k \boldsymbol{s}_k^T + \sigma^2 \boldsymbol{I} \tag{26}$$

$$= \boldsymbol{S}\boldsymbol{A}\boldsymbol{S}^T + \sigma^2 \boldsymbol{I} \tag{27}$$

where σ^2 is the noise variance. By performing an eigenvalue decomposition of the matrix $\boldsymbol{C}$, we get

$$\boldsymbol{C} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^T = \begin{bmatrix} \boldsymbol{U}_s & \boldsymbol{U}_n \end{bmatrix} \begin{bmatrix} \boldsymbol{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}_n \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_s^T \\ \boldsymbol{U}_n^T \end{bmatrix} \tag{28}$$

where $\boldsymbol{\Lambda}_s = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_K)$ contains the K largest eigenvalues of $\boldsymbol{C}$ in descending order and $\boldsymbol{U}_s = [\boldsymbol{u}_1, \cdots, \boldsymbol{u}_K]$ contains the corresponding orthonormal eigenvectors. $\boldsymbol{\Lambda}_n = \sigma^2 \boldsymbol{I}_{N-K}$ and $\boldsymbol{U}_n = [\boldsymbol{u}_{K+1}, \cdots, \boldsymbol{u}_N]$ contain the $N-K$ orthonormal eigenvectors that correspond to the eigenvalue σ^2.

Similar to the development in the previous section, a linear multiuser detector for the k^{th} user data stream is of the form of a linear operator followed be a hard limiter $\hat{b}_k = \mathrm{sgn}(\boldsymbol{T}_k^T r)$ where k is the user of interest.

WLS (Decorrelating) detector using Subspace approach

It is shown in [7] , that the WLS detector in terms of the signal subspace parameters is given by

$$\boldsymbol{T}_k = \frac{k}{[\boldsymbol{s_k^T U_s (\Lambda_s - \sigma^2 I_K)^{-1} U_s^T s_k}]} \boldsymbol{U_s (\Lambda_s - \sigma^2 I_K)^{-1} U_s^T s_k} \tag{29}$$

Linear MMSE detector using Subspace Approach

The linear MMSE detector for user k, is a the linear map $T_k \in R^N$ chosen so as to minimize

$$\mathrm{MSE} = \mathbf{E}\left\{(A_k b_k - \boldsymbol{T}_k^T \boldsymbol{r})^2\right\} \tag{30}$$

subject to $\boldsymbol{T}_k^T \boldsymbol{s}_k = 1$.

The detector T_k is given in terms of the signal subspace parameters by

$$\boldsymbol{T}_k = \frac{k}{[\boldsymbol{s_k^T U_s \Lambda_s^{-1} U_s^T s_k}]} \boldsymbol{U_s \Lambda_s^{-1} U_s^T s_k} \tag{31}$$

We refer the reader to [7] for a proof of the same.

Notice that the linear transformations (29) and (31) depend only the eigenvalues and corresponding eigenvectors constituting the signal subspace spanned by the signature waveforms of the K users. Thus, to be able to do blind detection the receiver must find out the eigenvalues and eigenvectors of the signal subspace. In general, the receiver will have to follow some signal subspace tracking algorithm to find out these eigenvectors and eigenvalues.

Note: If the signature waveforms of the interfering user's were known apriori then this becomes a trivial exercise as the signature vectors form the basis of the signal subspace (see (27)).

Tracking the signal subspace

As stated above, central to the formulation of the subspace based linear multiuser detectors, are the eigenvalues and the eigenvectors of the autocorrelation matrix $\boldsymbol{C}$ of the received vector $\boldsymbol{r}$. The rank $\boldsymbol{K}$ of the matrix $\boldsymbol{C}$ determines the number of users currently sharing the channel and the $\boldsymbol{K}$ largest eigenvalues and corresponding eigenvectors determine the realization of both the Decorrelating and MMSE detectors.

The authors of [7] have adopted the PASTd algorithm [9] (Projection Approximation Subspace Tracking with Deflation) for blind adaptive multiuser detection application. Advantages of this algorithm include almost sure convergence to the signal eigenvectors and eigenvalues, low computational complexity (O(NK)) and the rank tracking capability. We present the PASTd algorithm to determine the signal subspace in Table 1. For a detailed reading on the subject we refer the reader to [9]

Table 1: The PASTd Algorithm for updating the eigenvalues and eigenfunctions

	$\boldsymbol{x}_1(t)$	$=$	$\boldsymbol{v}(t)$
FOR	k=1:K		DO
	$\boldsymbol{y}_k(t)$	$=$	$\boldsymbol{u}_k^T(t-1)\boldsymbol{x}_k(t)$
	$\lambda_k(t)$	$=$	$\alpha\lambda_k(t-1) + \lvert y_k(t)\rvert^2$
	$\boldsymbol{u}_k(t)$	$=$	$\boldsymbol{u}_k(t-1) + [x_k(t) - \boldsymbol{u}_k(t-1)y_k(t)]y^*(t)/\lambda_k$
	$x_{k+1}(t)$	$=$	$x_k(t) - \boldsymbol{u}_k(t)y_k(t)$
END			
	$\sigma^2(t)$	$=$	$\alpha\sigma^2(t-1) + \lVert x_{K+1}(t)\rVert^2/(N-K)$

5 Simulation

In this section we give Monte Carlo simulation results for the performance of linear receivers in the presence of MUI and thermal noise. We also present the performance of blind adaptive receivers, showing how efficiently they track the signal subspace.

5.1 Linear Detectors

We perform two simulations to show that the linear receivers are effective in over coming the near far problem. In the first case, we consider the bit error probabilities (BER) of different receivers as a function of the signal-to-noise ratio (SNR) for $K = 18$. Pseudorandom noise (PN) sequences of length $N = 127$ are used as the signature waveforms of the users, the bit epochs of all the users are aligned and all users have same power. The results are shown in Fig. 2. As we can see the MMSE, Linear Equalizer and WLS receivers perform much better than the conventional receiver, as can they decorrelate the interference caused by other users. This in turn implies that their near-far resistance is better than that of the conventional matched filter. Also note that the linear equalizer performs slightly worse than the linear MMSE detector as is expected. The WLS detector, which is a linear implementation of the MAP receiver performs the best.

We show that the proposed receivers are efficient in eliminating the near-far problem by simulating a two user CDMA system. The cross-correlation coefficients are taken to be $\bar{H}_{1,2}(0) = 0.3$ and $\bar{H}_{1,2}(1) = 0.1$, ie.

$$\boldsymbol{H}(0) = \begin{bmatrix} 1 & 0.3 \\ 0.3 & 1 \end{bmatrix} \quad \text{and} \quad \boldsymbol{H}(1) = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix} \tag{32}$$

The SNR for user 1 is kept constant at 5 dB while the relative signal power of user 2 is varied. The results are included in Fig. 3. It can be seen that BER of user 1 does not change to any appreciable extent with increase in the signal power of user 2 for the MMSE, Linear Equalizer and WLS detectors. Moreover, the BER is comparable to that of just a single-user system with SNR 5dB. We can conclude that these receivers are able to reject the MUI and the perform as if only thermal noise were present in the system.

5.2 Blind Adaptive Receivers

A useful performance measure for blind adaptive schemes is average signal-to-interference ratio, introduced in [6], which is defined as

$$\text{SIR} = \frac{\mathbf{E}\{\boldsymbol{T}_k^T \boldsymbol{r}\}}{\text{var}\{\boldsymbol{T}_k^T \boldsymbol{r}\}} \tag{33}$$

where the expectation is with respect to the data bits of MUI's and the noise. For the simulation we calculate the expected value by averaging over different simulation runs,

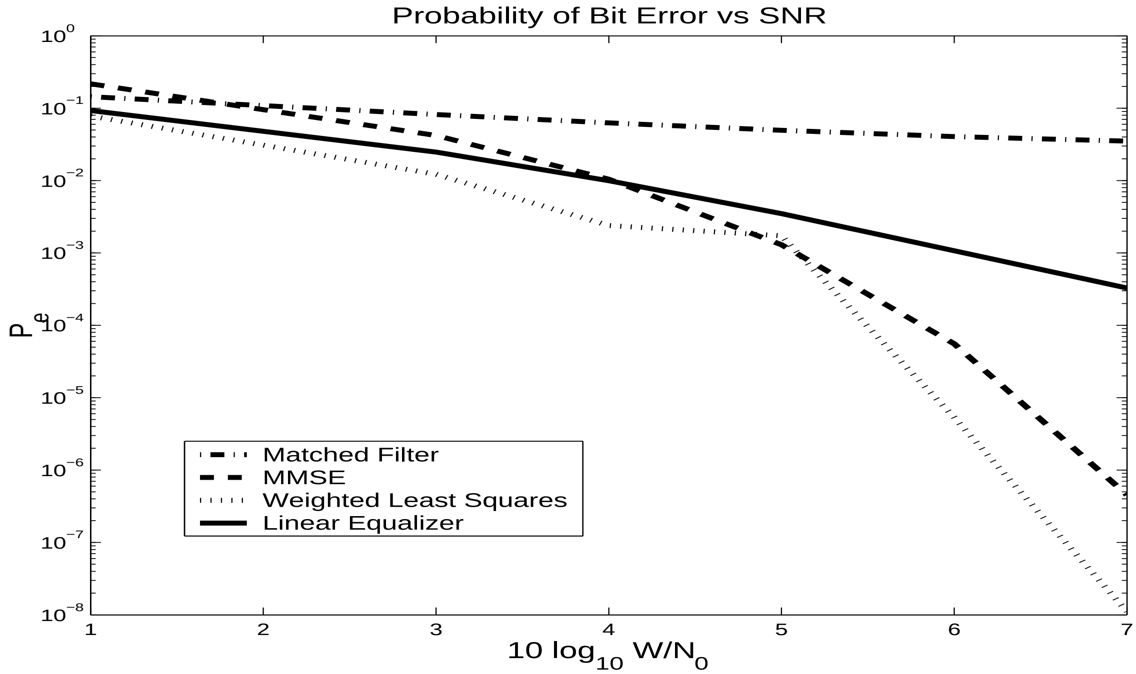

Figure 2: BER vs SNR for conventional matched filter, MMSE detector, linear detector and WLS receiver for $K = 18$ and $N = 127$

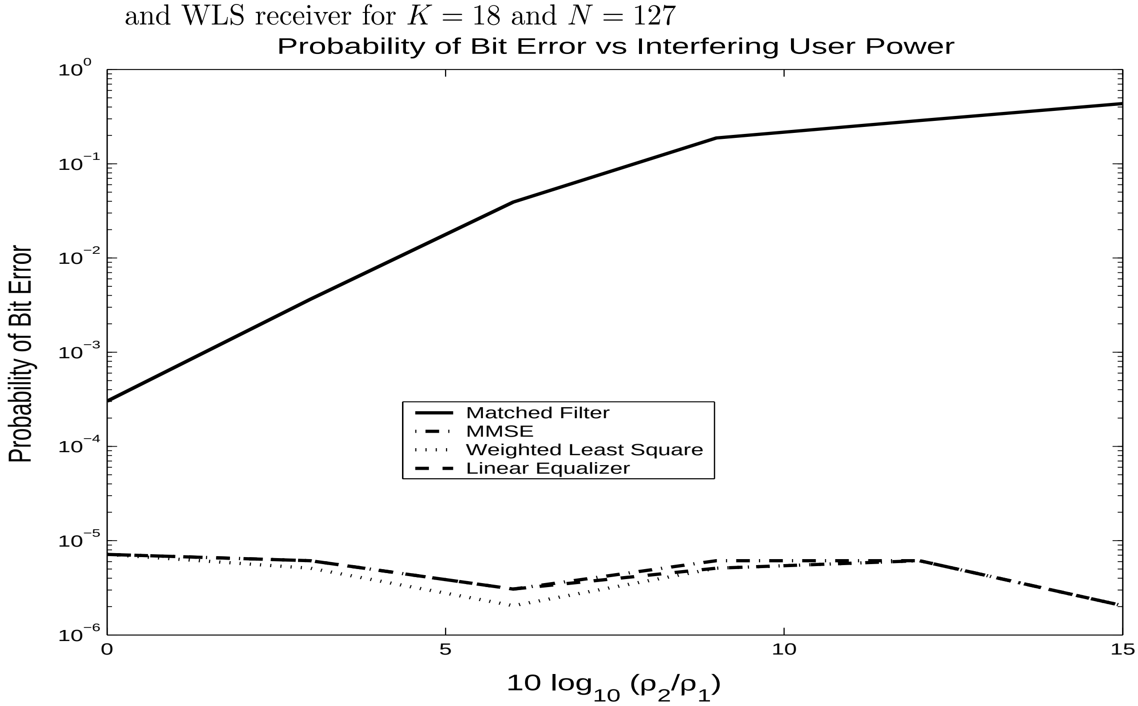

Figure 3: BER vs interfering user's power, for conventional matched filter, MMSE detector, linear detector and WLS receiver with $K = 2$, $N = 5$ and $w_1/N_0 = 5\text{dB}$

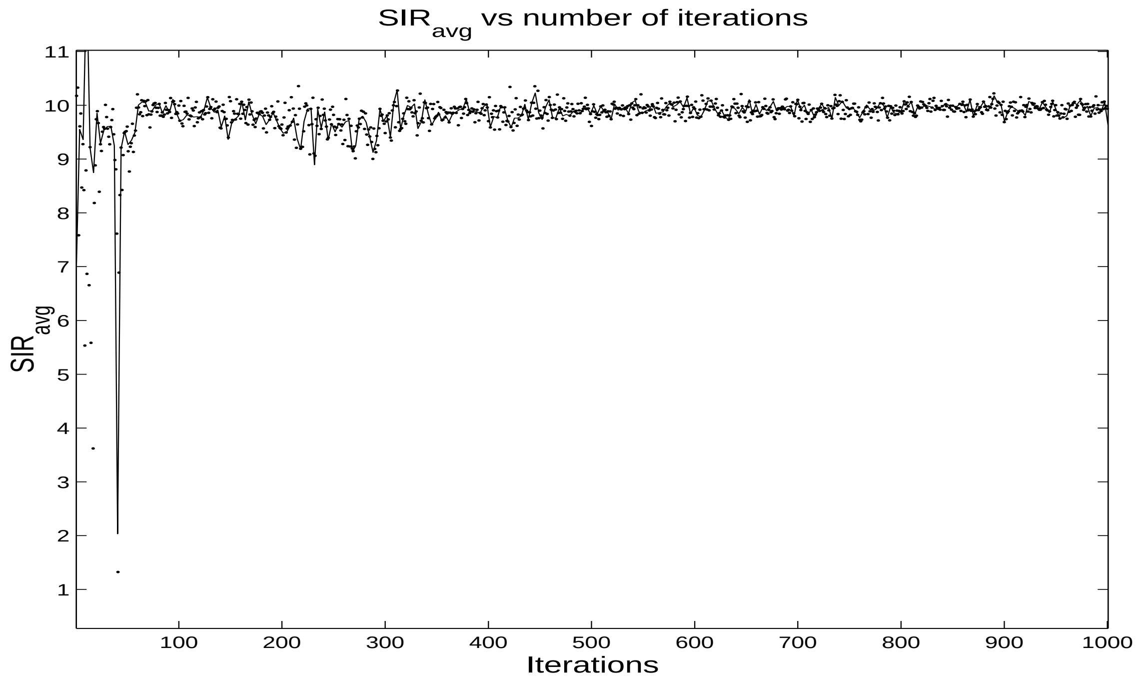

Figure 4: SIR as a function of number of iterations

giving

$$\mathrm{SIR}_{\mathrm{av}}[i] = \frac{\sum_{k=1}^{S} \boldsymbol{m}_1^T[i]\boldsymbol{s}_1}{\sum_{k=1}^{S} \left[\boldsymbol{m}_1^T[i](\boldsymbol{r}[i] - b_{1,k}[i]\boldsymbol{s}_1)\right]^2} \tag{34}$$

where S is the number of simulations over which SIR is averaged. In our simulations we have averaged over $S = 100$ simulations. We simulate a 6 user CDMA system with processing gain $N = 32$. The signal-to-noise ratio of user 1 is 20 dB. There are four interfering users with 10 dB MUI and one with 20 dB MUI, relative to the desired users's signal. The plot given in Fig. 4 shows that the SIR improves with the number of iterations. We have assumed that the channel does not change with time, thus our estimate of the channel improves with time, so the interference cancellation becomes better.

6 Conclusion

In this project we have looked into suboptimal receivers for multiuser detection in CDMA systems. The optimal ML receiver has too high a complexity to be used in practice. For real time implementation we require the detectors to have a reasonable degree of complexity. Hence, linear MMSE and WLS detectors whose complexity is linear in the number users are considered. There performance is comparable to that of the optimal ML receiver with respect to both the BER and near-far resistance offered. However, they suffer from the drawback of large detection delay. A suboptimal implementation of the MMSE detector, i.e. the linear equalizer having a finite delay is also considered. But due to edge effects, it suffers from a slight performance degradation.

All these receivers assume that there the signaling waveforms, power and delays of every other active users is known. At the mobile station, this is a luxury. In general, nothing is known about the signaling waveforms of other users and the receiver must estimate them adaptively. To keep the track of the changes in the channel, a subspace based blind implementation of the MMSE and WLS receivers is considered.

Thus there is a tradeoff between the performance measures(BER vs SNR) and the practicality measure (complexity and detection delay). Depending on the situations, a suboptimal receiver satisfying the implementation constraints can be chosen.

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