Economics of Rumours

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ECON 617: Game Theory

Motivation

Economic Reasons

- Volatility in asset markets.
- Strong co-movement among seeming unrelated financial assets.
- Herd-like or imitative behaviour of participants in asset markets.

Social Reasons

- Brand choice by customers
- Spreading of fads, fashions and ideas within society.
- Should we trust rumours?

The Model

- **Unknown Investment opportunity**

	Returns	Probability
Low	b	$1-p$
High	a	p

- **Two kinds of agents**

	Cost	Probability
Low	0	q
High	c	$1-q$

Basic Assumptions

- Investment opportunity is **unknown**
  - agents do not know whether the opportunity exists.
  - agents do not know the returns (i.e. the state of the opportunity).
- Cost of investment is **private information**.
- $a > c > b > 0$ (High-cost investors invest when returns are high)

Information Transmission

At Start of the Process

Some (randomly chosen) investors

- Learn of the opportunity
- and its state (i.e. its returns)

And decide whether to invest of not.

- Rest of the population does not **yet** know of the opportunity.
- A portion of the remaining population **hear** that someone has invested.

However, they **do not know**

- the return of the opportunity
- the costs of others who had invested

Have to decide whether to invest or not

Rumour Process

- Investor has to decide whether to invest or not.
- If he opts to invest, he becomes a source of information to others.

Decision to believe in the rumour and to pass it on is based on optimizing behaviour.

Solution Concept

Nash Equilibrium of a **Bayesian Extensive Game**.

- On hearing the rumour, the investors update their beliefs about the state of the opportunity. Let μ be the updated belief of the opportunity being in high return state, a.
- The high cost investor invests only if

$$\mu a + (1-\mu) b \geq c \quad \equiv \quad \frac{(c-b)}{(a-b)} < \mu$$

Different Models of Information Transmission

Information Transmission (Exogenous or Endogenous)

1. Probability of hearing the rumour does not depend on the number of agents who have invested in the past.
2. Probability of hearing the rumour is an increasing function of the number of people who have already invested (**a contagion process**).

Exogenous Information Transmission

- **Unknown Investment opportunity**

	Returns	Probability
Low	b	$1-p$
High	a	p

- **Two kinds of agents**

	Cost	Probability
Low	0	q
High	c	$1-q$

- Population is the set of positive integers, indexed by i.
- Discrete time process — at each time only one investor gets a chance to invest.
- Investor i gets his chance in period i.
- Only the first investor is told the state of the world.
- All other investors only observe the decision taken by the **previous** investor.

Existence of Investment Opportunity is Unknown

The First Model

- The first investor is told the opportunity exists and what the returns are.
  - Low Cost (i.e. 0) Always Invests
  - High Cost (i.e. c) Invests when the returns are high (i.e. a) but not when the returns are low (i.e. b)
- Investor 2 observes (or hears about) whether or not investor 1 had invested.
- If he did not invest, the second investor does not even find out that the opportunity exists and the rumour stops.
- If the first investor invested, investor 2 updates his beliefs using Baye's rule

$$\Pr\left(a \,|\, \text{investor 1 invests}\right) = \frac{p}{p + (1-p)q}$$

Two Assumptions

Assumption ⋆

$$\frac{p}{p+(1-p)q} \geq \frac{(c-b)}{(a-b)}$$

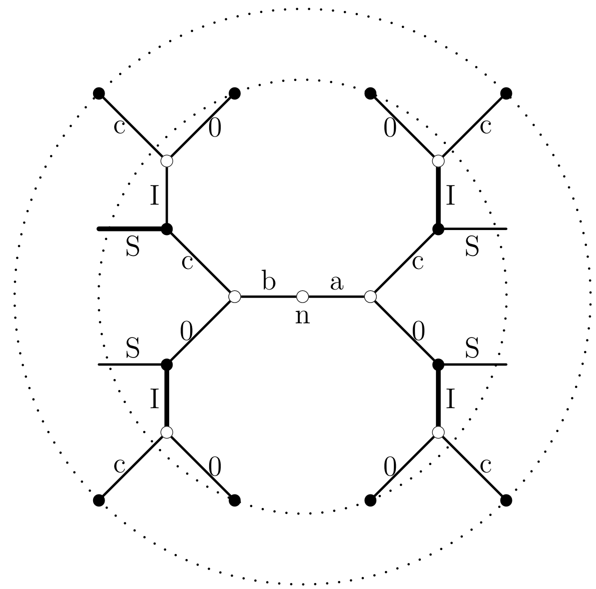
The second investor (even if he has high costs) will invest if he observes that the first investor has invested.

Assumption ⋆⋆

$$\frac{(c-b)}{(a-b)} > p$$

A high cost investor who only knows the ex-ante distribution will not invest

The Game Tree



Conclusion of the First Model

Proposition 2.1

The informativeness of the rumour does not change over time in the sense that the ex post (conditional on observing that the previous person invested) probability that the return is a remains the same.

Everyone invests if and only if the first person invests

Key assumptions for the proof

- Information transmission process is unaffected by the state of the world.
- Agents do not know that the opportunity exists.

Existence of Investment Opportunity is Known

The Second Model

- The first investor is told the state of the investment opportunity.
  - Low Cost (i.e. 0) — Always Invests
  - High Cost (i.e. c) — Invests when the returns are high (i.e. a) but not when the returns are low (i.e. b)
- **Everybody knows that the opportunity exists** — everyone who has low costs will invest in their respective periods
- The high-cost investor, however, will invest only if state a is sufficiently likely.

$$\frac{(c-b)}{(a-b)} < \mu(t+1) = \frac{p}{p+(1-p)z(t)}$$

where $$z(t) = \frac{\Pr(\text{investor } t \text{ invested} \mid b)}{\Pr(\text{investor } t \text{ invested} \mid a)}$$

Conclusion of the Second Model

Proposition 2.2
The informativeness of a rumour dies away over time in the sense that the ex post (conditional on observing that the previous person invested) probability that the return is *a* converges over time to the ex ante probability.

- If assumption ⋆⋆ holds, there is a time t^* after which high cost investors do not invest, even if the previous person had invested.
- If assumption ⋆⋆ does not hold, once anyone invests, all subsequent investors of either cost type will invest.

Conclusion of the Second Model

Key steps for the proof

- Initially, all investors invest if their predecessor had invested, but only the low cost investor invest if their predecessor did not invest.

$$z(t) = 1 - (1-q)^t \to 1 \quad \Longrightarrow \quad \mu(t) \to p$$

Informativeness of the rumour decays with time

- There exists some $t^* \geq 2$ such that high cost investor $t^* + 1$ will not invest even if investor t^* invests.
  Reason: If it were known that a particular investor had low costs, his decision to invest will provide no information to the next investor and from then on all subsequent investors will learn nothing from hearing the rumour.

Endogenous Information Transmission

Drawback with the Simple Models

- Fail to capture that speed of transmission should depend on the value of the information.
- If the information is such that it only affects relatively few people, it is unlikely to spread very fast.

Endogenous Information Transmission Model

- Each person has some probability of getting the information at any instant.
- This probability depends on how many people in the past have used this information to invest.
- The probability is proportional to the **total number of people** who have invested at any time in the past.
- There is no informational delay. Each instance of an investment remains a perpetual source of the rumour. (simple epidemic model)

Endogenous Information Transmission

The Rumour Process

- The rumour takes form of information that someone has invested
- This also tells the potential investor that the investment opportunity is available.
- This is **all** the information he gets to know.
- He does not know the number of people who have already invested
- He does not find out what information anyone else in the population had.
- He gets **no negative information**.
- He gets to hear the rumour only once.

The Model

- **Unknown Investment opportunity**

	Returns	Probability
Low	b	$1-p$
High	a	p

- **Two kinds of agents**

	Cost	Probability
Low	0	q
High	c	$1-q$

- Population given by the $[0,1]$ interval.
- Continuous time process.
- At time 0, each person has x probability of begin told that the opportunity exists, as well as whether returns are a or b.
- After that no one is told the true state of the world any more; the only source of information is the rumour.

Some Definitions

- For $i = a, b$ and any time instant s, define
  - $N(i,s)$ The proportion of population that has invested until instant s, given state i.
  - $P(i,s)$ The proportion of the population that has not heard the rumour until instant s given state i.
  - $\Pr(r \mid i, s)$ The probability that in state i agent hears the rumour for the first time between s and $s + ds$
- y is the rate of spread of information. At any instant, s, the probability that a person who ha not yet heard the rumour will hear it between s and $s + ds$ is given by

$$y \times N(i,s) \times ds$$

Initial Conditions

$$P(a,0) = 1 - x \quad P(b,0) = 1 - x, \qquad N(a,0) = x \quad N(b,0) = xq$$

The Decision Problem

Suppose an agent hears the rumour in some time interval $[s, s + ds]$.

- If the agent has low costs, she will always invest
- If the agent has high costs, she will invest only if

$$\mu(s) = \Pr\big(a \mid \text{hears rumour in } [s, s + ds]\big) \geq \frac{(c-b)}{(a-b)}$$

Updated Beliefs

By Baye's rule

$$\mu(s) = \frac{p}{p + (1-p)z(s)} \quad \text{where} \quad z(s) = \frac{\Pr(r \mid b, s)}{\Pr(r \mid a, s)}$$

Decision Rule

$$z(s) \leq \frac{p(a-c)}{(1-p)(c-b)} = z^* \tag{3.2}$$

$z(s) \leq z^*$ Regime 1	otherwise, Regime 2
Both types invest	Only low-cost invest

Dynamics of the Decision Rule

$$\Pr(r \mid i, s) = y \cdot N(i,s) \cdot P(i,s) \cdot ds$$

$$z(s) = \frac{y \cdot N(b,s) \cdot P(b,s)}{y \cdot N(a,s) \cdot P(a,s)} \tag{3.3}$$

Dynamics in Regime 1

$$\frac{dP(i,s)}{ds} = -yN(i,s)P(i,s), \qquad \frac{dN(i,s)}{ds} = yN(i,s)P(i,s) \tag{3.4}$$

Dynamics in Regime 2

$$\frac{dP(i,s)}{ds} = -yN(i,s)P(i,s), \qquad \frac{dN(i,s)}{ds} = y\boldsymbol{q}N(i,s)P(i,s) \tag{3.5}$$

Two Lemmas

Lemma 3.1 (Behaviour in Regime 1)

If the system has been in Regime 1 at all dates before t,

$$P(a,s) < P(b,s) \quad \text{and} \quad N(a,s) > N(b,s) \quad \text{for } 0 < s < t$$

Lemma 3.2 (Behaviour in Regime 2)

If the system has been in Regime 1 at all dates before and including t^* and has been in Regime 2 at all dates after t^* but before t,

$$P(a,s) < P(b,s) \quad \text{and} \quad N(a,s) > N(b,s) \quad \text{for } \mathbf{0} < s < t$$

Implications

Rumour travels faster in high return state

The Main Result

Proposition 3.1

$z(s)$ increases monotonically over time and is unbounded. Therefore for any value of z^* there will be an instant t^* at which there will be a transition from Regime 1 to Regime 2. After t^* the system will remain in Regime 2.

Implication of the Main Result

- ex post (after hearing the rumour) probability of state a goes to 0 over time.
- If the rumour is sufficiently old when one first hears it, the returns are almost certainly low.
- Rumour gives us very precise information.

Reason

$$z(s) = \frac{N(b,s)}{N(a,s)} \frac{P(b,s)}{P(a,s)}$$

$$\frac{N(b,s)}{N(a,s)} \nearrow k \leq 1 \qquad \frac{P(b,s)}{P(a,s)} \nearrow \text{ unbounded}$$

- **Observation of the rumour** becomes less and **less** informative.
- Information from the **time it takes for the rumour to first reach us** becomes very **precise** over time.

Comparison of the Three Models

First Model Informativeness of the rumour does not change over time.

Second Model Informativeness of the rumour decays over time till the information becomes essentially uninformative.

Main Model A rumour that has been around a long time becomes extremely precise.

Which Model is Closest to Main Model

First Model

- ✔ Agents do not know the opportunity exists till they hear the rumour.
- ✗ Once one person does not invest, no one else finds out that the opportunity exists.
- ✗ In the main model, there is always a source of rumour. Eventually everyone will hear the rumour.

Second Model

- ✔ Everyone knows the opportunity at outset.
- ✔ Informativeness of the act of hearing the rumour **conditional on not having heard it before** declines over time.
- ✗ In the main model, the date at which the rumour reaches someone is endogenous and therefore carries information.

Is the Model Realistic?

Variations Considered in the Paper

- **Information Decay:** In a time interval of length ds, a fraction $g \cdot ds$ will stop being a potential source of information.
- **True Information** available to a fraction of the remaining population, in each time instant.
- **Transmission process** is linear.
- **Payoffs** decline if lots of people invest. "Crowding" effects.

Variations not considered

- Number of people investing in future affects the payoff of present investor.
- Agents are allowed to **lie**. Cheap-talk behaviour.
- Different information content in signals.