

EECS 750 – Project Report

MAC with Feedback

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For point-to-point discrete memoryless channels, feedback does not increase capacity. However, for multiple-access discrete memoryless channels, feedback can increase capacity. The capacity expression for MAC with feedback, known in terms of directed information, is non-computable. Single-letter inner bounds have been given by Cover–Leung and Bross–Lapidoth. The Bross–Lapidoth region contains Cover–Leung region and in some cases, this inclusion is strict. When one channel input is a function of the other input and the channel output, Cover–Leung region is the feedback capacity region. Cover–Leung region is achievable when there is feedback to only one encoder. This report is a summary of various results about capacity of MAC with feedback.

1 Discrete Memoryless multiple access channel

A multiple access channel (MAC) is the simplest non-trivial many to one communications channel. In this report we will only consider two user discrete memoryless multiple access channel denoted by $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}, P_{Y|X_1, X_2})^1$, where $\mathcal{X}_1, \mathcal{X}_2$ are the channel input alphabets, \mathcal{Y} is the channel output alphabet and $P_{Y|X_1, X_2}$ is the channel transition matrix. The channel is memoryless, that is,

$$P(y_n | x_{1,1}^n, x_{2,1}^n) = P_{Y|X_1, X_2}(y_n | x_{1,n}, x_{2,n}) \quad (1)$$

where $x_{i,1}^n$ is a shorthand notation for $(x_{i,1}, \dots, x_{i,n})$.

To present all results in a common framework, we will first review channel coding problem for MAC without feedback and then present achievability results for MAC with feedback.

2 MAC without feedback

Consider a transmission system $(n, \Theta_1, \Theta_2, \tau)$ as shown in Figure 1, consisting of two encoders e_1, e_2 and one decoder d , where

$$e_1 : \{1, \dots, \Theta_1\} \rightarrow \mathcal{X}_1^n$$

$$e_2 : \{1, \dots, \Theta_2\} \rightarrow \mathcal{X}_2^n$$

$$d : \mathcal{Y}^n \rightarrow \{1, \dots, \Theta_1\} \times \{1, \dots, \Theta_2\}$$

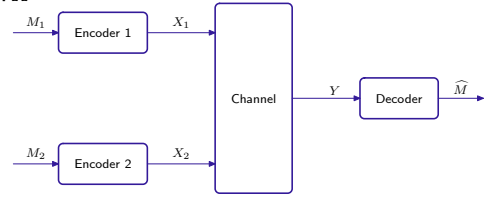


Figure 1 MAC without feedback

Encoder i , $i = 1, 2$ receives an equally likely message M_i taking values in $\{1, \dots, \Theta_i\}$, with $P(M_i = m) = 1/\Theta_i$, for all $m \in \{1, \dots, \Theta_i\}$. Further M_1 is independent of M_2 . The encoders use encoding rule e_1 and e_2 to generate codewords of block length n given by

$$X_{1,1}^n = e_1(M_1)$$

$$X_{2,1}^n = e_2(M_2)$$

The codewords $X_{1,1}^n$ and $X_{2,1}^n$ are transmitted to the decoder over the MAC. The decoder receives Y^n according to the channel transition probability given by (1). It generates an estimate $(\widehat{M}_1, \widehat{M}_2)$ according to the decision rule d , i.e.,

$$(\widehat{M}_1, \widehat{M}_2) = d(Y^n)$$

The event $\{(\widehat{M}_1, \widehat{M}_2) \neq (M_1, M_2)\}$ is called an “error”. Let P_e denote the probability of error. For the transmission system $(n, \Theta_1, \Theta_2, \tau)$, we want

$$P_e = P((\widehat{M}_1, \widehat{M}_2) \neq (M_1, M_2)) \leq \tau$$

¹ In this report we do not consider a cost function associated with the channel. Most results extend naturally when a cost function is w is also considered.

Achievable Rate Given a channel $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}, P_{Y|X_1, X_2})$ a pair (R_1, R_2) is said to be *achievable* if $\forall \epsilon > 0, \exists$ a transmission system $(n, \Theta_1, \Theta_2, \tau)$ such that for all sufficiently large n ,

$$\frac{1}{n} \log \Theta_i - \epsilon \geq R_i \quad i = 1, 2$$

$$\tau \leq \epsilon$$

Capacity Region Given a channel $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}, P_{Y|X_1, X_2})$, the collection of all achievable rates $C = \{(R_1, R_2) \mid (R_1, R_2) \text{ is achievable}\}$ is called *capacity region* for that channel.

For MAC without feedback, the capacity region is given by the convex closure of C^* where

$$C^* = \bigcup_{P_1(X_1)P_2(X_2)} \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq I(X_1; Y \mid X_2) \\ R_2 \leq I(X_2; Y \mid X_1) \\ R_1 + R_2 \leq I(X_1, X_2; Y) \end{array} \right\} \quad (2)$$

To “see” why the rate region given by (2) is achievable, observe that

- (i) The union is taken over independent distributions $P(X_1, X_2) = P(X_1)P(X_2)$ because the encoders can not cooperate.
- (ii) If encoder 2’s message is decoded correctly, the maximum rate at which encoder 1 can transmit is $I(X_1; Y \mid X_2)$. A similar interpretation holds for the maximum rate of encoder 2.
- (iii) The maximum sum rate of both the encoders can not exceed $I(X_1, X_2; Y)$.

3 MAC with feedback

Consider a transmission system $(n, \Theta_1, \Theta_2, \tau)$ as shown in Figure 2. It is similar to the model of Section 2 with a difference that the channel can be used with feedback. The encoders and the decoders are given by

$$e_{1,k} : \{1, \dots, \Theta_1\} \times \mathcal{Y}^{k-1} \rightarrow \mathcal{X}_1$$

$$e_{2,k} : \{1, \dots, \Theta_2\} \times \mathcal{Y}^{k-1} \rightarrow \mathcal{X}_2$$

$$d : \mathcal{Y}^n \rightarrow \{1, \dots, \Theta_1\} \times \{1, \dots, \Theta_2\}$$

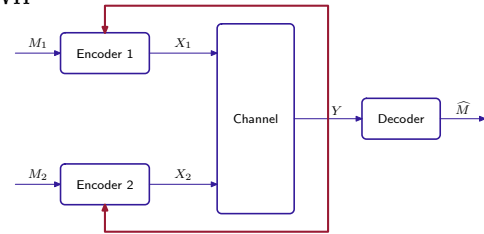


Figure 2 MAC with feedback

The rest of the system and the definitions of achievable rates and channel capacity remain the same as in Section 2. Notice that now the encoders can exploit the feedback for cooperating with one another. The capacity MAC without feedback was a single shot

problem, while the capacity of MAC *with* feedback is a sequential problem, and hence inherently more complicated.

3.1 Feedback increases capacity

The MAC channel is a discrete memoryless channel. For point-to-point discrete memoryless channels, feedback does not increase capacity. This may lead one to suspect that a similar result might hold for the multi-access case. However for discrete memoryless MAC feedback *can increase* capacity as shown by Gaarder and Wolf [1]. They considered a multiple access binary erasure channel which is described $(\{0, 1\}, \{0, 1\}, \{0, 1, 2\}, \delta_{[Y=X_1+X_2]})$ where δ is the Kronecker delta function. If this channel is used without feedback, the capacity is given by

$$C = \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq 1 \\ R_2 \leq 1 \\ R_1 + R_2 \leq 1.5 \end{array} \right\} \quad (3)$$

Gaarder and Wolf showed that the rate point $(0.76, 0.76)$, which lies outside the no feedback rate region of (3), is achievable when the channel is used with feedback. Their main idea was for the encoders to use the feedback channel to communicate with another. Suppose that both encoders have messages that take one of 2^m messages. For the first m transmissions, both encoders send the binary representation of their messages. Due to the structure of the channel, approximately $m/2$ bits will be perceived correctly by the decoder, while $m/2$ will be ambiguous. Each encoder knows which bits are ambiguous to the decoder. For the next n channel uses, both encoders communicate the correct value of the ambiguous bits to the receiver at the cooperative channel capacity of $\log_2 3$ bits/transmission. Thus, $n = (m/2)/\log_2 3$. Hence, the total average rate sum is given by $R_1 = R_2$, $R_1 + R_2 = (m + m)/(m + m/2 \log_2 3) = 2/(1 + 1/\log_2 9) = 1.52037$. Therefore, $(R_1, R_2) = (0.76, 0.76)$ is achievable. Hence, feedback can increase capacity of a discrete memoryless MAC. This raises the question of what is the feedback capacity of MAC. Till date, there is no precise answer to this question.

4 Capacity region for MAC with feedback

Kramer [2] used the notion of *directed information* to show that the capacity of MAC with feedback is given by

$$C = \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq I_\infty(X_1 \rightarrow Y | X_2) \\ R_2 \leq I_\infty(X_2 \rightarrow Y | X_1) \\ R_1 + R_2 \leq I_\infty(X_1 X_2 \rightarrow Y) \end{array} \right\}$$

where

$$I_\infty(X \rightarrow Y | Z) = \lim_{N \rightarrow \infty} \frac{1}{N} I(X^N \rightarrow Y^N | Z^N)$$

and

$$I(X^N \rightarrow Y^N | Z^N) = \sum_{n=1}^N I(X^n; Y_n | Y^{n-1}, Z^n)$$

This expression, however, is an incomputable non–single–letter expression. Inner bounds for capacity can be derived using this expression by restricting the code–trees to use finite history. In particular, for any L ,

$$R_K = \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq \frac{1}{L} I(X_1 \rightarrow Y | X_2) \\ R_2 \leq \frac{1}{L} I(X_2 \rightarrow Y | X_1) \\ R_1 + R_2 \leq \frac{1}{L} I(X_1 X_2 \rightarrow Y) \end{array} \right\}$$

is achievable.

Cover and Leung [3] gave a single letter expression for an inner bound of the capacity region for MAC with feedback. Willems [4] showed that the Cover Leung inner bound is tight if the channel satisfies the condition that at least one of the encoders can determine the symbol produced by the other encoder based on the channel output and the symbol it produced itself. Cover and Leung region is not always tight, as was demonstrated by [5]. Recently, Bross and Lapidoth [6] gave an inner bound with a single letter characterization, which contains the Cover–Leung region with the inclusion being, for some channels, strict. We next present the inner bounds of Cover–Leung and Bross–Lapidoth.

5 MAC with feedback — Cover–Leung achievable region

Gardner and Wolf had used an *ad hoc* two stage scheme to show that feedback can increase the capacity of a discrete memoryless MAC by allowing encoders to communicate. Cover and Leung [3] extended that idea by repeating the process for more than two blocks and using *superposition* and *block stationary* encoding and decoding. The total duration of transmission is divided into blocks, each block consisting of a “fresh information” phase and a “resolution” phase. Suppose M_1 and M_2 are the messages to be sent in the fresh information phase and U is the message to be sent for the resolution phase. M_1 and M_2 are “private messages” known only to the respective encoders while U is a “common message” known to both the encoders. The two messages are superimposed using binning, U determines the bin, and M_i determines the codeword from the chosen bin. The rates are chosen such that the encoders can decode each other’s private message. The decoder can decode the common message U , but can only create a list of *feasible* private messages. Both encoders can also create this list. Moreover, as the encoders could decode each other’s messages, they know which message from the feasible list was actually sent. The index of this message is used as the common message for the next block. Using such a scheme, Cover and Leung showed that the following rate region is achievable.

$$R_{CL} = \bigcup_{P_1(X_1|U)P_2(X_2|U)P(U)} \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq I(X_1; Y | X_2, U) \\ R_2 \leq I(X_2; Y | X_1, U) \\ R_1 + R_2 \leq I(X_1, X_2; Y) \end{array} \right\} \quad (4)$$

They also show that $\|U\| \leq \min\{\|X_1\| \|X_2\| + 1, \|Y\| + 2\}$ is sufficient to generate the entire region². Thus, (4) is single letter inner bound for the feedback capacity of MAC.

To “see” why the rate region given by (4) is achievable, observe that,

- (i) The union is over all distributions where conditioned on U (the common message) X_1 and X_2 are chosen independently. The common message is known to both the encoders and can be used for cooperation. However, the encoders can not share any other form of randomness.
- (ii) It is assumed that the decoder is able to decode the common message U . If encoder 2’s message is also decoded correctly, the maximum rate at which encoder 1 can communicate is $I(X_1; Y | X_2, U)$. A similar explanation holds for maximum rate of encoder 2.
- (iii) The maximum sum rate of both the encoders can not exceed $I(X_1, X_2; Y)$.

Willems [4] showed that for the class of MAC’s for which at least one input (say X_1) is a function of the output Y and the other input (X_2), the Cover–Leung region is the feedback capacity region. In a subsequent paper, Willems and van der Meulen [7] showed that Cover Leung region is achievable when there is feedback to only one encoder.

6 MAC with feedback — Bross–Lapidoth achievable region

Bross and Lapidoth [6] use the same basic idea as used by Cover and Leung. The total duration of transmission is divided into blocks as before. However, each block now consists of three phases — “fresh information” phase, a “two-way” phase and “resolution” phase. The encoders have a “common message” U and a private message M_1 and M_2 . They use binning to generate codewords $X_{1,1}^n$ and $X_{2,1}^n$ and send them over the channel. From the feedback from the channel, they generate extra codewords $V_{1,1}^n$ and $V_{2,1}^n$ such that $V_{i,k}$ is a function of $X_{i,k}$ and Y_k . In the two-way phase, they exchange this extra information. At the end of the two-way phase, each encoder has the other encoder’s extra message, and the decoder has acquired (V_1, V_2) . Under certain conditions (see [6] for details)

$$I(X_1; Y | X_2, U, V_1) \geq I(X_1; Y | X_2, U)$$

$$I(X_2; Y | X_1, U, V_2) \geq I(X_2; Y | X_1, U)$$

² Cover and Leung actually state that $\min\{\|X_1\| \|X_2\|, \|Y\|\}$ is sufficient to generate the entire rate region. However, Willems [4] and Bross and Lapidoth [6] state that $\min\{\|X_1\| \|X_2\| + 1, \|Y\| + 2\}$ is needed. All of them say that this follows from the results in a technical report from Salehi. This is a minor issue, as the important point is that the cardinality of U is bounded. The exact value of the bound is not that important.

The idea is that for channels where one input is not the function of the other input and the channel output, an encoder (say 1) should not try to communicate its message to the other encoder (say 2) directly. Rather, encoder 1 should transmit at a higher rate, so that at the end of the fresh information phase, encoder 2 does not know the exact message sent by encoder 1. Then both encoders generate extra codeword $V_{i,1}^n$ and communicate it to each other in the two-way phase. The two-way phase actually consists of L sub-phases, where the uncertainty about V_1, V_2 is sequentially reduced over the L phases, each sub-phase is of geometrically smaller duration. Assume that the rates are chosen such that knowing, $u^n, x_{2,1}^n, y^n$ and $v_{1,1}^n$, encoder 2 can decoder $x_{1,1}^n$ correctly and the same also holds for encoder 1.

The decoder knows y^n and $(v_{1,1}^n, v_{2,1}^n)$. Using this it can generate a list of feasible messages. Both encoders also have this information and can generate the same list. With high probability, one message from this list was actually transmitted. Both the encoders know the index of that message. This is the common information U for the next block. When, in the next block, the decoder receives U , it can correctly decode $X_{1,1}^n$ and $X_{2,1}^n$.

Thus, the idea is not to immediately try to communicate $X_{i,1}^n$ to the other encoder. Send a block of data and wait for the channel outputs. This tells the encoder what the other encoder thinks about it. Then use the two-way phase to disambiguate within this smaller region of uncertainty. Thus, the encoders can take advantage of the feedback while communicating with one another.

The achievable rate region using this scheme is given by

$$R_{BL} = \bigcup_{\substack{P(U)P(X_1|U)P(X_2|U) \\ P(\tilde{X}_1)P(\tilde{X}_2) \\ g_1, g_2}} \left\{ (R_1, R_2) : \begin{array}{l} \eta \geq \hat{\eta}^{(TW)} \\ R_L \geq \tilde{R}_L^{(TW)} \\ R_1 \leq (1 + \eta)^{-1} I(X_1; Y | X_2, U, V_1) \\ R_2 \leq (1 + \eta)^{-1} I(X_2; Y | X_1, U, V_2) \\ R_1 + R_2 \leq (1 + \eta)^{-1} [I(X_1, X_2; Y | V_1, V_2) - R_L] \end{array} \right\} \quad (5)$$

where $g_i : \mathcal{X}_1 \times \mathcal{Y} \rightarrow \mathcal{V}$ and $V_i = g_i(X_i, Y)$ and

$$\theta = \max \left\{ \frac{H(V_1 | X_2, Y)}{I(\tilde{X}_1; \tilde{Y} | \tilde{X}_2, \tilde{V}_1)}, \frac{H(V_2 | X_1, Y)}{I(\tilde{X}_2; \tilde{Y} | \tilde{X}_1, \tilde{V}_2)} \right\} < 1$$

$$\eta^{(TW)} = \frac{\theta}{1 - \theta} \quad \omega_1 = \frac{I(\tilde{X}_1; \tilde{Y} | \tilde{V}_1, \tilde{V}_2)}{I(\tilde{X}_1; \tilde{Y} | \tilde{V}_1, \tilde{X}_2)} \quad \omega_2 = \frac{I(\tilde{X}_2; \tilde{Y} | \tilde{V}_2, \tilde{V}_1)}{I(\tilde{X}_2; \tilde{Y} | \tilde{V}_2, \tilde{X}_1)}$$

$$R_L^{(TW)} = \min_{i=1,2} \left\{ \begin{array}{l} H(V_i | Y) - \omega_i H(V_i | X_i, Y) \\ + H(V_i | V_i, Y) - \omega_i H(V_i | X_i, Y) \\ + \eta^{(TW)} \cdot [H(\tilde{V}_i | \tilde{Y}) - \omega_i H(\tilde{V}_i | \tilde{X}_i, \tilde{Y})] \\ + H(\tilde{V}_i | \tilde{V}_i, \tilde{Y}) - \omega_i H(\tilde{V}_i | \tilde{X}_i, \tilde{Y}) \end{array} \right\}$$

This is a single-letter characterization of the inner bound for capacity region of MAC with feedback. The cardinality of the auxiliary random variable U is bounded by $\min\{\|X_1\| \|X_2\| +$

$1, \|Y\| + 2\}$. Bross and Lapidoth do not give any bound on the cardinality of \mathcal{V}_i . However, as $g_i : \mathcal{X}_i \times \mathcal{Y} \rightarrow \mathcal{V}_i$, the cardinality of \mathcal{V}_i is bounded by $\|\mathcal{X}_i\| \|\mathcal{Y}\|$.

There is no straight-forward way to “see” why the rate region given by (5) is achievable. Some of the artifacts of the result can be explained intuitively.

- (i) There are two probability distributions, $P(U)P(X_1 | U)P(X_2 | U)$ and $P(\tilde{X}_1)P(\tilde{X}_2)$. The first represents the distribution to be chosen for the fresh information phase, the second the distribution for the two-way phase. During the fresh information phase, U is superimposed on X_1 and X_2 using binning. The encoders share no other source of randomness, and hence conditioned on U , X_1 and X_2 must be independent. During each sub-phase of the two-way phase, the encoders do not cooperate, hence their codebooks must be generated independently.
- (ii) $nH(V_1 | X_2, Y)$ is the exponent of the number of V_1 that are jointly typical with X_2 and Y . That is, after the end of the fresh information phase, this is the size of the list at encoder 2. During the first sub-phase of the two-way phase, encoder 2 knows \tilde{V}_1 and $nI(\tilde{X}_1, \tilde{Y} | \tilde{X}_2, \tilde{V}_1)$ is the reduction in the size of the list at encoder 2. A similar interpretation holds for the other term in θ . Thus θ represents the size of the larger lists after the first sub-phase of the two-way phase. Only those g_i are allowed for which this is less than 1. This ensures that during the subsequent sub-phases of the two-way phase, the uncertainty about V_1 and V_2 reduces.
- (iii) $\eta^{(TW)}$ represents the maximum fraction of time spent in the two-way phase. If the two-way phase consists of L sub-phases then $\eta^{(TW)} = \theta + \theta^2 + \dots + \theta^L < \theta/(1 - \theta)$. Since the two-way phase does not convey any fresh information, the effective rate is scaled by $1/(1 + \eta)$.
- (iv) When $\omega_1 = 1$, that is there exists a mapping $\psi : \mathcal{Y} \times \mathcal{V}_1 \times \mathcal{V}_2 \rightarrow \mathcal{X}_2$ such that $\psi(y, v_1, v_2) = x_2$ for any pair $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$ with $p(x_1, x_2) > 0$, then

$$H(V_1 | Y) - \omega_1 H(V_1 | X_2, Y) = I(V_1; X_2 | Y)$$

which denotes the exponent of the size of list of uncertainty at the decoder about X_2 . $\omega \neq 1$ is a generalization when such a ψ does not exist. $R_L^{(TW)}$ denotes the size of the list of uncertainty at the decoder.

The rate region given by (5) contains the Cover-Leung region. This can be seen as follows. Choose g_1, g_2 to take only one value. In this case it can be shown that $\tilde{\eta}^{(TW)} = 0$ and $\tilde{R}_L^{(TW)} = 0$. Setting these values in (5), we recover the rate region of (4).

7 Examples

In this section, we present some examples to illustrate the different rate regions.

7.1 Multiple-Access Binary Erasure Channel

Consider a MAC with $\mathcal{X}_1 = \mathcal{X}_2 = \{0, 1\}$ and $\mathcal{Y} = \{0, 1, 2\}$ with $Y = X_1 + X_2$. The capacity of this channel when used without feedback is given by

$$C_{\text{no feedback}} = \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq 1 \\ R_2 \leq 1 \\ R_1 + R_2 \leq 1.5 \end{array} \right\}$$

Gaarder and Wolf used the example of this channel to show that feedback can increase capacity for MAC. Cover-Leung gave an achievable region for this channel, for which $R_1 + R_2 = 1.58496$ is achievable. Willems [4] showed that for this channel Cover-Leung region is the capacity region.

7.2 Binary MAC

Consider a MAC with $\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{Y} = \{0, 1\}$ where the probability of receiving the symbol “1” is proportional to the sum of the channel inputs, that is,

$$p(1|1, 0) = p(1|0, 1) = q$$

$$p(1|1, 1) = 2q$$

$$p(1|0, 0) = 0$$

where $0 < q < 1/2$. This channel does not meet Willems’ condition: no encoder can determine the symbol produced by the other encoder based on the channel output and the symbol it produced itself. However, Willems’ condition is “partially fulfilled” in the sense that if the encoder produces “0” and if it observes that the channel output is “1”, it can infer that the other encoder produced a “1”. Bross-Lapidoth exploit this fact to show that their rate region is larger than Cover-Leung’s region for this channel.

For this channel, the maximal symmetric sum rate in Cover-Leung region is $(R_1 + R_2)/q = 0.499426 + o(1)$, which is attained with $\mathcal{U} = \{0, 1, 2\}$ and $P_U = [0.446808, 0.09, 0.463192]$ and

$$P(X_1 = 0 | U = 0) = P(X_2 = 1 | U = 0) = 0.214$$

$$P(X_1 = 0 | U = 1) = P(X_2 = 1 | U = 1) = 1 - \varepsilon, \quad \varepsilon \ll 1$$

$$P(X_1 = 0 | U = 2) = P(X_2 = 1 | U = 2) = 0.242$$

Choose $\mathcal{V}_1 = \mathcal{V}_2 = \{1, e\}$ and g_1, g_2 as

$$g_i(x_i, y) = \begin{cases} 1, & x_i = 0 \text{ and } y = 1 \\ e, & \text{otherwise} \end{cases}$$

Here “e” denotes an “erasure” meaning that encoder i can not infer what encoder \bar{i} sent during that time. Choose $\mathcal{U} = \{0, 1\}$ $P_U(1) = 0.0045$,

$$P_{X_i|U}(0 | 0) = 0.731$$

$$P_{X_i|U}(0 | 1) = 1 - \varepsilon, \quad \varepsilon \ll 1$$

and $P_{\tilde{X}_1}(1) = P_{\tilde{X}_2}(1) = 0.2601$. This yields a symmetric sum rate of $(R_1 + R_2)/q = 0.553 + o(1)$, which is larger than the sum rate of Cover–Leung for small values of q .

7.3 Noisy Binary Adder

Consider a MAC with $\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{Y} = \mathcal{Z} = \{0, 1\}$ with $Y = X_1 + X_2 + Z$ where $P_Z(0) = P_Z(1) = 1/2$. For this channel, the maximal symmetric sum rate for the Cover–Leung region is $(R_1 + R_2) = 0.87242$ which is achieved using $\mathcal{U} = \{0, 1\}$ (The transition probability matrix achieving this rate was not given). Kramer shows that $R_1 + R_2 = 0.87758$ is achievable using a eight state Markov coding scheme.

8 Conclusion

The feedback capacity of discrete memoryless MAC is still an open problem and needs further investigation. Scenarios where single-letter characterization of the capacity region can be obtain need to be identified.

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