

# Space Time Coding for DS-CDMA

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EECS 555: Digital Communication Theory

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# Outline

- 1 Multiple Antenna Downlink Schemes
- 2 Space Time Spreading
- 3 Main Results of Hochwald et. al. (2001)
  - Imperfect Channel Estimation
  - Single User — Multipath Components
- 4 Multiuser Interference
  - Effect of Number of users
  - Bounds as Compared to Actual Performance
- 5 Conclusion

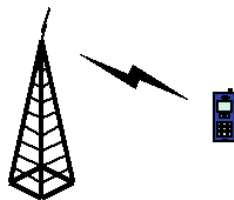
# References

-  B. Hochwald, T. Marzetta and C. Papadias  
A Transmitter Diversity Scheme for Wideband CDMA  
Systems Based on Space-Time Spreading  
*IEEE JSAC*, Vol. 19, No. 1, January 2001
-  A. J. Viterbi  
CDMA : Principles of Spread Spectrum  
Communication  
Prentice Hall PTR, 17 April, 1995
-  Proposed TDCO #662/98 TO ETSI SMG2 UMTS  
standards, Dec. 1998  
Space Time Block Coded Transmit Antenna Diversity  
Scheme for WCDMA
-  V. Tarokh, H. Jafarkhani and A.R. Calderbank,  
Space-time block codes from orthogonal design  
*IEEE Trans. Inform. Theory*, Vol. 45, July 99

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## Single Antenna System

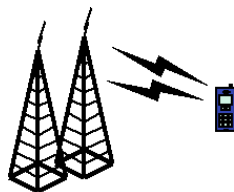


(Antenna 1)

$$\mathbf{s}_1 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} b(k) \mathbf{c}(k)$$

$$\mathbf{c}^\dagger(k) \mathbf{r} = N \sqrt{\frac{E_c(k)}{2}} h_1 b(k) + \mathbf{c}^\dagger(k) \mathbf{n}$$

## Two Antenna System



(Antenna 2)

$$\mathbf{s}_2 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} b(k) \mathbf{c}(k)$$

$$\mathbf{r} = h_1 \mathbf{s}_1 + h_2 \mathbf{s}_2 + \mathbf{n}$$

$$\mathbf{c}^\dagger(k) \mathbf{r} = N \sqrt{\frac{E_c(k)}{2}} (h_1 + h_2) b(k) + \mathbf{c}^\dagger(k) \mathbf{n}$$

# Two Distinct Spreading Codes for Each User

(Antenna 1)

$$\mathbf{s}_1 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} b(k) \mathbf{c}_1(k)$$

(Antenna 2)

$$\mathbf{s}_2 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} b(k) \mathbf{c}_2(k)$$

(Received Signal)

$$\mathbf{r} = h_1 \mathbf{s}_1 + h_2 \mathbf{s}_2 + \mathbf{n}$$

$$y = (h_1^* \mathbf{c}_1^\dagger(k) + h_2^* \mathbf{c}_2^\dagger(k)) \mathbf{r} = N \sqrt{\frac{E_c(k)}{2}} \overbrace{(|h_1|^2 + |h_2|^2)}^{\text{Diversity} = 2} b(k) + \eta$$

$$y \underset{b(k)=-1}{\overset{b(k)=+1}{\geq}} 0$$

## Advantages

- Rate = 1 code
- Optimal two-fold diversity gain over single antenna case
- Linear Complexity Receiver

## Disadvantages

- Two code sequences per user  
Effective number of users halved

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## Objective

Achieve two-fold diversity with rate = 1 code **without reducing the total number of users in the system**

## Solution

Space-Time Spreading (STS)

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# Space Time Spreading

$$\text{(Antenna 1)} \quad \mathbf{s}_1 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} (b_1(k)\mathbf{c}_1(k) + b_2(k)\mathbf{c}_2(k))$$

$$\text{(Antenna 2)} \quad \mathbf{s}_2 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} (b_2(k)\mathbf{c}_1(k) - b_1(k)\mathbf{c}_2(k))$$

$$\text{(Rcvd Signal)} \quad \mathbf{r} = h_1\mathbf{s}_1 + h_2\mathbf{s}_2 + \mathbf{n}$$

$$y_1 = \text{Re} \left\{ (h_1^* \mathbf{c}_1^\dagger(k) - h_2^* \mathbf{c}_2^\dagger(k)) \mathbf{r} \right\}$$

$$y_2 = \text{Re} \left\{ (h_2^* \mathbf{c}_1^\dagger(k) + h_1^* \mathbf{c}_2^\dagger(k)) \mathbf{r} \right\}$$

$$\Rightarrow y_i = 2N \sqrt{\frac{E_c(k)}{2}} \underbrace{(|h_1|^2 + |h_2|^2)}_{\text{Diversity} = 2} b_i(k) + \eta_i \quad \begin{matrix} b_i(k) = +1 \\ \geq \\ b_i(k) = -1 \end{matrix} \quad 0$$

# Practical Considerations

## Assumptions So Far

- Synchronous system
  - Orthogonal Code Sequences
  - Frequency non-selective fading (no multipaths)
- ⇒ no inter-user interference
- Perfect channel estimates

## In a Realist Scenario

- Frequency selective fading (multipaths)
- Imperfect channel estimates

## System Performance

- With multipath components **and no other user**
- With multipath components **and other users**

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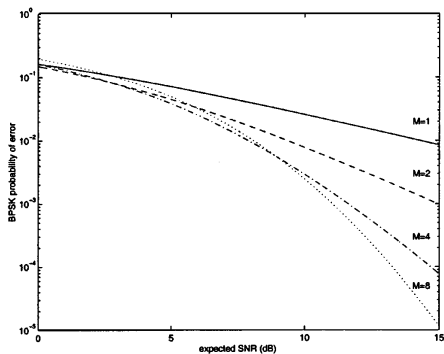
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# Effect of Channel Estimation Error

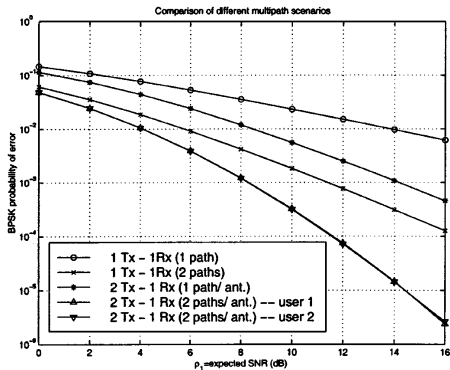


Probability of error for BPSK modulation vs. expected message SNR for 1, 2, 4 and 8 antennas, where errors in estimating the propagation coefficients are accounted for; total pilot power is 10 dB stronger than the total message power for each user i.e.  $E_p = 10E_b$ .

$$P_e = \frac{1}{(4 + 4\beta)^M} \sum_{j=0}^{M-1} \frac{\Gamma(2M - 1 - j)}{\Gamma(M)\Gamma(M - j)} \left[ \frac{2\sqrt{4 + 4\beta}}{\sqrt{4\beta} + \sqrt{4 + 4\beta}} \right]^{1+j}$$

$$\text{where } \beta = \frac{E_p E_b}{N_0 M (N_0 M + E_p + E_b)}$$

# Effect of Multipath (Single User)



Probability of error for BPSK modulation vs. expected message SNR for 1 and 2 transmit antennas in flat fading and in multipath with perfect knowledge of fading coefficients. Codes  $\mathbf{c}_1$  and  $\mathbf{c}_2$  are Walsh codes of length 128 and delay between two paths is 10 chips.

$$P_{e|H}^{(1)} = \frac{1}{2} \mathcal{Q} \left[ \operatorname{Re}\{g_{11} + g_{12}\} \sqrt{\frac{E_b}{N_0 g_{11}}} \right] + \frac{1}{2} \mathcal{Q} \left[ \operatorname{Re}\{g_{11} - g_{12}\} \sqrt{\frac{E_b}{N_0 g_{11}}} \right]$$

$$P_{e|H}^{(2)} = \frac{1}{2} \mathcal{Q} \left[ \operatorname{Re}\{g_{22} + g_{21}\} \sqrt{\frac{E_b}{N_0 g_{22}}} \right] + \frac{1}{2} \mathcal{Q} \left[ \operatorname{Re}\{g_{22} - g_{21}\} \sqrt{\frac{E_b}{N_0 g_{22}}} \right]$$

# What is Missing?

- Accepted as optional diversity mode in release A of IS-2000 wideband CDMA standard
- Effect on performance of other user interference

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# Multiuser Interference

## System

- Two transmit antennas — one receive antenna
- $J$  multipaths from each antenna
- BPSK modulation
- Each user is modulated using a random **carrier phase**
- Coherent demodulation

## Assumptions

- 1 Each path experiences independent Rayleigh fading
- 2 Multipath delay an integral multiple of chip interval
- 3 Same set of delays from both transmit antennas
- 4 Intersymbol interference can be neglected

# Bound on Probability of Error

$$P_{e|H}^{(1)} \leq \min_{\rho > 0} \mathbb{E} \left\{ \exp \left( \rho \sqrt{\frac{E_c(1)}{2}} \tilde{g}_{1,1}(1) b_1(1) \right) \exp \left( \rho \sqrt{\frac{E_c(1)}{2}} g_{1,2}(1) b_2(1) \right) \times \prod_{k=2}^{K_u} \prod_{i=1}^2 \exp \left( \rho \sqrt{\frac{E_c(k)}{2}} \tilde{g}_{1,i}(k) b_i(k) \cos \theta_k \right) \exp(\rho \eta_1) \right\} \left. \begin{array}{l} \mathbf{b}_1(1) = -1 \\ \mathbf{h}_1, \mathbf{h}_2 \end{array} \right\}$$

$$\mathbb{E} \left\{ \exp \left( \rho \sqrt{\frac{E_c(1)}{2}} \tilde{g}_{1,1}(1) b_1(1) \right) \right\} = \exp \left( -\rho \sqrt{\frac{E_c(1)}{2}} g_{1,1}(1) \right)$$

$$\mathbb{E} \left\{ \exp \left( \rho \sqrt{\frac{E_c(1)}{2}} g_{1,2}(1) b_2(1) \right) \right\} \leq \exp \left( \rho \sqrt{\frac{E_c(k)}{2}} |\tilde{g}_{1,2}(1)| \right)$$

$$\mathbb{E} \left\{ \exp \left( \rho \sqrt{\frac{E_c(k)}{2}} \tilde{g}_{1,i}(k) b_i(k) \cos \theta_k \right) \right\} \leq ?$$

$$\mathbb{E} \left\{ \exp(\rho \eta_1) \right\} = \exp \left( \frac{1}{2} \rho^2 g_{1,1}(1) \frac{N_0}{2} \right)$$

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# Fixed Codewords for All Users

$$\mathbb{E} \left\{ \exp \left( \rho \sqrt{\frac{E_c(k)}{2}} \tilde{g}_{1,i}(k) b_i(k) \cos \theta_k \right) \right\} \leq \exp \left( \frac{1}{4} \rho^2 \frac{E_c(k)}{2} (\tilde{g}_{1,i}(k))^2 \right)$$

$$P_{e|H}^{(1)} \leq \exp \left\{ - \frac{\left( \sum_{j=1}^J h_{1,j}^2 + \sum_{j=1}^J h_{2,j}^2 \right) \frac{E_b(1)}{2} \left[ 1 - \left| \frac{g_{12}(1)}{g_{11}(1)} \right| \right]^2}{\frac{1}{2Ng_{11}(1)} \sum_{k=2}^{K_H} \frac{E_c(k)}{2} \left( (g_{1,1}(k))^2 + (g_{1,2}(k))^2 \right) + N_0} \right\}$$

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## Averaged over All Codewords

$$\mathbb{E} \left\{ \exp \left( \rho \sqrt{\frac{E_c(k)}{2}} \tilde{g}_{1,i}(k) b_i(k) \cos \theta_k \right) \right\} \leq \exp \left\{ \frac{1}{2} \rho^2 \frac{E_c(k)}{2} N \left( \sum_{j=1}^J h_{1,j} h_{1,j} + \sum_{j=1}^J h_{2,j} h_{2,j} \right)^2 \right\}$$

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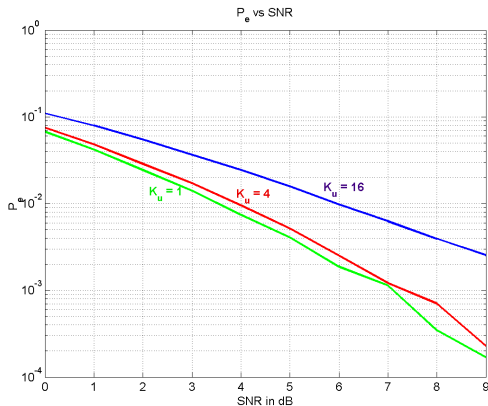
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$$\mathbb{E} \left\{ \exp \left( \rho \sqrt{\frac{E_c(k)}{2}} \tilde{g}_{1,i}(k) b_i(k) \cos \theta_k \right) \right\} \leq \exp \left\{ \frac{1}{2} \rho^2 \frac{E_c(k)}{2} N \left( \sum_{j=1}^J h_{1,j} h_{1,j} + \sum_{j=1}^J h_{2,j} h_{2,j} \right)^2 \right\}$$

$$P_{e|H}^{(1)} \leq \exp \left\{ - \frac{\left( \sum_{j=1}^J h_{1,j}^2 + \sum_{j=1}^J h_{2,j}^2 \right) \frac{E_b(1)}{2} \left[ 1 - \left| \frac{g_{12}(1)}{g_{11}(1)} \right| \right]^2}{\frac{1}{2N} \left( \sum_{j=1}^J h_{1,j}^2 + \sum_{j=1}^J h_{2,j}^2 \right) \sum_{k=2}^{K_u} \frac{E_b(k)}{2} + N_0} \right\}$$

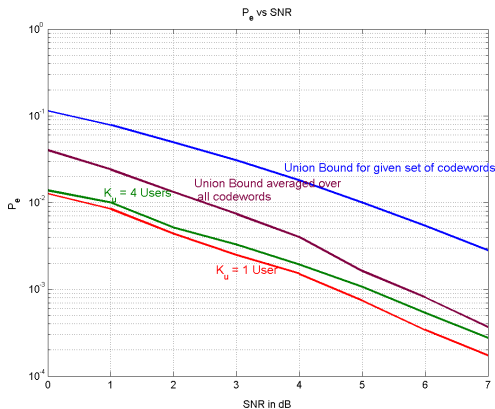
# Effect of Number of Users



- $P_e$  vs SNR for  $K_u = 1$ ,  $K_u = 4$  and  $K_u = 16$
- Walsh Code of length  $2N = 128$
- $J = 2$  multipaths
- Path delay,  $\tau = 10$  chips
- Rayleigh fading
- Perfect CSI available

$$P_{e|H}^{(1)} \leq \exp \left\{ - \frac{\left( \sum_{j=1}^J h_{1,j}^2 + \sum_{j=1}^J h_{2,j}^2 \right) \frac{E_b(1)}{2} \left[ 1 - \left| \frac{g_{12}(1)}{g_{11}(1)} \right| \right]^2}{\frac{1}{2N} \left( \sum_{j=1}^J h_{1,j}^2 + \sum_{j=1}^J h_{2,j}^2 \right) \sum_{k=2}^{K_u} \frac{E_b(k)}{2} + N_0} \right\}$$

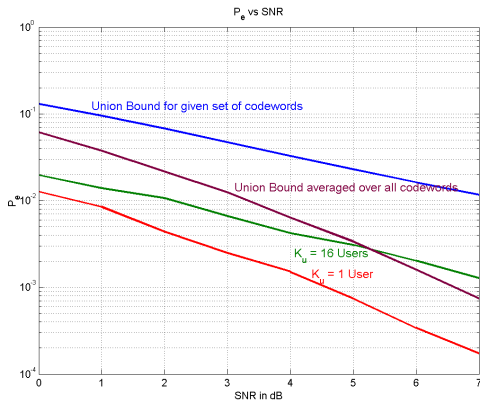
# Bound vs Performance



- $P_e$  vs SNR for  $K_u = 1$  and  $K_u = 4$ , union bound for given set of codes and union bound averaged over all codes
- Walsh Code of length  $2N = 128$
- $J = 2$  multipaths
- Path delay,  $\tau = 10$  chips
- Rayleigh fading
- Perfect CSI available

$$P_{e|H}^{(1)} \leq \exp \left\{ - \frac{\left( \sum_{j=1}^J h_{1,j}^2 + \sum_{j=1}^J h_{2,j}^2 \right) \frac{E_b(1)}{2} \left[ 1 - \left| \frac{g_{12}(1)}{g_{11}(1)} \right| \right]^2}{\frac{1}{2Ng_{11}(1)} \sum_{k=2}^{K_u} \frac{E_c(k)}{2} \left( (g_{1,1}(k))^2 + (g_{1,2}(k))^2 \right) + N_0} \right\}$$

# Bound vs Performance



- $P_e$  vs SNR for  $K_u = 1$  and  $K_u = 16$ , union bound for given set of codes and union bound averaged over all codes
- Walsh Code of length  $2N = 128$
- $J = 2$  multipaths
- Path delay,  $\tau = 10$  chips
- Rayleigh fading
- Perfect CSI available

$$P_{e|H}^{(1)} \leq \exp \left\{ - \frac{\left( \sum_{j=1}^J h_{1,j}^2 + \sum_{j=1}^J h_{2,j}^2 \right) \frac{E_b(1)}{2} \left[ 1 - \left| \frac{g_{12}(1)}{g_{11}(1)} \right| \right]^2}{\frac{1}{2Ng_{11}(1)} \sum_{k=2}^{K_u} \frac{E_c(k)}{2} \left( (g_{1,1}(k))^2 + (g_{1,2}(k))^2 \right) + N_0} \right\}$$

# Outline

- 1 Multiple Antenna Downlink Schemes
- 2 Space Time Spreading
- 3 Main Results of Hochwald et. al. (2001)
  - Imperfect Channel Estimation
  - Single User — Multipath Components
- 4 Multiuser Interference
  - Effect of Number of users
  - Bounds as Compared to Actual Performance
- 5 Conclusion

# Conclusion

## Space Time Spreading

- 1 Provides optimal two-fold diversity
- 2 Rate = 1 scheme
- 3 Requires linear complexity receiver
- 4 No extra spreading codes required
- 5 Acceptable performance in MAI.

# Thank You