Space Time Coding for DS-CDMA systems

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Project Report

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1 Introduction

Unlike Additive White Gaussian Noise (AWGN) channels, wireless channels are subject to severe attenuation due to destructive addition of multipaths arising from propagation delay in the medium. This phenomenon commonly referred to as *fading*, leads to random fluctuations in the signal strength thereby degrading the quality of the signal. Fading can lead to a decrease in the overall capacity of the system by sometimes upto 35 dB.

An effective means to mitigate the effects of fading on a wireless channel is known as *diversity*. As the name suggests, diversity is a method to provide the receiver with multiple (diverse) copies of the same symbol in the hope that one the copies would undergo comparatively lesser attenuation, which would help the receiver in the correct decoding of the symbol.

Diversity can be provided in different ways, for example, (i) *time-diversity* where multiple copies of same symbol are transmitted over time, (ii) *frequency diversity*, where multiple copies are transmitted over different frequencies or (iii) *Space (antenna) diversity*, where multiple copies are obtained from spatially separated antennas and combined using Maximal Ratio Combining (MRC), Equal Gain Combining (EGC) or any other such combining schemes at the receiver.

The utilization of time/frequency diversity incurs an expense — time in the case of time diversity and bandwidth in case of frequency diversity to introduce redundancy. As compared to these schemes, antenna (space) diversity is an attractive proposition because it does not incur any penalty on bandwidth efficiency.

However, traditional receiver side antenna diversity is difficult to obtain on the forward link because of the problems associated with deploying multiple antennas and r-f circuitry on the mobile receiver which is required to be small and computationally less intensive. Thus, there has been recent interest in improving the capacity of wireless link by employing multiple antennas at the base station.

In this respect, space-time codes have been extensively studied as a means of providing transmitter side antenna diversity on the forward link. The coding is carried over space, by transmitting redundant symbols from more than one antenna, and time, by distributing the information symbols over several time intervals of transmission. More recently, a particular form of space-time coding for DS-CDMA systems, known as Space-Time Spreading (STS) has been accepted as an optional diversity mode in IS-2000 standard. STS was first introduced in [1]. Herein, the authors provided a general framework for STS code construction and evaluated the performance of a particular STS scheme, known as the Alamouti scheme, in non-perfect channel state information and multipath fading scenarios. However, the analysis was carried out assuming the model of a single user CDMA system and did not take into account the effect of multiuser interference. Without such an analysis, their scheme can not be considered practical for a multiuser cellular CDMA system. In this project, we evaluate the performance of STS in the presence of MAI and compare the results with those presented in [1].

The rest of the report is divided as follows: In section 2, we describe some of the existing dowlink tranmit diversity schemes; introducing the concept of Space-Time Spreading (STS). In section 3, we consider a generalized version of STS as presented in [1]. We provide some of the main results of the paper without going into the proofs. In the next section, we analyze the performance of STS in presence of MAI and derive an upper bound for the error probability. In section 5, we compare our analysis with simulation results.

2 Multiple Antenna Downlink Schemes

We begin by describing some existing downlink proposals and explain their advantages and limitations. In this section we follow the development as presented in [1].

2.1 Transmit Same Signal from Two Antennas

An ineffective two antenna scheme has both the antennas transmitting the same signal using the same spreading codes. Let $\underline{s}_l$ be the baseband representation of the signal transmitted from antenna l, $l \in \{1, 2\}$. Then we can write

$$\underline{s}_1 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} b(k)\underline{c}(k) \qquad \underline{s}_2 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} b(k)\underline{c}(k) \tag{1}$$

where $b(k) \in \{\pm 1\}$ is the data bit of the kth user, $\underline{c}(k)$ is a $N \times 1$ column vector representing the spreading code of the kth user, $E_c(k)$ is the energy per chip for the kth user and the factor of $\sqrt{2}$ is used to normalize the total transmitted power per bit. Assuming flat fading, the down-converted received signal $\underline{\mathrm{r}}$ at the desired user 1 can be written as

$$\underline{r} = h_1\underline{s}_1 + h_2\underline{s}_2 + \underline{n} \tag{2}$$

where h_l is a zero mean complex normal random variable, representing the fading coefficient of the channel from antenna l to user 1, and $\underline{n}$ is a vector of independent samples from a complex AWGN process. Assuming that the users are perfectly orthogonal, after despreading the user 1 received signal we get

$$\underline{c}^\dagger(1)\underline{r} = N\sqrt{\frac{E_c(1)}{2}}\big(h_1 + h_2\big)b(1) + \underline{c}^\dagger(1)\underline{n}$$

Notice that because of the wide separation between the two antennas, h_1 and h_2 are independent and, therefore $(1/\sqrt{2})(h_1 + h_2)$ has the same distribution as either h_1 or h_2. This makes the scheme ineffective as no diversity gain is realized.

2.2 Two Different Spreading Codes for Each User

Consider a slight variation of the scheme presented in section 2.1. Here, we use different spreading codes $\underline{c}_1(k)$ and $\underline{c}_2(k)$ to transmit the same bit from the two different antennas for each user. Then, we have the transmitted signals as

$$\underline{s}_1 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} b(k)\underline{c}_1(k) \qquad \underline{s}_2 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} b(k)\underline{c}_2(k) \tag{3}$$

The received signal remains the same as (2), but user 1 separately despreads $\underline{c}_1(1)$ and $\underline{c}_2(1)$ which are the two sequences corresponding to $b(1)$. This, yields two effective received signals

$$d_1 = \underline{c}_1^\dagger(1)\underline{r} = N\sqrt{\frac{E_c(k)}{2}} h_1 b(1) + \underline{c}_1^\dagger(1)\underline{n}$$
$$d_2 = \underline{c}_2^\dagger(1)\underline{r} = N\sqrt{\frac{E_c(k)}{2}} h_2 b(1) + \underline{c}_2^\dagger(1)\underline{n}$$

The receiver then combines d_1 and d_2 using Maximal Ratio Combining (MRC) to get the decision statistic for b(1), given by

$$\begin{aligned} y &= \mathrm{Re}\{h_1^* d_1 + h_2^* d_2\} \\ &= N\sqrt{\frac{E_c(k)}{2}} \left(|h_1|^2 + |h_2|^2\right) b(1) + \mathrm{Re}\{\eta\} \end{aligned} \tag{4}$$

Either hard or soft decision of $b(1)$ can now be done. The statistical distribution of of $\left(|h_1|^2 + |h_2|^2\right)$ is proportional to χ_4^2, which provides a two fold diversity gain over over single antenna case. Even though, this is the best one can hope for in the absence of feedback information at the transmitter, this scheme comes with the prohibitive penalty of of requiring two spreading codes per user. This could potentially lead to a reduction in the effective number of users the system can simultaneously support by a factor of two.

2.3 Orthogonal Transmit Diversity

An early inclusion in IS-2000 standard, *orthogonal transmit diversity* (OTD) is an open loop transmit diversity scheme which offers some diversity gain without requiring extra resources. Here, each user's bit stream $\{b_n(k)\}$ is first split into two independent data streams, namely the even bit stream $\{b_{n1}(k)\}$ and the odd bit stream $\{b_{n2}(k)\}$. For convenience of notation, we drop the index n and represent the two bit streams of the kth user as $\{b_1(k)\}$ and $\{b_2(k)\}$. The transmitted signals from the two antennas for OTD are

$$\underline{s}_1 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} b_1(k)\underline{c}_1(k) \qquad \underline{s}_2 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} b_2(k)\underline{c}_2(k) \tag{5}$$

where

$$\underline{c}_1(k) = \begin{bmatrix} \underline{c}(k) \\ 0_{N\times 1} \end{bmatrix}_{2N\times 1}, \qquad \underline{c}_2(k) = \begin{bmatrix} 0_{N\times 1} \\ \underline{c}(k) \end{bmatrix}_{2N\times 1}$$

Notice that $\underline{s}_1$ and $\underline{s}_2$ both span two symbol periods and that $\underline{c}_1(k)$ and $\underline{c}_2(k)$ are of length $2N$ and mutually orthogonal, i.e., $\underline{c}_1^\dagger(i)\underline{c}_2(j) = 0,\ \forall\ i,j \in \{1,2\ldots,K_u\}$. Hence, there is no waste of resources in OTD. Again, the received signal $\underline{r}$ is given by (2), which after despreading with $\underline{c}_1(1)$ and $c_2(1)$, gives

$$d_1 = \underline{c}_1^\dagger(1)\underline{r} = N\sqrt{\frac{E_c(k)}{2}}h_1 b_1(1) + \underline{c}_1^\dagger(1)\underline{n}$$
$$d_2 = \underline{c}_2^\dagger(1)\underline{r} = N\sqrt{\frac{E_c(k)}{2}}h_2 b_2(1) + \underline{c}_2^\dagger(1)\underline{n}$$

The decision statistics for the substreams $\{b_1(1)\}$ and $\{b_2(1)\}$ are obtained as

$$\begin{aligned} y_1 &= \mathrm{Re}\{h_1^* d_1\} = N\sqrt{\frac{E_c(k)}{2}}|h_1|^2 b_1(1) + \mathrm{Re}\{\eta_1\} \\ y_2 &= \mathrm{Re}\{h_2^* d_2\} = N\sqrt{\frac{E_c(k)}{2}}|h_2|^2 b_2(1) + \mathrm{Re}\{\eta_2\} \end{aligned} \tag{6}$$

where $\eta_i = h_i^* \underline{c}_i^\dagger(1)\underline{n}\,,\ i \in \{1,2\}$. If we compare (6) and (4), we notice that OTD provides only a one-fold diversity. The advantage of this lop-sided diversity is that if the channel from one of the antennas goes into a deep fade then the receiver may be able to recover atleast half of the bit stream coming from the other antenna. In contrast, however, using two distinct codes per user in (4) recovers both the even and odd data stream provided both h_1 and h_2 are not in deep fade simultaneously.

2.4 Space-Time Spreading with Two Transmitter Antennas

Using the same notation as the previous section, i.e., splitting the user's data stream into even and odd streams $\{b_1(k)\}$ and $\{b_2(k)\}$ respectively, we let the signal transmitted from antenna 1 be

$$\underline{s}_1 = \sum_{k=1}^{K_u}\sqrt{\frac{E_c(k)}{2}}\big(b_1(k)\underline{c}_1(k) + b_2(k)\underline{c}_2(k)\big) \tag{7}$$

and the signal transmitted from antenna 2 be

$$\underline{s}_2 = \sum_{k=1}^{K_u}\sqrt{\frac{E_c(k)}{2}}\big(b_2(k)\underline{c}_1(k) - b_1(k)\underline{c}_2(k)\big) \tag{8}$$

where $\underline{c}_1(k),\ \underline{c}_2(k)$ are any set of orthogonal $2N \times 1$ spreading codes, i.e., $c_1^\dagger(i)c_2(j) = 0,\ \forall\ i,j \in \{1,2,\ldots,K_u\}$.

Again, observe that we are using two spreading codes of length $2N$ for each user, but employing both codes for the two bits. Like OTD, there is no waste of resources in

Space-Time Spreading (STS). The received signal (2), after despreading with $\underline{c}_1(1)$ and $\underline{c}_2(1)$ gives

$$d_1 = \underline{c}_1^\dagger(1)\underline{r} = N\sqrt{\frac{E_c(k)}{2}}\big(h_1 b_1(1) + h_2 b_2(1)\big) + \underline{c}_1^\dagger(1)\underline{n}$$
$$d_2 = \underline{c}_2^\dagger(1)\underline{r} = N\sqrt{\frac{E_c(k)}{2}}\big(-h_2 b_2(1) + h_1 b_2(1)\big) + \underline{c}_2^\dagger(1)\underline{n}$$

respectively. The bit stream decision variables y_1 and y_2 for this case are

$$\begin{aligned} y_1 &= \mathrm{Re}\left\{h_1^\dagger d_1 - h_2^\dagger d_2\right\} = N\sqrt{\frac{E_c(k)}{2}}\big(|h_1|^2 + |h_2|^2\big)b_1(1) + \mathrm{Re}\left\{\eta_1\right\} \\ y_2 &= \mathrm{Re}\left\{h_2^\dagger d_1 + h_1^\dagger d_2\right\} = N\sqrt{\frac{E_c(k)}{2}}\big(|h_1|^2 + |h_2|^2\big)b_2(1) + \mathrm{Re}\left\{\eta_2\right\} \end{aligned} \tag{9}$$

where η_1, η_2 are just complex Gaussian noise terms. We see that (9) provides the optimal two-fold diversity gain. Also compared to the scheme given in section 2.2, STS does not incur any extra expenses in the form of code sequence utilization.

Discussion : The Space-Time Spreading scheme described above has been tailored to the case of two transmit antennas. In [1], the authors are concerned with obtaining diversity gains for more than two antennas and for complex $\{b\}$. They generalize the above scheme for an arbitrary number of transmit antennas M. Let Q be the number of substreams in which each user is split and let $L \geq Q$ be the number of spreading codes per Q substreams. The above scheme works for $M = 2$ and $L = Q$, but for $M > 2$, they show that we cannot always find a scheme for which $L = Q$. In fact, for complex $\{b\}$, there exits an $M = 2$ antenna scheme with $L = Q$, but for $M > 2$, we must always have $L > Q$.

The derivation of these results is concerned to a problem in orthogonal designs studied in [3] and applied to multiple antenna block coding in [4]. In this report we are not concerned with code design and construction and restrict attention to the $M = 2$, which had been accepted as an optional diversity mode in IS-2000 standard. In the following sections, we evaluate the performance of this STS scheme in presence of Multipath Fading and Multiple Access Interference (MAI). First, we summarize some of the results presented in [1].

3 Performance Analysis Results As Presented in [1]

3.1 Flat Fading, Non-Perfect Channel Estimates

Decoding is predicated on the knowledge of the fading coefficients for each of the M transmitter antennas that is provided to the mobiles through the transmission of M

pilot signals. In [1], authors consider the effect of non-perfect channel state information on the probability of making an error. In particular, they obtain exact expressions for raw (uncoded) probability of error that directly incorporates the effect of the error in estimating the fading coefficients. The expressions disclose two counteracting effects: Increasing the number of transmitter antennas tends to give greater diversity gains, but if the total pilot power is fixed, the individual estimates for the fading coefficients deteriorate, and crosstalk increases among the subusers.

3.1.1 Real Signals

Consider a STS scheme with $M = L = Q$ and real valued BPSK (± 1) signals. Assume that the spreading codes of the users are perfectly orthogonal and there are no multipath components. Then, the exact expression for the probability of bit error for non-perfect channel estimates as given in [1, (43)] is

$$P_e = \frac{1}{(4+4\beta)^M} \sum_{j=0}^{M-1} \frac{\Gamma(2M-1-j)}{\Gamma(M)\Gamma(M-j)} \left[\frac{2\sqrt{4+4\beta}}{\sqrt{4\beta}+\sqrt{4+4\beta}} \right]^{1+j} \tag{10}$$

where $\beta = \frac{E_p E_b}{N_0 M(N_0 M + E_p + E_b)}$. $\frac{E_b}{N_0}$ and $\frac{E_p}{N_0}$ denote the SNRs of the message bearing and pilot signals respectively.

Notice that the probability of error (10) depends only of M and β. The effect of imperfect knowledge of the fading coefficients, as manifested by a finite value for the expected SNR of the pilot $\frac{E_p}{N_0}$, is an effective reduction in the SNR of the message bearing signal $\frac{E_b}{N_0}$.

Fig. 1 shows the plots of the probability of error, according to (10) for $M = 1, 2, 4, 8$, as functions of $\frac{E_b}{N_0}$, where $\frac{E_p}{N_0} = 10\frac{E_b}{N_0}$ (i.e. 10dB greater). Holding the total transmitter power fixed, while increasing the number of transmitter antennas, trades off two opposing effects: gain in diversity versus reduction in the quality of the channel estimates due to reduced power per pilot. As can be seen from Fig. 1, for low SNRs, increasing the number of antennas can actually increase the probability of error.

3.1.2 Complex Signals

Assuming $M = L = Q$ and complex valued signals with $|b_k| = 1$, the exact expression for the pairwise probability of error between signals $\underline{b}$, $\underline{b}'$ is given by

$$P_e = \frac{1}{(4+\delta^2\beta)^M} \sum_{j=0}^{M-1} \frac{\Gamma(2M-1-j)}{\Gamma(M)\Gamma(M-j)} \left[\frac{2\sqrt{4+\delta\beta}}{\sqrt{\delta\beta}+\sqrt{4+\delta\beta}} \right]^{1+j} \tag{11}$$

where $\delta = |\underline{b} = \underline{b}'|$ and β is as given for the real case. For derivation of results (10) and (11) we refer the reader to [1].

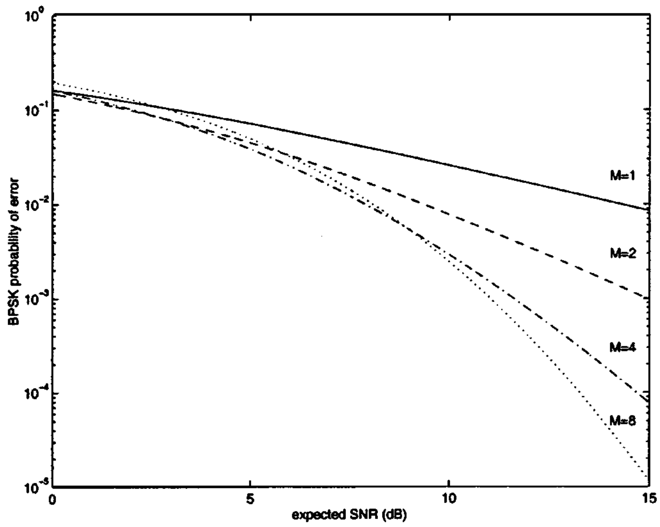

Figure 1: Probability of error for BPSK modulation versus expected message SNR for one, two, four, and eight antennas, where errors in estimation the propagation coefficients are accounted for; total pilot power is 10 dB stronger than the total message power for each user.

3.2 Frequency Selective Fading, Perfect Channel Estimates

In the presence of multipath, the same transmission scheme as (7),(8) can be used, but a RAKE-type architecture is required for reception. When favorable multipaths conditions exist, a RAKE-type receiver can yield additional diversity gains, beyond those attained by STS. For the performance analysis, the authors of [1] make the following assumptions

A1 Two transmitter antennas ($M = 2$) and one receiver antenna.

A2 Codes $\{\underline{c}_l\}$ are real with gain $2N$ so that $2NE_c = E_b$.

A3 Input data stream $\{b_k\}$ are BPSK.

A4 Channel for each transmitter antenna to the receiver antenna comprises of J distinct paths.

A5 The J paths from antenna m experience independent Rayleigh fading, expressed through the channel coefficients $h_{m,j},\ m = 1, 2,\ j = 1, \ldots, J$.

A6 Each pair of paths from the two transmitter antennas arrives with the same set of delays to the receiver antenna.

A7 Path delays are approximately a few chips in duration and small compared with the symbol period so that intersymbol interference (ISI) can be neglected.

A8 The spreading codes $\underline{c}_l(1)$ of the desired user 1 and all its delayed versions are orthogonal to the spreading codes of all the other users, i.e., $\underline{c}_{l,j}(1)^\dagger \underline{c}_{m,i}(k) = 0,\ \forall\ l, m \in \{1, 2\},\ \forall\ i, j \in \{1, \ldots, J\}$ and $\forall\ k \in \{2, \ldots, K_u\}$

Assumption A8 is justified because of the development in [1], where the authors assume the absence of interfering users. Assuming the spreading codes (and all its delayed versions) of the desired user 1 to be completely orthogonal to the codes (and all their delayed versions) of all the other users is equivalent to neglecting the effect of MAI. We drop A8 in the next section where we consider the effect of multiple access interference on STS.

The transmitted signals are described again by (7) and (8). A spreading code $\underline{c}_l$ arrives at the receiver as a set of J signature vectors $\{\underline{c}_{l,1}, \underline{c}_{l,2}, \ldots, c_{l,J}\}$, which are just the delayed versions of $\underline{c}_l$. As a result of multipath, the received signal is

$$\underline{r} = \sum_{k=1}^{K_u} \sqrt{\tfrac{E_c(k)}{2}} \Bigg[\sum_{j=1}^{J} h_{1,j} \Big(b_1(k)\underline{c}_{1,j}(k) + b_2(k)\underline{c}_{2,j}(k) \Big) + \sum_{j=1}^{J} h_{2,j} \Big(b_2(k)\underline{c}_{1,j}(k) - b_1(k)\underline{c}_{2,j}(k) \Big) \Bigg] + \underline{n} \tag{12}$$

Under assumption A8, the probability of error for bits $b_1(1)$ and $b_2(1)$ of the desired user, conditioned on the fading coefficients are shown in [1, (50)] to be

$$P_{e|H}^{(1)} = \frac{1}{2} Q \left[\mathrm{Re}\{g_{11}(1) + g_{12}(1)\} \sqrt{\frac{E_b}{g_{11}(1) N_0}} \right] + \frac{1}{2} Q \left[\mathrm{Re}\{g_{11}(1) - g_{12}(1)\} \sqrt{\frac{E_b}{g_{11}(1) N_0}} \right] \tag{13}$$

$$P_{e|H}^{(2)} = \frac{1}{2} Q \left[\mathrm{Re}\{g_{22}(1) + g_{21}(1)\} \sqrt{\frac{E_b}{g_{22}(1) N_0}} \right] + \frac{1}{2} Q \left[\mathrm{Re}\{g_{22}(1) - g_{21}(1)\} \sqrt{\frac{E_b}{g_{22}(1) N_0}} \right] \tag{14}$$

respectively, where

$$G(k) = \begin{bmatrix} (\mathbf{C}_1(1)\underline{h}_1 - \mathbf{C}_2(1)\underline{h}_2)^\dagger \\ (\mathbf{C}_2(1)\underline{h}_1 + \mathbf{C}_1(1)\underline{h}_2)^\dagger \end{bmatrix} \cdot \left[(\mathbf{C}_1(k)\underline{h}_1 - \mathbf{C}_2(k)\underline{h}_2) \quad (\mathbf{C}_2(k)\underline{h}_1 - \mathbf{C}_1(k)\underline{h}_2) \right] \tag{15}$$

Here, $\mathbf{C}_l(k) := [\underline{c}_{l,1}(k) \ldots \underline{c}_{l,J}(k)]$, $l \in \{1,2\}$ is a $N \times J$ matrix, which is a collection of all delayed versions of $\underline{c}_l(k)$ and $\underline{h}_l := [h_{l,1} \ldots h_{l,J}]$.

4 Multiuser Interference

As mentioned previously, the sytem model studied in [1] is a single user system. In any spread spectrum scheme, multiuser interference contributes significantly to the overall interference and can not be ignored. In this section, we analyze the performance of the scheme proposed in section 2.4 in the presence of multipath and multiple access interference from other users. In particular, we drop the assumption A8 of the previous section.

4.1 System Model

We use the same model as presented in Section 2.4 with the exception that we now consider each user to be modulated by an additional carrier phase. Thus (7) and (8) change slightly, and the transmitted signals are now given by

$$\underline{s}_1 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} \big(b_1(k)\underline{c}_1(k) + b_2(k)\underline{c}_2(k)\big) \cos\phi_k \tag{16}$$

$$\underline{s}_2 = \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} \big(b_2(k)\underline{c}_1(k) - b_1(k)\underline{c}_2(k)\big) \cos\phi_k \tag{17}$$

Similar to the earlier case, the down-converted received signal is

$$\begin{aligned}\underline{r} = \sum_{k=1}^{K_u} \sqrt{\tfrac{E_c(k)}{2}} \Bigg[\sum_{j=1}^{J} h_{1,j} \Big(b_1(k)\underline{c}_{1,j}(k) + b_2(k)\underline{c}_{2,j}(k)\Big) \cos\theta_k \\ + \sum_{j=1}^{J} h_{2,j} \Big(b_2(k)\underline{c}_{1,j}(k) - b_1(k)\underline{c}_{2,j}(k)\Big) \cos\theta_k \Bigg] + \underline{n}\end{aligned} \tag{18}$$

where, $\theta_k = \phi_k - \phi_1$ and $\underline{c}_{i,j}$ is the shifted version of $\underline{c}_i$. Here we have assumed that the multipaths are delayed by a multiple of chip intervals. The above equation can be rewritten as

$$\underline{r} = \sum_{k=1}^{K_u} \sqrt{\tfrac{E_c(k)}{2}} \left[(\mathbf{C}_1(k)\underline{h}_1 - \mathbf{C}_2(k)\underline{h}_2) \quad (\mathbf{C}_2(k)\underline{h}_1 + \mathbf{C}_1(k)\underline{h}_2) \right] \begin{bmatrix} b_1(k) \\ b_2(k) \end{bmatrix} \cos\theta_k + \underline{n} \tag{19}$$

where $\mathbf{C}_i = \left[\underline{c}_{i,1}(k) \quad \underline{c}_{l,2}(k) \quad \dots \quad \underline{c}_{i,J}(k)\right]$, $\underline{h}_i = \left[h_{i,1} \quad \dots \quad h_{i,J}\right]^T$, $i \in \{1,2\}$.
After despreading for the first user we get,

$$\underline{d}_1 = \mathbf{C}_1^\dagger(1) \cdot \underline{r} \tag{20}$$

$$\underline{d}_2 = \mathbf{C}_2^\dagger(1) \cdot \underline{r} \tag{21}$$

The bit decision variables y_1 and y_2 are

$$\begin{aligned} y_1 &= \mathrm{Re}\left\{\underline{h}_1^\dagger \underline{d}_1 - \underline{h}_2^\dagger \underline{d}_2\right\} \\ y_2 &= \mathrm{Re}\left\{\underline{h}_2^\dagger \underline{d}_1 + \underline{h}_1^\dagger \underline{d}_2\right\} \end{aligned} \quad \text{and} \quad \begin{aligned} y_1 &\underset{b_1(1)=-1}{\overset{b_1(1)=+1}{\gtrless}} 0 \\ y_2 &\underset{b_2(1)=-1}{\overset{b_2(1)=+1}{\gtrless}} 0 \end{aligned} \tag{22}$$

Defining $\underline{y} = [y_1 \ y_2]^T$, we have

$$\underline{y} = \mathrm{Re}\left\{ \sum_{k=1}^{K_u} \sqrt{\tfrac{E_c(k)}{2}} \mathbf{D}^\dagger(1)\mathbf{D}(k)\underline{b}\cos\theta_k + \mathbf{D}(1)^\dagger \underline{n} \right\} \tag{23}$$

where $\mathbf{D}(k) = \big[(\mathbf{C}_1(k)\underline{h}_1 - \mathbf{C}_2(k)\underline{h}_2) \quad (\mathbf{C}_2(k)\underline{h}_1 + \mathbf{C}_1(k)\underline{h}_2)\big]$

$$\Rightarrow \underline{y} = \mathrm{Re}\left\{ \sum_{k=1}^{K_u} \sqrt{\tfrac{E_c(k)}{2}} \mathbf{G}(k)\underline{b}\cos\theta_k + \underline{\eta}_1 \right\} \tag{24}$$

where $\mathbf{G}$ is a 2×2 matrix given by $\mathbf{G}(k) \triangleq \mathbf{D}^\dagger(1)\mathbf{D}(k)$ and $\underline{\eta}_1 \triangleq \mathbf{D}^\dagger(1)\underline{n}$.

4.2 Probability of error analysis

In this section we find bounds on the error probability when the channel state information is known to the receiver.

Consider the probability of making an error while decoding $b_1(1)$

$$P_{e|H}^{(1)} = \Pr\{\, \mathrm{Re}\{Y_1\} > 0 \mid b_1(1) = -1,\, \underline{h}_1,\, \underline{h}_2 \,\} \tag{25}$$

$$= \Pr\left\{ \mathrm{Re}\left\{ \sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} \Big(g_{1,1}(k)b_1(1) + g_{1,2}(k)b_2(1) \Big) \cos\theta_k + \eta_1 \right\} > 0 \;\middle|\; b_1(1) = -1,\, \underline{h}_1,\, \underline{h}_2 \right\} \tag{26}$$

$$= \Pr\left\{ \left[\sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} \Big(\tilde{g}_{1,1}(k)b_1(1) + \tilde{g}_{1,2}(k)b_2(1) \Big) \cos\theta_k + \tilde{\eta}_1 \right] > 0 \;\middle|\; b_1(1) = -1,\, \underline{h}_1,\, \underline{h}_2 \right\} \tag{27}$$

where $\tilde{g}_{i,j}(1) = \mathrm{Re}\{g_{i,j}(1)\}$ and $\tilde{\eta}_1 = \mathrm{Re}\{\eta_1\}$. Applying Chernoff's Bound we have

$$P_{e|H}^{(1)} \leq \min_{\rho>0} \mathbf{E}\left\{ \exp\rho \left[\sum_{k=1}^{K_u} \sqrt{\frac{E_c(k)}{2}} \Big(\tilde{g}_{1,1}(k)b_1(k) + \tilde{g}_{1,2}(k)b_2(k) \Big) \cos\theta_k + \tilde{\eta}_1 \right] \;\middle|\; \underline{b}_1(1) = -1, \underline{h}_1, \underline{h}_2 \right\} \tag{28}$$

which can be rewritten as

$$P_{e|H}^{(1)} \leq \min_{\rho>0} \mathbf{E}\left\{ \begin{array}{c} \exp\left(\rho\sqrt{\dfrac{E_c(1)}{2}}\tilde{g}_{1,1}(1)b_1(1)\right) \exp\left(\rho\sqrt{\dfrac{E_c(1)}{2}}g_{1,2}(1)b_2(1)\right) \\ \times \displaystyle\prod_{k=2}^{K_u}\prod_{i=1}^{2} \exp\left(\rho\sqrt{\dfrac{E_c(k)}{2}}\tilde{g}_{1,i}(k)b_i(k)\cos\theta_k\right) \exp\left(\rho\eta_1\right) \end{array} \;\middle|\; \underline{b}_1(1) = -1, \underline{h}_1, \underline{h}_2 \right\} \tag{29}$$

Now there are two ways in which we can evaluate the expectation in the above expression. We can assume that the codes used for different users are fixed and try to find an upper bound on the performance of the system. Or we can assume that only the spreading codes of the first user is fixed and try to find a bound on the best that we can hope to achieve. Both these developments are presented here.

4.2.1 Bound on Probability of Error for a Given Set of Codes for Users

In this section, we assume that the spreading codes of all the users are fixed, implying that the correlations are fixed. Looking at each of the terms of (29) one by one we have

$$\mathbf{E}\left\{\exp\left(\rho\sqrt{\frac{E_c(1)}{2}}\tilde{g}_{1,1}(1)b_1(1)\right)\right\} = \exp\left(-\rho\sqrt{\frac{E_c(1)}{2}}g_{1,1}(1)\right) \tag{30}$$

$$\begin{aligned}\mathbf{E}\left\{\exp\left(\rho\sqrt{\frac{E_c(1)}{2}}g_{1,2}(1)b_2(1)\right)\right\} &= \frac{1}{2}\left\{\exp\left(\rho\sqrt{\frac{E_c(k)}{2}}g_{1,2}(1)\right) + \exp\left(-\rho\sqrt{\frac{E_c(k)}{2}}g_{1,2}(1)\right)\right\} \\ &\leq \exp\left(\rho\sqrt{\frac{E_c(k)}{2}}\,|\tilde{g}_{1,2}(1)|\right)\end{aligned} \tag{31}$$

$$\begin{aligned}\mathbf{E}\left\{\exp\left(\rho\sqrt{\frac{E_c(k)}{2}}\tilde{g}_{1,i}(k)b_i(k)\cos\theta_k\right)\right\} &= \frac{1}{2}I_o\left(\rho\sqrt{\frac{E_c(k)}{2}}\tilde{g}_{1,i}(k)\right) + \frac{1}{2}I_o\left(-\rho\sqrt{\frac{E_c(k)}{2}}\tilde{g}_{1,i}(k)\right) \\ &\leq \exp\left(\frac{1}{4}\rho^2\frac{E_c(k)}{2}\left(\tilde{g}_{1,i}(k)\right)^2\right)\end{aligned} \tag{32}$$

η_1 is a Gaussian variable with $\mu = 0$ and $\sigma^2 = g_{1,1}(1)\frac{N_0}{2}$. Note that $\tilde{g}_{1,1}(1) = g_{1,1}(1)$

$$\mathbf{E}\left\{\exp\left(\rho\eta_1\right)\right\} = \exp\left(\frac{1}{2}\rho^2 g_{1,1}(1)\frac{N_0}{2}\right) \tag{33}$$

Substituting (30)-(33) in the (29) we get,

$$P_{e|H}^{(1)} \leq \min_{\rho>0}\left\{\begin{array}{r}\exp\left\{-\rho\sqrt{\frac{E_c(1)}{4}}\left(g_{1,1}(1) - |g_{1,2}(1)|\right)\right\} \\ \times\exp\left\{\frac{1}{2}\rho^2\left[\sum_{k=2}^{K_u}\frac{E_c(k)}{2}\left((\tilde{g}_{1,1}(k))^2 + (\tilde{g}_{1,2}(k))^2\right) + g_{1,1}(1)\frac{N_0}{2}\right]\right\}\end{array}\right\} \tag{34}$$

Note that

$$\min_{\rho>0}\exp\left\{A\rho + B\rho^2\right\} = \exp\left\{\frac{-A^2}{4B}\right\} \tag{35}$$

Therefore

$$P_{e|H}^{(1)} \leq \exp\left\{-\frac{\frac{E_c(1)}{2}\left(g_{1,1}(1) - |g_{1,2}(1)|\right)^2}{g_{1,1}(1)N_0 + \sum_{k=2}^{K_u}\frac{E_c(k)}{2}\left((\tilde{g}_{1,1}(k))^2 + (\tilde{g}_{1,2}(k))^2\right)}\right\} \tag{36}$$

Observing that $g_{1,1}(1) = 2N\Big(\sum_{j=1}^{J} \hat{h}_{1,j}h_{1,j} + \sum_{j=1}^{J} \hat{h}_{2,j}h_{2,j}\Big)$ and rearranging we get,

$$P_{e|H}^{(1)} \leq \exp\left\{ -\frac{\left(\sum_{j=1}^{J} h_{1,j}^2 + \sum_{j=1}^{J} h_{2,j}^2\right)\frac{E_b(1)}{2}\left[1 - \left|\frac{g_{12}(1)}{g_{11}(1)}\right|\right]^2}{\frac{1}{2Ng_{11}(1)}\sum_{k=2}^{K_u}\frac{E_c(k)}{2}\big((g_{1,1}(k))^2 + (g_{1,2}(k))^2\big) + N_0} \right\} \tag{37}$$

Notice that the scheme provides a two-fold diversity as expected. Also the interference from other users depends on the correlation coefficients $g_{1,1}(k)$ and $g_{1,2}(k)$. Knowing the spreading sequences used for all the users, the cross correlations $\mathbf{G}(k)$ are known. So we can evaluate the probability of error. The actual performance of this system has been simulated in Section 5 and as it turns out, the bound is not very tight.

4.2.2 Bound on Probability of Error for Fixed Codes for the Desired User

We want to know what is the best performance that we can achieve for given channel conditions for a particular choice of codes for a single user. We are free to choose the codes for other users to optimize the performance. To evaluate the error probability, we bound the performance of the system averaged over all possible code assignments. Then we know that there exists atleast one system that performs better than this average.

To evaluate this, we take the expectation over all possible values of $g_{1,i}(k)$ in (29). The terms (30) and (31) which depend only on the spreading sequence of the desired user remain the same. But now the interference due to other users, (32) will change. This can be bound by considering what happens at the chip level. Let us look again at the correlation matrix $\mathbf{G}(k)$

$$\mathbf{G}(k) \triangleq \mathbf{D}^{\dagger}(1)\mathbf{D}(k) \tag{38}$$

$$= \begin{bmatrix} \begin{pmatrix} \underline{h}_1^{\dagger}\mathbf{C}_1^{\dagger}(1)\mathbf{C}_1(k)\underline{h}_1 - \underline{h}_1^{\dagger}\mathbf{C}_1^{\dagger}(1)\mathbf{C}_2(k)\underline{h}_2 \\ +\underline{h}_2^{\dagger}\mathbf{C}_2^{\dagger}(1)\mathbf{C}_2(k)\underline{h}_2 - \underline{h}_2^{\dagger}\mathbf{C}_2^{\dagger}(1)\mathbf{C}_1(k)\underline{h}_1 \end{pmatrix} & \begin{pmatrix} \underline{h}_1^{\dagger}\mathbf{C}_1^{\dagger}(1)\mathbf{C}_2(k)\underline{h}_1 - \underline{h}_2^{\dagger}\mathbf{C}_2^{\dagger}(1)\mathbf{C}_2(k)\underline{h}_1 \\ +\underline{h}_1^{\dagger}\mathbf{C}_1^{\dagger}(1)\mathbf{C}_1(k)\underline{h}_2 - \underline{h}_2^{\dagger}\mathbf{C}_2^{\dagger}(1)\mathbf{C}_1(k)\underline{h}_2 \end{pmatrix} \\ \begin{pmatrix} \underline{h}_1^{\dagger}\mathbf{C}_2^{\dagger}(1)\mathbf{C}_1(k)\underline{h}_1 - \underline{h}_1^{\dagger}\mathbf{C}_2^{\dagger}(1)\mathbf{C}_2(k)\underline{h}_2 \\ +\underline{h}_2^{\dagger}\mathbf{C}_1^{\dagger}(1)\mathbf{C}_1(k)\underline{h}_1 - \underline{h}_2^{\dagger}\mathbf{C}_1^{\dagger}(1)\mathbf{C}_2(k)\underline{h}_2 \end{pmatrix} & \begin{pmatrix} \underline{h}_1^{\dagger}\mathbf{C}_2^{\dagger}(1)\mathbf{C}_2(k)\underline{h}_1 + \underline{h}_2^{\dagger}\mathbf{C}_1^{\dagger}(1)\mathbf{C}_2(k)\underline{h}_1 \\ +\underline{h}_1^{\dagger}\mathbf{C}_2^{\dagger}(1)\mathbf{C}_1(k)\underline{h}_2 + \underline{h}_2^{\dagger}\mathbf{C}_1^{\dagger}(1)\mathbf{C}_1(k)\underline{h}_2 \end{pmatrix} \end{bmatrix} \tag{39}$$

Thus,

$$g_{1,1}(k) = \begin{Bmatrix} \underline{h}_1^{\dagger}\mathbf{C}_1^{\dagger}(1)\mathbf{C}_1(k)\underline{h}_1 - \underline{h}_1^{\dagger}\mathbf{C}_1^{\dagger}(1)\mathbf{C}_2(k)\underline{h}_2 \\ +\underline{h}_2^{\dagger}\mathbf{C}_2^{\dagger}(1)\mathbf{C}_2(k)\underline{h}_2 - \underline{h}_2^{\dagger}\mathbf{C}_2^{\dagger}(1)\mathbf{C}_1(k)\underline{h}_1 \end{Bmatrix} \tag{40}$$

To evaluate each of these four terms we consider

$$\mathbf{R}_{p,q}(1,k) \triangleq \mathbf{C}_p^\dagger(1)\mathbf{C}_q(k) \tag{41}$$

$$[\mathbf{R}_{p,q}(1,k)]_{i,j} = \underline{c}_{p,i}^\dagger(1)\underline{c}_{q,j}(k) \tag{42}$$

$$= \sum_{n=1}^{2N} c_n \tag{43}$$

where c_n are the i.i.d. chips $\{1,-1\}$. Thus one term of (40) is

$$\underline{h}_l\mathbf{C}_p^\dagger(1)\mathbf{C}_q(k)\underline{h}_m = \sum_{i=1}^{J}\sum_{j=1}^{J} h_{l,i}\,[\mathbf{R}_{p,q}(1,k)]_{i,j}\,h_{m,j} \tag{44}$$

$$= \sum_{i=1}^{J}\sum_{j=1}^{J} h_{l,i}\left(\sum_{n=1}^{2N} c_n\right) h_{m,j} \tag{45}$$

$$= \sum_{i=1}^{J}\sum_{j=1}^{J}\sum_{n=1}^{2N} h_{l,i}c_n h_{m,j} \tag{46}$$

Assuming the four terms of $g_{1,1}(k)$ given by (40) are independent, we get

$$\mathbf{E}\left\{\exp\left\{\rho\sqrt{\frac{E_c(k)}{2}}\tilde{g}_{1,1}(k)b_i(k)\cos\theta_k\right\}\right\} = \prod_{l,p,q,m}\mathbf{E}\left\{\exp\left\{\rho\sqrt{\frac{E_c(k)}{2}}\underline{h}_l\mathbf{C}_p^\dagger(1)\mathbf{C}_q(k)\underline{h}_m b_1(k)\cos\theta_k\right\}\right\} \tag{47}$$

$$= \prod_{l,p,q,m}\mathbf{E}\left\{\exp\left\{\rho\sqrt{\frac{E_c(k)}{2}}\sum_{i=1}^{J}\sum_{j=1}^{J}\sum_{n=1}^{2N} h_{l,i}c_n h_{m,j}b_1(k)\cos\theta_k\right\}\right\} \tag{48}$$

$$= \prod_{l,p,q,m}\prod_{i=1}^{J}\prod_{j=1}^{J}\prod_{n=1}^{2N}\mathbf{E}\left\{\exp\left\{\rho\sqrt{\frac{E_c(k)}{2}}h_{l,i}c_n h_{m,j}b_1(k)\cos\theta_k\right\}\right\} \tag{49}$$

$$= \prod_{l,p,q,m}\prod_{i=1}^{J}\prod_{j=1}^{J}\prod_{n=1}^{2N}\mathbf{E}\left\{I_0\left\{\rho\sqrt{\frac{E_c(k)}{2}}h_{l,i}c_n h_{m,j}b_1(k)\right\}\right\} \tag{50}$$

$$\leq \prod_{l,p,q,m}\prod_{i=1}^{J}\prod_{j=1}^{J}\prod_{n=1}^{2N}\mathbf{E}\left\{\exp\left\{\rho^2\frac{E_c(k)}{2}h_{l,i}^2c_n^2h_{m,j}^2b_1^2(k)\right\}\right\} \tag{51}$$

$$= \prod_{l,p,q,m}\prod_{i=1}^{J}\prod_{j=1}^{J}\prod_{n=1}^{2N}\exp\left\{\frac{1}{4}\rho^2\frac{E_c(k)}{2}h_{l,i}^2h_{m,j}^2\right\} \tag{52}$$

$$= \prod_{l,p,q,m} \exp\left\{\frac{1}{4}\rho^2\frac{E_c(k)}{2}2N\underbrace{\left(\sum_{i=1}^{J} h_{l,i}^2\right)}_{H_l}\underbrace{\left(\sum_{j=1}^{J} h_{m,j}^2\right)}_{H_m}\right\} \tag{53}$$

$$= \exp\left\{\frac{1}{2}\rho^2\frac{E_c(k)}{2}N\Big(H_1^2 + 2H_1H_2 + H_2^2\Big)\right\} \tag{54}$$

We will get the same expression for the term corresponding to $g_{1,2}(k)$ in (29). Combining all these we get

$$P_{e,1} \le \min_{\rho>0}\left\{\begin{array}{c}\exp\left\{-\rho\sqrt{\frac{E_c(1)}{2}}\big(g_{1,1}(1) - |g_{1,2}(1)|\big)\right\} \\ \times\exp\left\{\frac{1}{2}\rho^2\left[N\big(H_1+H_2\big)^2\sum_{k=2}^{K_u}\frac{E_c(k)}{2} + g_{1,1}(1)\frac{N_0}{2}\right]\right\}\end{array}\right\} \tag{55}$$

Using (35), we have

$$P_{e,1} \le \exp\left\{-\frac{\frac{E_c(1)}{2}\left(g_{1,1}(1) - |g_{1,2}(1)|\right)^2}{2N(H_1+H_2)^2\sum_{k=2}^{K_u}\frac{E_c(k)}{2} + g_{1,1}(1)N_0}\right\} \tag{56}$$

Observe that $g_{1,1}(1) = 2N\Big(\sum_{j=1}^{J} h_{1,j}h_{1,j} + \sum_{j=1}^{J} h_{2,j}h_{2,j}\Big)$. Rearranging we have,

$$\implies P_{e,1} \le \exp\left\{-\frac{\frac{E_c(1)}{2}4N^2(H_1+H_2)^2\left[1 - \left|\frac{g_{12}(1)}{g_{11}(1)}\right|\right]^2}{2N(H_1+H_2)^2\sum_{k=2}^{K_u}\frac{E_c(k)}{2} + 2N_0N\big(H_1+H_2\big)}\right\} \tag{57}$$

Rewriting this in terms of bit energies rather than chip energies we have

$$\boxed{P_{e|H}^{(1)} \le \exp\left\{-\frac{\big(\sum_{j=1}^{J} h_{1,j}^2 + \sum_{j=1}^{J} h_{2,j}^2\big)\frac{E_b(1)}{2}\left[1 - \left|\frac{g_{12}(1)}{g_{11}(1)}\right|\right]^2}{\frac{1}{2N}\big(\sum_{j=1}^{J} h_{1,j}^2 + \sum_{j=1}^{J} h_{2,j}^2\big)\sum_{k=2}^{K_u}\frac{E_b(k)}{2} + N_0}\right\}} \tag{58}$$

Notice that space-time coding is also providing a diversity gain to the interference by other users. But this interference is protected by the processing gain of $2N$, so this diversity gain of the interference does not effect the performance adversely.

Also note that there is a factor of $1 - \left|\frac{g_{1,2}(1)}{g_{1,1}(1)}\right|$ in the numerator. This means that the inteference caused by the signal for the desired user from the second antenna decreases

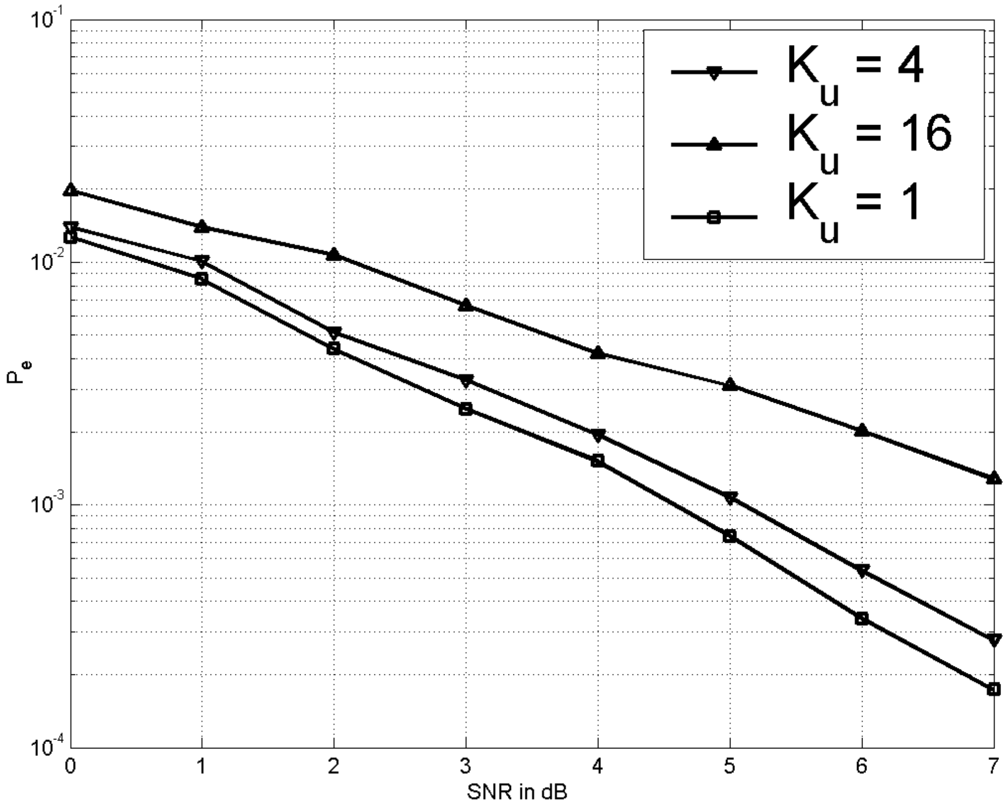

Figure 2: Performance simulation for the case of 1, 4 and 16 users

the signal power rather than increasing the interference. Generally we will choose $\underline{c}_1(1)$ and $\underline{c}_2(1)$ to be orthogonal, so that the correlation between the multipaths will be small, and will not affect the performance too much. Figure 3 compares this bound with the simulated performance of the system. We find that this bound is quite close to the actual performance.

5 Numerical Results

We simulated the performance of the system in absence of other users, as done in [1] and compared it with the performance in the presence of 4 and 16 users. Each user was randomly assigned two Walsh sequences of length 128. We assume two multipaths from each antenna and the delay between the multipaths is 10 chip intervals (which is less than 10% of the symbol duration, so ISI will be negligible). The result is shown in Figure. 2, which illustrates that the introduction of other users degrades performance.

In Figure. 3 we compare the union bounds given by (37) and (58) to the actual performance. As can be seen that the bound given by (37) is pretty loose and as such is

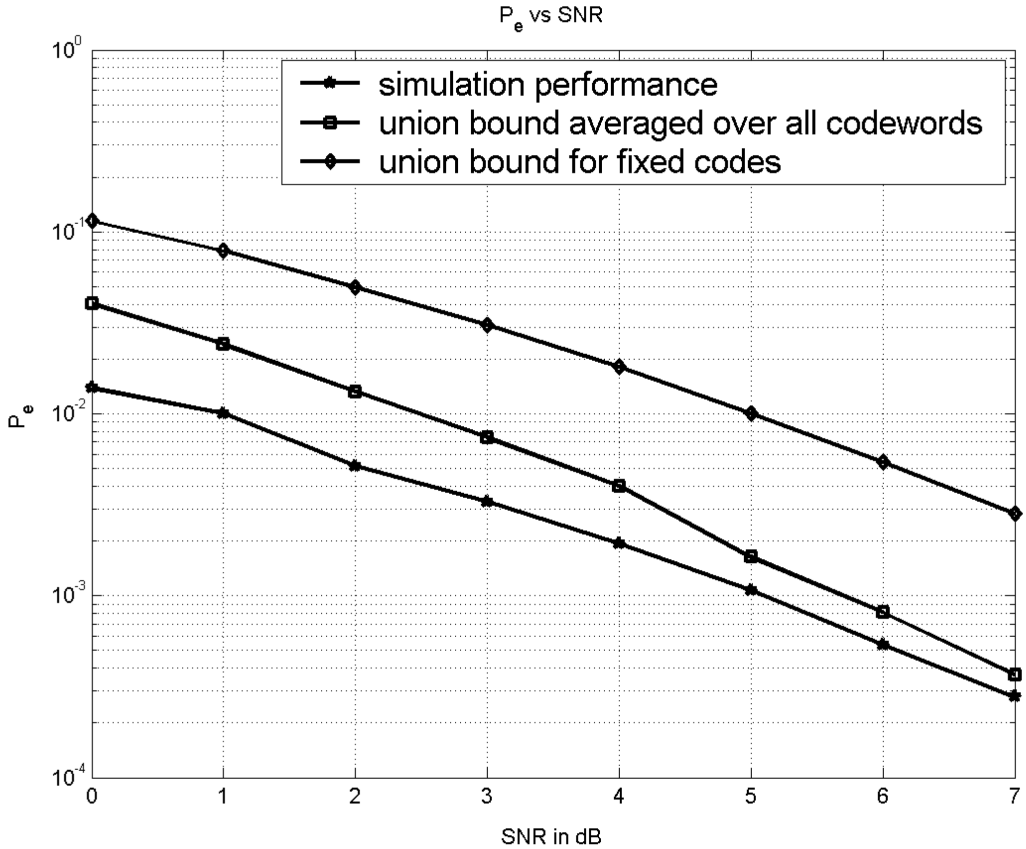

Figure 3: Union bound for probability of error averaged over all codewords, union bound for the given set of codewords and actual performance

not of much use. The bound given by (58) is much tighter, but is not an upper bound for any particular system. As explained earlier, this bound only tells us that there exists code assignments, such that the system can perform at least as well as the bound. But it is not necessary that the codes chosen for the system satisfy this criteria, so it can happen that a particular system performs worse than the bound.

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