# Optimal Performance of Feedback Control Systems with Limited Communication over Noisy Channels

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# Organization

1. Motivation
2. Problem Formulation
3. Explanation of the Solution Methodology
4. Extensions
5. Conclusion

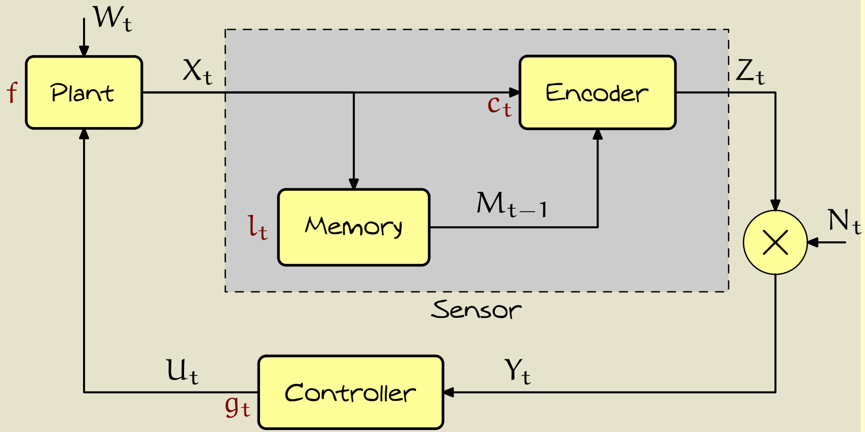
# Motivation

- Recently, there has been a focus on networked controlled systems (NCS).
- The simplest NCS has
  1. one sensor/encoder and one controller, and
  2. a noisy channel between the sensor and the controller.
- Two classes of problems have been studied:
  1. Identifying necessary and sufficient conditions for stability.
  2. Generalizations of the Bode Integral.

# Motivation

- Some applications, like vehicular traffic control, need stronger performance guarantees than stability.
- We consider the class of additive performance metric: total cost is the sum of costs along the entire path.

# *Problem Formulation*




$$\text{cost} = E\left\{ \sum_{t=1}^{T} \rho(X_t, U_t) \;\middle|\; c^T, l^T, g^T \right\}$$

# Common Knowledge



- Aumann's Notion of Common Knowledge

$$X : (\Omega, \mathcal{F}, P) \to (X, \mathcal{G}, P_X)$$

$$Y : (\Omega, \mathcal{F}, P) \to (Y, \mathcal{H}, P_Y)$$

Common Knowledge between X and Y is $\sigma(X) \cap \sigma(Y)$

# Information State

- One possible information state

$$\sigma\big(X_t, M_{t-1}, \Pr(X_t, M_{t-1} \mid Y^t, U^{t-1}, \gamma^t)\big) \supseteq \sigma(X_t, M_{t-1}) \cap \sigma(Y^t, U^{t-1}).$$

where $\gamma^t = (c^t, l^{t-1}, g^{t-1})$

# Information State



Let

$$\underline{B}_t(x, m) := \Pr(X_t = x, M_{t-1} = m \mid Y^{t-1}, U^{t-1}, c^{t-1}, l^{t-1}, g^{t-1})$$
$$B_t(x, m) := \Pr(X_t = x, M_{t-1} = m \mid Y^{t}, U^{t-1}, c^{t}, l^{t-1}, g^{t-1})$$
$$\overline{B}_t(x, m) := \Pr(X_t = x, M_{t-1} = m \mid Y^{t}, U^{t-1}, c^{t}, l^{t}, g^{t-1})$$

## Information States

$$\underline{\pi}_t := \Pr(X_t, M_{t-1}, \underline{B}_t)$$
$$\pi_t := \Pr(X_t, M_{t-1}, B_t)$$
$$\overline{\pi}_t := \Pr(X_t, M_{t-1}, \overline{B}_t)$$

# Information States

## Lemma

1. There exist linear transformations $\underline{Q}(c_t)$, $Q(l_t)$ and $\overline{Q}(g_t)$ such that

$$\pi_t := \underline{Q}(c_t)\underline{\pi}_t,$$

$$\overline{\pi}_t := Q(l_t)\pi_t,$$

$$\underline{\pi}_{t+1} := \overline{Q}(g_t)\overline{\pi}_t.$$

2. The conditional expected cost can be expressed as

$$E\{\rho(X_t, U_t) \mid c^t, l^t, g^t\} = \tilde{\rho}(\overline{\pi}_t, g_t),$$

where $\tilde{\rho}$ is a deterministic function.

# Sequential Decomposition

### Theorem

An optimal design $(C^*, L^*, G^*)$ can be obtained by the following nested optimality equations:

$$\overline{V}_T(\overline{\pi}) = \inf_{g_T} \tilde{\rho}(\overline{\pi}, g_T),$$

and for $t = 1, \ldots, T$

$$\underline{V}_t(\underline{\pi}) = \min_{c_t} V_t\big(\underline{Q}(c_t)\underline{\pi}\big),$$

$$V_t(\pi) = \min_{l_t} \overline{V}_t\big(Q(l_t)\pi\big),$$

$$\overline{V}_t(\overline{\pi}) = \inf_{g_t} \big\{\tilde{\rho}(\overline{\pi}, g_t) + \underline{V}_{t+1}\big(\overline{Q}(g_t)\overline{\pi}\big)\big\}.$$

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# Extensions

* Infinite Horizon Problems

  1. Expected Discounted Cost Per Unit Time
     *If $\rho$ is uniformly bounded, stationary designs are optimal.*
  2. Average cost per unit time
     *Under a technical condition (A1), stationary designs are $\epsilon$–optimal.*

* Sensors with imperfect observations

* No feedback link $\Longrightarrow$ real–time communication problem.

# *Conclusion & Remarks*

# Comclusions + Remarks



- **Computational Issues**

  1. Similarity to POMDPs — nature of information state.
  2. Difference from POMDPs — nature of “action space”.

- Identify special cases that are easy to solve

  1. Information state restricted to a parametric family.
  2. LQG does not work (cf. Witsenhausen’s Counterexample).

*Thank You*