On-time diagnosis of discrete event systems

Aditya Mahajan and Demosthenis Teneketzis

Dept. of EECS,
University of Michigan,
Ann Arbor, MI. USA.

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Fault Diagnosis in DES

1. Asymptotic (accuracy is critical; delay is important but not critical)
2. On-time (delay is critical; accuracy is important but not critical)

Most of the literature on diagnosis of DES has concentrated on asymptotic fault diagnosis.

Contribution of this paper

- Formulate on-time fault diagnosis as a minimax optimization problem.
- Use decision theory to provide a solution methodology.

Modelling questions

- What do we mean by "time"?
- What should the diagnoser/monitor do?
- How do we model performance?

When it is time to take a decision but the monitor is not sure that a fault has occurred, it will **make mistakes.**

Preliminaries

Language, Monitor, and Costs

Language

- Language L is prefix-closed, **finite, and bounded**

$$L = L_T \cup L_{NT}$$

- Terminal Strings: $L_T := \{s \in L : L \setminus s \neq \emptyset\}$
- Non-terminal Strings: $L_{NT} := L \setminus L_T$.

- Event Set $\Sigma = \Sigma_o \cup \Sigma_{uo} \implies$ natural projections.
- Observable events: Σ_o
- Unobservable events: Σ_{uo}.
- Fault event $f \in \Sigma_{uo}$.

Monitor

- Observes $P(L)$
- Upon observing an event, the monitor can:
  - **raise an alarm**, $\implies$ the system is shut down immediately.
  - **do nothing**, $\implies$ the system continues to operate.
- Monitoring policy $g : P(L) \to \{0, 1\}$
- Monitored sub-language $L|_g$

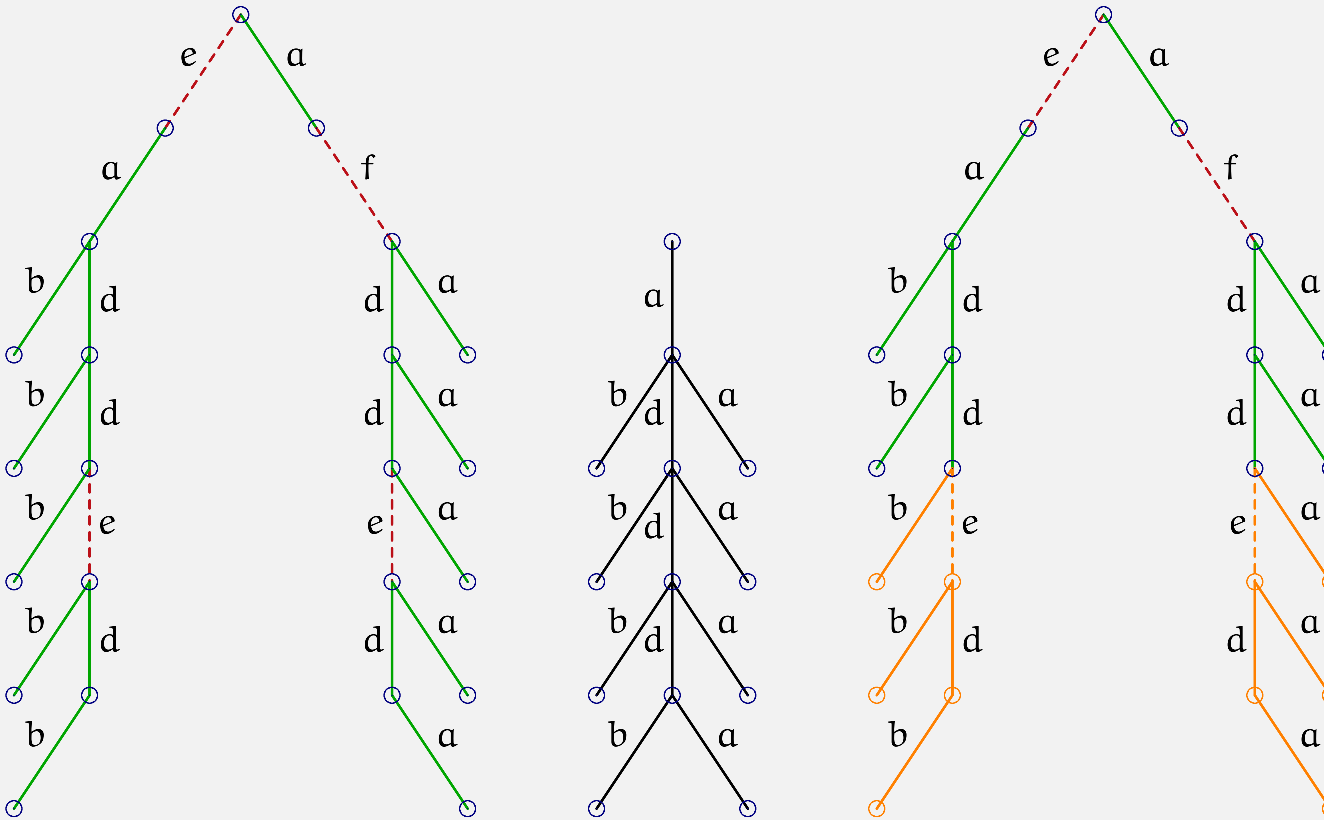
Sub-language where the system can stop

- Monitor raises an alarm $\implies$ system stops in $L^S_{NT} \cup L^S_T$

$$L^S_{NT} = \{s \cdot \sigma \in L_{NT} : \sigma \in \Sigma_o\}, \qquad L^S_T = \{s \cdot \sigma \in L_T : \sigma \in \Sigma_o\}$$

- Monitor does not raise an alarm $\implies$ system stops in L_T
- System can stop in $L^S = L^S_{NT} \cup L_T$
- For any g, $(L|_g)_T \subseteq L^S$

Example



Language L P(L) $L|_g$ for $g(add) = 1$

Quantifying timeliness

- After a fault has occurred, each event incurs a cost c.
- System is stopped in a non-faulty state $\Longrightarrow$ false alarm penalty of H_{NT}.
- System executes a terminal trace in a faulty state $\Longrightarrow$ additional terminal penalty of H_T.

Cost of stopping

- For $s \in L$, let
  - $\tau(s)$ be the first stage when a fault occurs in s.
  - n be the "length" of s
- for $s \in L^S_{NT}$, $C(s) = \begin{cases} (n - \tau(s))c, & \text{if } s \text{ contains a fault,} \\ H_{NT}, & \text{otherwise;} \end{cases}$
- for $s \in L_T$, $C(s) = \begin{cases} (n - \tau(s))c + H_T, & \text{if } s \text{ contains a fault,} \\ 0, & \text{otherwise.} \end{cases}$

Problem Formulation

The on-time diagnosis problem

- **Given**
  - Prefix-closed, finite, and bounded language L,
  - Observable events Σ_o, unobservable events Σ_{uo}, and fault event f
  - Cost c, fault alarm penalty H_{NT}, and a terminal penalty H_T.
- **Define**
  - $\mathcal{G}$ family of functions from $P(L)$ to $\{0, 1\}$
  - Performance of a monitoring policy $g \in \mathcal{G}$

$$\mathcal{J}(g) := \max_{s \in (L|_g)_T} C(s).$$

- **Choose**
  - A monitoring rule $g^* \in \mathcal{G}$ to minimize $\mathcal{J}(g)$

$$\mathcal{J}^* = \mathcal{J}(g^*) = \min_{g \in \mathcal{G}} \max_{s \in (L|_g)_T} C(s)$$

Centralized minimax optimization problem

Can be solved by dynamic programming

Some Notation

- $Q(t) := \{s \cdot \sigma \in P^{-1}(t) : \sigma \in \Sigma_o\}$
- $Q_T(t) := P^{-1}(t) \cap L_T$

Optimal monitoring rule

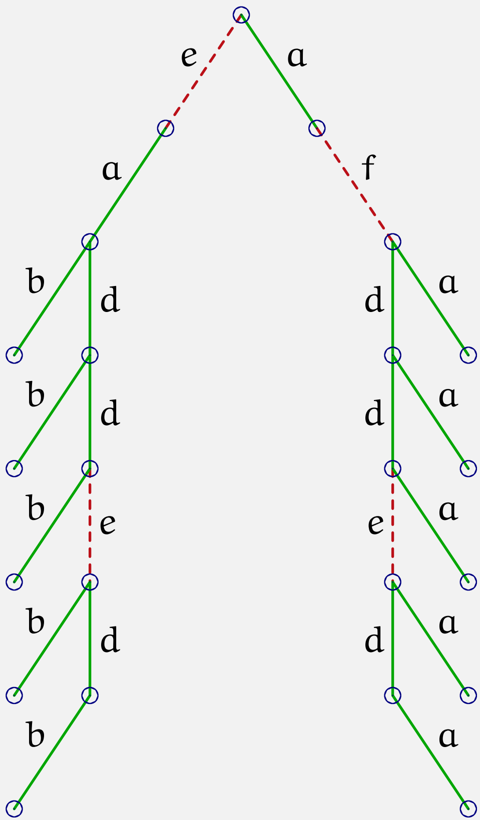
- For $t \in (P(L))_T$

$$\underbrace{V(t)}_{\substack{\text{minimum}\\ \text{worst case}\\ \text{cost to go at } t}} = \min\Big\{ \underbrace{\max_{s \in Q(t)} C(s)}_{\substack{\text{worst case cost}\\ \text{of stopping}}}, \underbrace{\max_{s \in Q_T(t)} C(s)}_{\substack{\text{worst case cost}\\ \text{of continuing}}} \Big\}$$

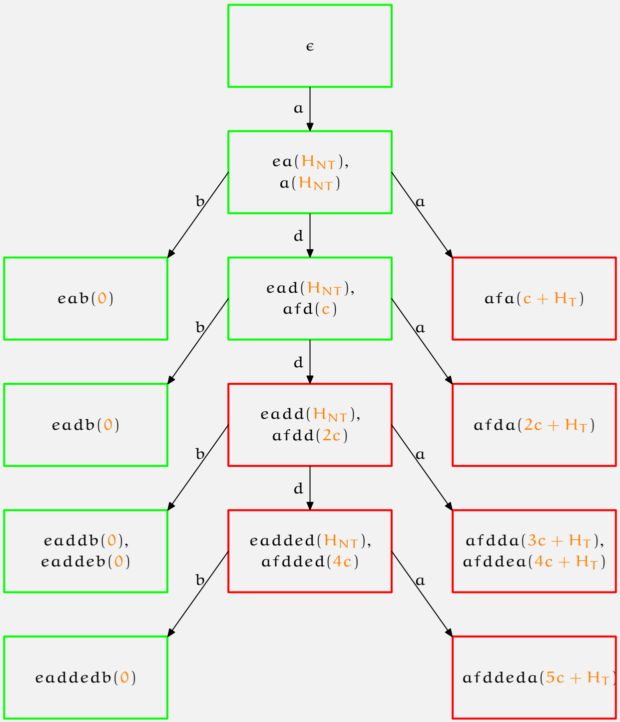
- For $t \in (P(L))_{NT}$, let $O_C(t) := \{e \in \Sigma : t \cdot e \in P(L)\}$, and

$$\underbrace{V(t)}_{\substack{\text{minimum}\\ \text{worst case}\\ \text{cost to go at } t}} = \min\Big\{ \underbrace{\max_{s \in Q(t)} C(s)}_{\substack{\text{worst case cost}\\ \text{of stopping}}}, \underbrace{\max\Big\{ \max_{s \in Q_T(t)} C(s), \max_{e \in O_C(t)} V(t \cdot e) \Big\}}_{\text{worst case cost of continuing}} \Big\}$$

Example



Language L



Optimal monitor for

$H_T = c$, $H_{NT} = 3c$

Relaxing some modelling assumptions

- **Live** languages

  Should be possible. Working on the details.

- Generalized costs

  Use a trace dependent cost in the paper

- Generalized projections

  Use **prefix-preserving** projections in the paper

Summary

- Formulate and solve on-time fault diagnosis problem.
- Penalize false alarm and (trace dependent) amount of delay in fault detection.
- Equivalent to a minimax optimization problem.

Thank you