ASSIGNED: Jan. 27, 2006. **READ:** Part 3c of Official Lecture Notes (available on-line).
DUE DATE: Feb. 03, 2006. **TOPICS:** Fourier series expansions of periodic functions.

Show work on separate sheets of paper. Include all hand and Matlab plots and code.

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[50] 1. $x(t) = t/2, 0 \leq t < 4$ is a single period of a sawtooth signal $x(t)$ with period=4.

Using the integral $\int_0^4 te^{-j\pi kt/2}dt = (8j)/(\pi k)$ for $k \neq 0$ (and $= 8$ for $k = 0$), compute constants $\{a_k, b_k, c_k, \theta_k, x_k\}$ in the following three Fourier series expansions of $x(t)$:

[5] (a) $x(t) = a_0 + a_1 \cos(\frac{2\pi}{T}t) + a_2 \cos(\frac{4\pi}{T}t) + \ldots + b_1 \sin(\frac{2\pi}{T}t) + b_2 \sin(\frac{4\pi}{T}t) + \ldots$

[5] (b) $x(t) = c_0 + c_1 \cos(\frac{2\pi}{T}t + \theta_1) + c_2 \cos(\frac{4\pi}{T}t + \theta_2) + c_3 \cos(\frac{6\pi}{T}t + \theta_3) + \ldots$

[5] (c) $x(t) = x_0 + x_1 e^{j\frac{2\pi}{T}t} + x_2 e^{j\frac{4\pi}{T}t} + \ldots + x_1^* e^{-j\frac{2\pi}{T}t} + x_2^* e^{-j\frac{4\pi}{T}t} + \ldots$

[5] (d) Using Matlab, plot the sum of the first 05 terms of (b), for $0 \leq t \leq 8$.

[5] (e) Using Matlab, plot the sum of the first 10 terms of (b), for $0 \leq t \leq 8$.

[5] (f) By hand, plot the line spectrum of $x(t)$ for frequencies up to 2 Hz.

[5] (g) $x(t) \rightarrow \boxed{\text{FILTER REJECTS} > 1\,\text{Hz; PASSES} \leq 1\,\text{Hz}} \rightarrow y(t)$
By hand, plot the line spectrum of $y(t)$ for frequencies up to 2 Hz.

[5] (h) Plot $y(t)$ itself for $0 \leq t \leq 8$. **Hint:** You may already have this plot!

[5] (i) Compute the average power of $y(t)$. **Hint:** Parseval's theorem.

[5] (j) What fraction of average power gets through the filter? That is, compute $\frac{MS(y)}{MS(x)}$.

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[20] 2. *Fourier series with only a finite number of terms:*

[10] (a) Compute the Fourier series expansion of $16\cos^4(t)$. **Hint:** $\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$.

[10] (b) Compute $\int_0^\pi \cos^8(t)\,dt$. **Hint:** Use Parseval's theorem and your answer to (a).

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[30] 3. *Truncating infinite Fourier series to a finite number of terms:*
$x(t)$ is periodic with period=6 and Fourier coefficients $x_k = 3^{-|k|}$ for integers k.
That is, $x_0 = 1, x_{-1} = x_1 = \frac{1}{3}, x_{-2} = x_2 = \frac{1}{9}$, etc. Note x_k real $\rightarrow x(t) = x(-t)$ even.

$\hat{x}_K(t) = \sum_{k=-K}^{k=K} x_k e^{j\frac{2\pi}{6}kt}$ is defined by truncating the Fourier series of $x(t)$.
Note that for large K, $\hat{x}_K(t) \approx x(t)$; the approximation gets better as K increases.
Significance: $\hat{x}_K(t)$ is specified using only $K+1$ real numbers $\{x_k, |k| \leq K\}$.

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[5] (a) By hand, plot the line spectrum of $x(t)$ (do **not** plot $x(t)$ itself, just its spectrum).

[5] (b) By hand, plot the line spectrum of $\hat{x}_2(t)$ (this should be *part* of your answer to (a)).

[5] (c) By hand, plot the line spectrum of $x(t) - \hat{x}_4(t)$ on same scale as your answer to (a).

[5] (d) Show that $RMS(x(t)) = \sqrt{5}/2$. **Hints:** Parseval and $1 + r + r^2 + \ldots = \frac{1}{1-r}$ if $|r| < 1$.

[5] (e) Show that $RMS(x(t) - \hat{x}_K(t)) = \frac{1}{2}3^{-|K|}$. **Hint:** Multiply above hint by r^{K+1}.
In (d) and (e), note that $\sum_{k=1}^{\infty} = \sum_{-\infty}^{-1}$ by symmetry; don't forget $k = 0$ term!

[5] (f) Compute the smallest K such that $RMS(x(t) - \hat{x}_K(t)) < 0.0005\,RMS(x(t))$, i.e., the percent or normalized RMS error in replacing $x(t)$ with $\hat{x}_K(t)$ is less than 0.05%.
Significance: $\hat{x}_K(t)$ is specified using only $K+1$ real numbers $\{x_k, |k| \leq K\}$.

Excuse heard in a genetic engineering class: "My homework ate the dog."