EECS 210 Winter 1997

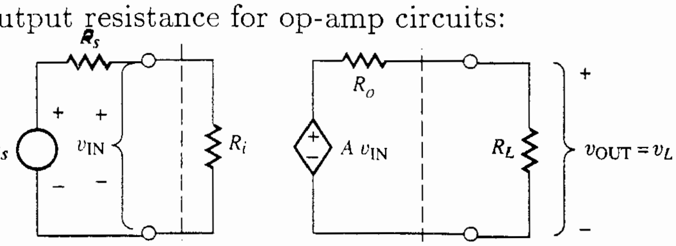
INPUT AND OUTPUT RESISTANCE FOR OP-AMP CIRCUITS

SIGNIFICANCE of input and output resistance for op-amp circuits:

Overall voltage gain $A' = \frac{v_{OUT}}{v_s}$

$= A\frac{R_i}{R_i+R_s}\frac{R_L}{R_o+R_L} < A$

$\rightarrow A$ as $R_i \rightarrow \infty$ and $R_o \rightarrow 0$.



DETERMINING input and output resistance for op-amp circuits:

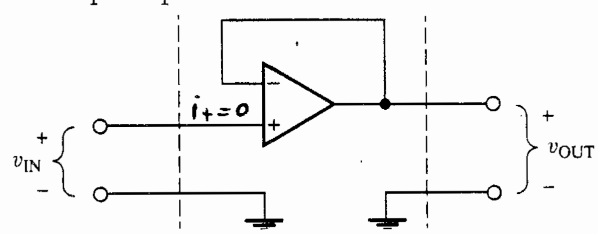
INPUT: Connect a V_{test} to *input* and compute resulting I_{test}. $R_i' = \frac{V_{test}}{I_{test}}$.
OUTPUT: Connect a V_{test} to *output* and compute resulting I_{test}. $R_o' = \frac{V_{test}}{I_{test}}$.
For output impedance R_o', set *input* voltage $v_s = 0$ and omit load R_L.
Note this is just the Thevenin resistance of the amplifier output.
GAIN: Connect a V_{test} to *input* and compute v_{OUT}. $A' = \frac{v_{OUT}}{v_s}$. Omit load R_L.

EXAMPLES of input and output resistance for op-amp circuits:

THE IDEAL $R_i' = \frac{V_{test}}{I_{test}} = \frac{V_{test}}{i_+} \rightarrow \infty$

FOLLOWER $R_o' = \frac{V_{test}}{I_{test}}|_{v_{IN}=0} = \frac{0}{I_{test}} = 0$

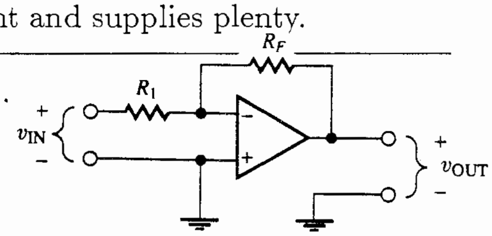
(since $v_{out} = 0$ we can't connect a $V_{test} \neq 0$ without producing an explosion!)



A follower is an ideal "front end": it draws no current and supplies plenty.

INVERTING $R_i' = \frac{V_{test}}{I_{test}} = R_i$ (obvious since $v_- = 0$).

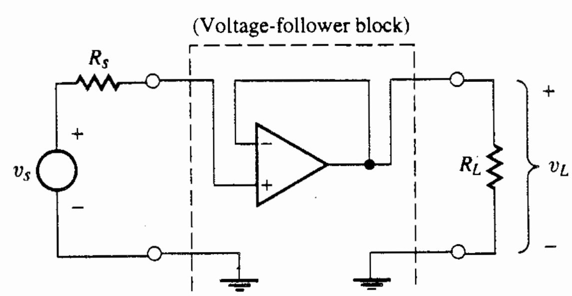
AMPLIFIER $R_o' = \frac{V_{test}}{I_{test}}|_{v_{IN}=0} = \frac{0}{I_{test}} = 0$



NON-IDEAL $R_i' = \frac{V_{test}}{I_{test}} = R_i\frac{(A+1)R_L+R_o}{R_L+R_o} + \frac{R_oR_L}{R_o+R_L}$

FOLLOWER $R_o' = \frac{V_{test}}{I_{test}} = \frac{R_o(R_i+R_s)}{R_o+(A+1)R_i+R_s}$

$A' = \frac{v_{OUT}}{v_s} = \frac{R_o+AR_i}{R_o+(A+1)R_i}$.



SIGNIFICANCE: Using typical op-amp values $A = 10^5, R_i = 2M\Omega, R_o = 75\Omega$, and a load $R_L = 1\Omega$ and source resistance $R_s = 1M\Omega$, the follower *still* acts almost ideally.
But other circuits (see p.161) may not!

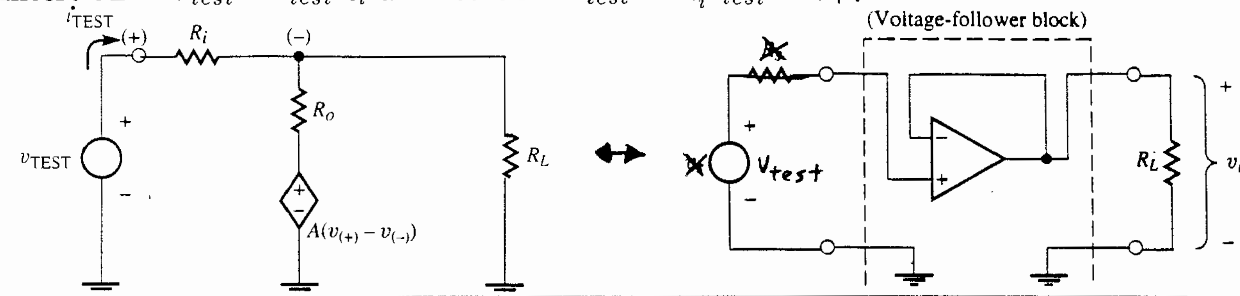
Let $R_i \rightarrow \infty$, $R_o \rightarrow 0$, $A \rightarrow \infty$. Then non-ideal results$\rightarrow$ideal results.

DETAILS OF ANALYSIS (also see Schwarz and Oldham p. 154-6):

INPUT: In figure below, write node equation at $v_- = v_{out}$ and note $v_+ = V_{test}$:

$$\frac{v_- - V_{test}}{R_i} + \frac{v_- - A(V_{test} - v_-)}{R_o} + \frac{v_-}{R_L} = 0$$

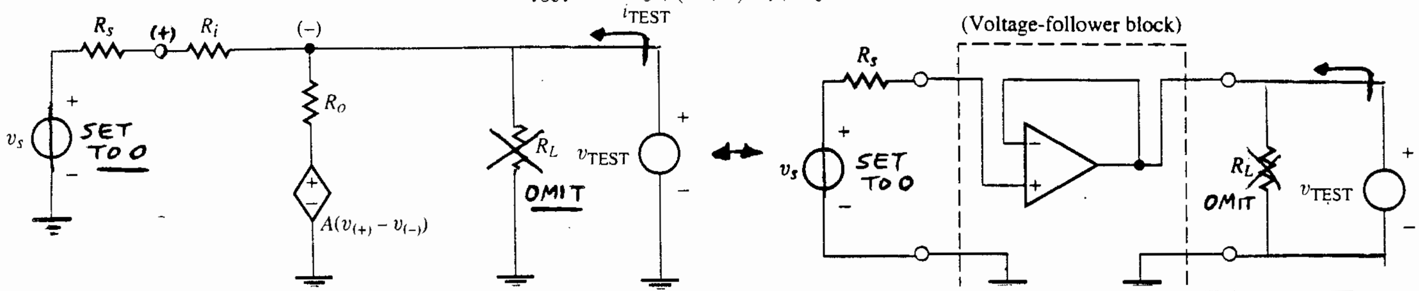
Insert $v_- = V_{test} - I_{test}R_i$ and solve for $V_{test} = R_i' I_{test} \rightarrow R_i'$.



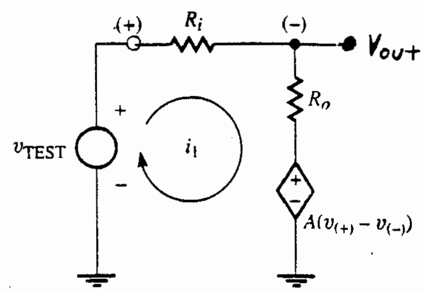
OUTPUT: See figure below. We have (note voltage divider)

$$I_{test} = \frac{V_{test}}{R_i+R_s} + \frac{V_{test} - A(-V_{test}\frac{R_i}{R_i+R_s})}{R_o} = V_{test}\frac{R_o+R_i+R_s+AR_i}{R_o(R_i+R_s)}$$

from which we get $R_o' = \frac{V_{test}}{I_{test}} = \frac{R_o(R_i+R_s)}{R_o+(A+1)R_i+R_s}$.



GAIN: See figure at right. We have



$$i_1 = \frac{V_{test} - A(V_{test} - v_{out})}{R_o + R_i}$$

and $v_{out} = V_{test} - i_1 R_i$. Substituting gives

$$A' = \frac{v_{OUT}}{v_{test}} = \frac{R_o + AR_i}{R_o + (A+1)R_i}.$$