

DEF: Node: A point to which two or more components are connected.

DEF: Ground: Reference node to which all other nodes are compared with regard to voltage. Think of it as “sea level” for node voltages.

DEF: Node voltage: Potential difference between the node and ground.

KVL: Satisfied since KVL states that node voltages are path-independent.

Note: Node analysis works for *non*-planar circuits (unlike mesh eqns).

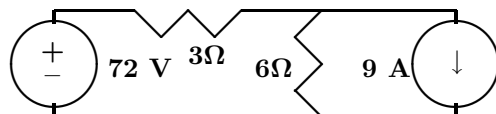
PROCEDURE FOR WRITING NODE EQUATIONS:

1. Select the *ground node*.

Usually this is the node that has the most components connected to it. Often circuits are drawn so this node is at bottom; don't count on it!

2. Define *node voltages* $\{V_1, V_2 \dots V_N\}$ at the other nodes.
3. Write **KCL** at each node: sum of currents leaving the node is zero. Do for each node except ground; Currents in terms of node voltages.
4. Each voltage source not connected to ground is regarded as a *supernode*: Write KCL for supernode, not the nodes voltage source connects.
5. *Dependent sources*: Express indpt variables in terms of node voltages.
6. Solve the linear system of equations for the unknown node voltages. Compute other voltages and currents of interest from node voltages.

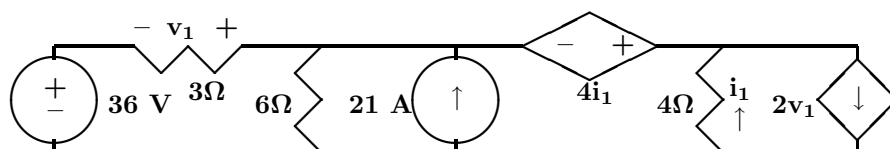
SIMPLE EXAMPLE



- Define *ground* as the node at the bottom of the diagram above.
- Define *node V* as the node at upper right of the diagram above.
- Write KCL at node *V*: Sum of currents *leaving* the node is zero: $(V - 72)/3 + (V/6) + 9 = 0 \rightarrow V = 30$.
- Compute other voltages and currents and check conservation of power:

ELEMENT	VOLTAGE	CURRENT	POWER
72 V :	72 (<i>source</i>)	$42/3 = 14$	$(72)(14) = 1008$
3 Ω :	$72 - 30 = 42$	$42/3 = 14$	$(42)(14) = 588$
6 Ω :	30 (<i>node</i>)	$30/6 = 5$	$(30)(05) = 150$
9 A :	30 (<i>node</i>)	9 (<i>source</i>)	$(30)(09) = 270$

- Power conserved: $1008 = 588 + 150 + 270$ checks.
- Note that the 9 A current source *dissipates* power (not unusual).

**COMPLEX
EXAMPLE**


Note: This example contains all four types of sources. Shows: supernodes; and dealing with dependent sources depending on voltage and current.

- Define *ground* as the node at the bottom of the diagram above.
- Define *node V* as the node at middle top of the diagram above.
- Write KCL at the *supernode*=dependent (on $4i_1$) voltage source:
 $(V - 36)/3 + V/6 - 21 + (V + 4i_1)/4 + 2v_1 = 0$
- Express indpt variables v_1 and i_1 in terms of node voltage V :
 $v_1 = V - 36$; $i_1 = -(V + 4i_1)/4 \rightarrow i_1 = -V/8$
- Substitute these into the supernode equation for V :
 $(V - 36)/3 + V/6 - 21 + (V - \frac{4}{8}V)/4 + 2(V - 36) = 0$
- Solve one equation in one unknown for V :
 $V[(1/3) + (1/6) + (1/8) + 2] = 36/3 + 21 + 2(36) \rightarrow V = 40$.
- Compute indpt voltages and currents from $V = 40$:
 $v_1 = V - 36 = 40 - 36 = 4$; $i_1 = -V/8 = -40/8 = -5$
- Compute current through dependent (on $4i_1$) voltage source:
 $i_{4i_1} = 2v_1 - i_1 = 2(4) - (-5) = 13$
- Compute other voltages and currents and check conservation of power:

ELEMENT	VOLTAGE	CURRENT	POWER
36 V :	36 (<i>source</i>)	$4/3 = 1.33$	$(36)(1.33) = 48$
3Ω :	$40 - 36 = 4$	$4/3 = 1.33$	$(4)(1.33) = 5.33$
6Ω :	40 (<i>node</i>)	$40/6 = 6.67$	$(40)(6.67) = 266.67$
21 A :	40 (<i>node</i>)	21 (<i>source</i>)	$(40)(21) = 840$
4i₁ :	$4(-5) = -20$	$i_{4i_1} = 13$	$(20)(13) = 260$
4Ω :	$-4(-5) = 20$	$i_1 = -5$	$(20)(5) = 100$
2v₁ :	$-4(-5) = 20$	$2v_1 = 8$	$(20)(8) = 160$

- Power conserved: $840 = 48 + 5.33 + 266.67 + 260 + 100 + 160$ checks.
- Note that three out of the four sources *dissipate* power (unusual).