
Given: **Circuit with:** resistors, inductors, capacitors, sinusoidal sources.
Goal: Compute any voltage or current in the circuit. **Know:** also sinusoids.
Want: Amplitude and phase of the sinusoid for each voltage or current.

EX: Voltage source $5\cos(6t)$ in series with 4Ω resistor and $\frac{1}{2}H$ inductor.
Goal: Compute the current $i(t)$ which flows through all three devices.

Hard Way: KVL$\rightarrow$ $5\cos(6t) - 4i - \frac{1}{2}\frac{di}{dt} = 0$. How do we solve this for $i(t)$?
Trial solution: $i(t) = A\cos(6t) + B\sin(6t)$. Substitute this in KVL:
$5\cos(6t) - 4A\cos(6t) - 4B\sin(6t) + \frac{1}{2}6A\sin(6t) - \frac{1}{2}6B\cos(6t) = 0$.
$[5 - 4A - 3B]\cos(6t) + [-4B + 3A]\sin(6t) = 0$. Set $t = 0$ and $\frac{\pi}{12} \rightarrow$
$5 = 4A + 3B$ and $0 = 3A - 4B$. Solving$\rightarrow$ $A = 0.8$ and $B = 0.6$.
Soln: $i(t) = 0.8\cos(6t) + 0.6\sin(6t) = \cos(6t - 37^o)$ using Problem Set #1.

Easy Way: **Trial sol'n:** $I(t) = Ie^{j6t}$. Substitute in KVL with $5\cos(6t) \rightarrow 5e^{j6t}$:
$5e^{j6t} - 4Ie^{j6t} - \frac{1}{2}6jIe^{j6t} = 0 \rightarrow I = 5/(4 + j3) = 1e^{-j37^o}$ (e^{j6t} cancels).
$I(t) = 1e^{j(6t-37^o)} \rightarrow i(t) = Re[I(t)] = \cos(6t - 37^o)$. MUCH easier!

Easier: $I = 5/(4 + j6\frac{1}{2}) = 5/(4 + j3) = 1e^{-j37^o} \rightarrow i(t) = \cos(6t - 37^o)$.

Q: What does $5e^{j6t}$ voltage source mean? Complex number voltage?
A: $5e^{j6t} = 5\cos(6t) + j5\sin(6t) = 2$ voltage sources connected in series.
Then: **Superposition**$\rightarrow$find response to $5\cos(6t)$ by setting $j5\sin(6t)$ to 0.
Means: Set $j = 0$ $\Leftrightarrow$take the **real part** of response (which has form Ie^{j6t}).
Here: $I(t)$=response to $5e^{j6t} \rightarrow Re[I(t)]$=response to $Re[5e^{j6t}] = 5\cos(6t)$.
Also: $Im[I(t)]$=response to $Im[5e^{j6t}] = 5\sin(6t)$: Solve 2 problems at once!

Phasors: Represent sinusoid $x(t) = M\cos(\omega t + \theta)$ with complex no. $X = Me^{j\theta}$.
Note: $x(t) = Re[Xe^{j\omega t}] = Re[Me^{j\theta}e^{j\omega t}] = Re[Me^{j(\omega t+\theta)}] = M\cos(\omega t + \theta)$.
EX#1: Simplify $x(t) = 3\cos(\omega t) + 3\cos(\omega t + 120^o) + 3\cos(\omega t + 240^o)$.
Hard: Use cosine addition formula$\rightarrow$mess. If do it right, get $x(t) = 0$ (?!)
Easy: Phasors: $X = 3e^{j0} + 3e^{j120^o} + 3e^{j240^o} = 0 \rightarrow x(t) = Re[Xe^{j\omega t}] = 0$!
Why? Draw picture in complex plane: easy to see resultant of these=0!

EX#2: Show that $5\cos(\omega t + 53^o) + \sqrt{2}\cos(\omega t + 45^o) = 6.4\cos(\omega t + 51^o)$.
Soln: $5e^{j53^o} + \sqrt{2}e^{j45^o} = (3 + j4) + (1 + j) = (4 + j5) = 6.4e^{j51^o}$. QED.

Note: $e^{\pm j\pi} = -1$. $j = e^{j\pi/2}$ and $-j = e^{j3\pi/2} \Leftrightarrow \cos(\omega t \pm 90^o) = \mp\sin(\omega t)$.

EECS 210 PHASORS, IMPEDANCE AND ADMITTANCE Winter 2001

Idea: Suppose voltage across and current through device are both sinusoids.
What: *Gain* and *phase shift* in going from current to voltage (and vice-versa).

Phasors: $i(t) = I_o \cos(\omega t) \Leftrightarrow I = I_o$ and $v(t) = V_o \cos(\omega t + \theta) \Leftrightarrow V = V_o e^{j\theta}$.
DEF: **Impedance** $Z = \frac{V}{I} = \frac{V_o}{I_o} e^{j\theta}$; **Admittance** $Y = \frac{1}{Z} = \frac{I}{V} = \frac{I_o}{V_o} e^{-j\theta}$.
Point: Can apply circuit analysis techniques to Z and Y, not just R and G.
Note: $e^{j\omega t}$ cancels throughout–don't even bother writing it at all!

Device name :	**Resistor**	**Inductor**	**Capacitor**
Its formula :	$v(t) = Ri(t)$	$v(t) = L\frac{di}{dt}$	$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$
Current i(t) :	$I_o \cos(\omega t)$	$I_o \cos(\omega t)$	$I_o \cos(\omega t)$
Voltage v(t) :	$RI_o \cos(\omega t)$	$-\omega L I_o \sin(\omega t)$	$\frac{I_o}{\omega C} \sin(\omega t)$
Gain; phase :	R; 0	ωL; $+90^o$	$\frac{1}{\omega C}$; -90^o
Impedance Z :	R	$j\omega L$	$\frac{1}{j\omega C} = \frac{-j}{\omega C}$
Admittance Y :	$\frac{1}{R}$	$\frac{1}{j\omega L} = \frac{-j}{\omega L}$	$j\omega C$
Z at DC($\omega = 0$) :	R	0	∞

DEF: $Z = R + jX$ where R=resistance and X=reactance (in Ω). $G \neq 1/R$.
DEF: $Y = G + jB$ where G=conductance and B=susceptance. $B \neq 1/X$.
Note: Impedances in series add; admittances in parallel add. $Z_1 || Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$.
EX: Capacitors in parallel: $Y = j\omega C_1 + \ldots + j\omega C_N = j\omega(C_1 + \ldots + C_N)$.

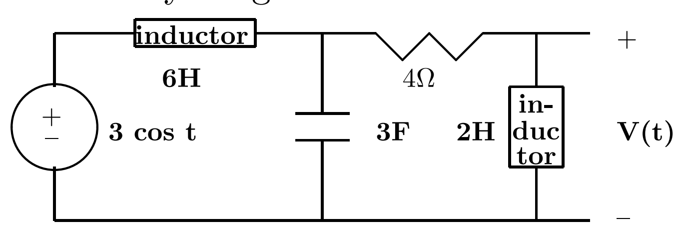
- **Phasors** are complex nos. that represent voltages, currents and sources.
- **Impedances** are nos. that represent resistors, inductors, capacitors.
- **Circuit analysis** includes KVL, KCL, node eqns, Thevenin/Norton.

EX: Illustrate various circuit techniques in the phasor domain:

1. Take **Thevenin equivalent** of everything left of the 2H inductor:

$$\mathbf{V_{OC}} = 3\frac{1/(j3)}{j6 + 1/(j3)} = \frac{3}{1-18} = -\frac{3}{17}.$$

$$\mathbf{R_{EQ}} = \frac{(j6)/(j3)}{j6 + 1/(j3)} + 4 = 4 - j\frac{6}{17}.$$



2. Now use **voltage divider** to compute voltage across 2H inductor:

$$V_L = -\frac{3}{17}\left[\frac{j2}{j2 + (4 - j6/17)}\right] = \frac{-j6}{68 + j28} = \frac{6e^{-j90^o}}{75e^{j20^o}} = 0.080e^{-j110^o}.$$

3. Convert from phasor to time domain: $v(t) = 0.080\cos(t - 110^o)$.