Given: Box containing resistors, inductors, capacitors with voltage and current phasors
$v(t) = |V|\cos(\omega t + \theta_v) \Leftrightarrow V = |V|e^{j\theta_v}; \quad i(t) = |I|\cos(\omega t + \theta_i) \Leftrightarrow I = |I|e^{j\theta_i}$.

Power: $p(t) = i(t)v(t) = |V||I|\cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$. Cosine addition formula:
$p(t) = \frac{1}{2}|V||I|\cos(\theta_v - \theta_i) + \frac{1}{2}|V||I|\cos(2\omega t + \theta_v + \theta_i)$= average+fluctuating power.

Note: Compare to resistor: $p(t) = i(t)^2R = |I|^2R\cos^2(\omega t) = \frac{1}{2}|I|^2R(1 + \cos(2\omega t))$.

AVERAGE POWER: $\bar{p} = \frac{1}{2}|V||I|\cos(\theta_v - \theta_i) = \frac{1}{2}RE[VI^*]$.
$\bar{p} = RE[V_{rms}I^*_{rms}] = |V_{rms}||I_{rms}|\cos(\theta_v - \theta_i)$, where $V_{rms} = \frac{V}{\sqrt{2}}, I_{rms} = \frac{I}{\sqrt{2}}$.
$\bar{p} = \frac{1}{2}|I|^2RE[Z] = \frac{1}{2}|V|^2RE[Y]$ where $Z = \frac{V}{I}$ and admittance $Y = \frac{I}{V}$ for the box.

EXAMPLE: Sinusoidal voltage source with phasor $V_{rms} = 120v$ at 60Hz drives a blender: 30Ω resistor in series with a $\frac{40}{377}H$ inductor. What power is dissipated?

We have $Z = R + j\omega L = 30 + j(2\pi 60)\frac{40}{377} = 30 + j40\Omega$ and then
$\bar{p} = |V_{rms}|^2RE[Y] = (120)^2RE[\frac{1}{30+j40}] = (120)^2RE[\frac{30-j40}{(50)^2}] = 172.8$ Watts.
Note that the voltage and current are $\tan^{-1}(\frac{40}{30}) = 53^o$ out of phase.

POWER FACTOR: $pf = \cos(\theta_v - \theta_i) = \cos(53^o) = 0.6$ for this example.

SIGNIFICANCE OF POWER FACTOR: $|I_{rms}| = \frac{|V_{rms}|}{|Z|} = \frac{120}{50} = 2.4$ Amps.

The *apparent power* delivered by source is $|V_{rms}||I_{rms}| = 288$ voltamps.

Hence it requires more current than it "should" to deliver 172.8 Watts to the load.

This causes the following problems:

1. Extra current must be generated by the source;
2. Wires must be bigger to carry the extra current without overloading;
3. Losses due to wire resistance dissipating power are greater.
4. Wire resistance→voltage drop→source voltage must be bigger.

Detroit Edison applies surcharges to industrial users if $pf < 0.9$ or so,
since it costs them money to build more power lines and supply more power.

What can we, as electrical engineers, do about this? **SEE OVER.**

MAXIMUM POWER TRANSFER:

Fixed source: Thevenin equivalent voltage phasor V_s; impedance Z_s. **Load:** Z_L.

What should (variable) Z_L be to maximize the power dissipated in the load Z_L?

SOLUTION: Let $Z_s = R_s + jX_s$ and $Z_L = R_L + jX_L$ where X_i =reactance.

$\bar{p} = \frac{1}{2}|I|^2RE[Z_L] = \frac{1}{2}|\frac{V_s}{(R_s+R_L)+j(X_s+X_L)}|^2R_L = \frac{|V_s|^2}{2}\frac{1}{(R_s+R_L)^2+(X_s+X_L)^2}R_L$.

Clearly $X_L = -X_s$; this is possible since reactance, unlike resistance, can be negative!

The problem is now a purely resistive problem; from before we know $R_L = R_s$.
We can combine $X_L = -X_s$ and $R_L = R_s$ into the single equation $Z_L = Z_s^*$.

CORRECTION OF POWER FACTOR:

FACT: Almost all loads "look inductive": voltage leads current.
This is because so many loads include coils (motors, speakers) which can be modelled as an inductor (the coil) in series with a resistor (coil resistance).

IDEA #1: Connect a capacitor C in *series* with the load:

Choose C so that $j\omega L + \frac{1}{j\omega C} = 0 \rightarrow C = \frac{1}{\omega^2 L}$.
Then $Z = R + j\omega L + \frac{1}{j\omega C} = R$ and the power factor is now 1.

EXAMPLE: For above example we want $C = 1/[(377)^2 \frac{40}{377}] = 67\mu F$.

PROBLEM: The power dissipated is now $\frac{(120)^2}{30} = 480$ Watts
since the current has jumped from $|I_{rms}| = 2.4$ Amp to $|I_{rms}| = 4$ Amp.

CONCLUSION: If load is driven by a *current source*, do this.
The current remains constant, so the power dissipated does not increase.

ASIDE: In fact this is a *resonant series RLC circuit*,
which is interesting in its own right and will be studied later.

IDEA #2: Connect a capacitor C in *parallel* with the load:

Then $\bar{p} = |V_{rms}|^2 RE[Y] = |V_{rms}|^2 RE[Y_{load} + j\omega C] = |V_{rms}|^2 RE[Y_{load}]$ =same.

Since $Y_{load} = \frac{1}{R+j\omega L} = \frac{R-j\omega L}{R^2+(\omega L)^2}$, we want $\frac{-j\omega L}{R^2+(\omega L)^2} + j\omega C = 0 \rightarrow C = \frac{L}{R^2+(\omega L)^2}$.

Then total load $Y = \frac{R}{R^2+(\omega L)^2}$ is real$\rightarrow pf = 1 \rightarrow \bar{p} = |V_{rms}|^2 \frac{R}{R^2+(\omega L)^2}$ =same.

The capacitor draws additional current at a different phase to make the total current drawn from the voltage source be in phase with the voltage.

EXAMPLE: Continuing the above example, $C = \frac{40/377}{(30)^2+(40)^2} = 42.4\mu F$.

The current is now $I_{rms} = V_{rms} Y = 120[\frac{1}{30+j40} + j(2\pi)(60)(42.4 \times 10^{-6})]$
$= 2.4e^{-j53^o} + j1.92 = 1.44 + j0$ Amp$\rightarrow$in phase with voltage 120 v.

Power dissipated is $V_{rms} I_{rms} = (120v)(1.44Amp) = 172.8$ Watts.
Since V_{rms} and I_{rms} are in phase, we just multiply them to get power:
Apparent power (in voltamps) and power dissipated (in watts) are equal since $pf = 1$.

CONCLUSION: If load is driven by a *voltage source*, do this.
The voltage remains constant, so the power dissipated does not increase.

COMPLEX POWER: We can write $S = V_{rms} I_{rms}^* = |S| e^{j\theta} = P + jQ$ where

$S =$ *complex* power and $|S| =$ *apparent* power (in voltamps);
$P = |S|\cos(\theta) =$ *power dissipated* (in watts);
$Q = |S|\sin(\theta) =$ *reactive power* (in vars [VoltAmpsReactive]).

Correcting the power factor to 1 is equivalent to making $Q = 0$.