Given: Transfer function $H(j\omega)$ having *complex* poles and zeros.

Solve: Single factor of form $\frac{\omega_n^2}{-\omega^2+2\zeta\omega_n(j\omega)+\omega_n^2} = \frac{\omega_n}{j\omega+p}\frac{\omega_n}{j\omega+p^*}$ (note sign of p)

where: Pole $p = |p|e^{j\theta} = \omega_n e^{j\cos^{-1}\zeta} \Leftrightarrow \omega_n = |p|, \zeta = \cos\theta, Re[p] = \omega_n\zeta$.

Then: Since dB add, we can just add the results for each term of this form.

$\omega \to 0$: $|H(j\omega)| \simeq 1$=0 dB. Horizontal line at 0 dB. Corner frequency: $\omega = \omega_n$

$\omega \to \infty$: $|H(j\omega)| \simeq \frac{\omega_n^2}{\omega^2}$. Slope: -40 dB/decade=-12 dB/octave through $\omega = \omega_n$

Peak: $|H(j\omega)| = \frac{\omega_n^2}{\sqrt{(\omega^2-\omega_n^2)^2+(2\zeta\omega_n\omega)^2}} = \frac{\omega_n^2}{\sqrt{(2\zeta\omega_n\omega_n)^2}} = \frac{1}{2\zeta} = \frac{1}{2}\sec\theta$ at $\omega = \omega_n$

Height: $|20\log_{10}(2\zeta)| = |20\log_{10}(2\cos\theta)|$ dB (need $|\cdot|$ since < 0; see below).

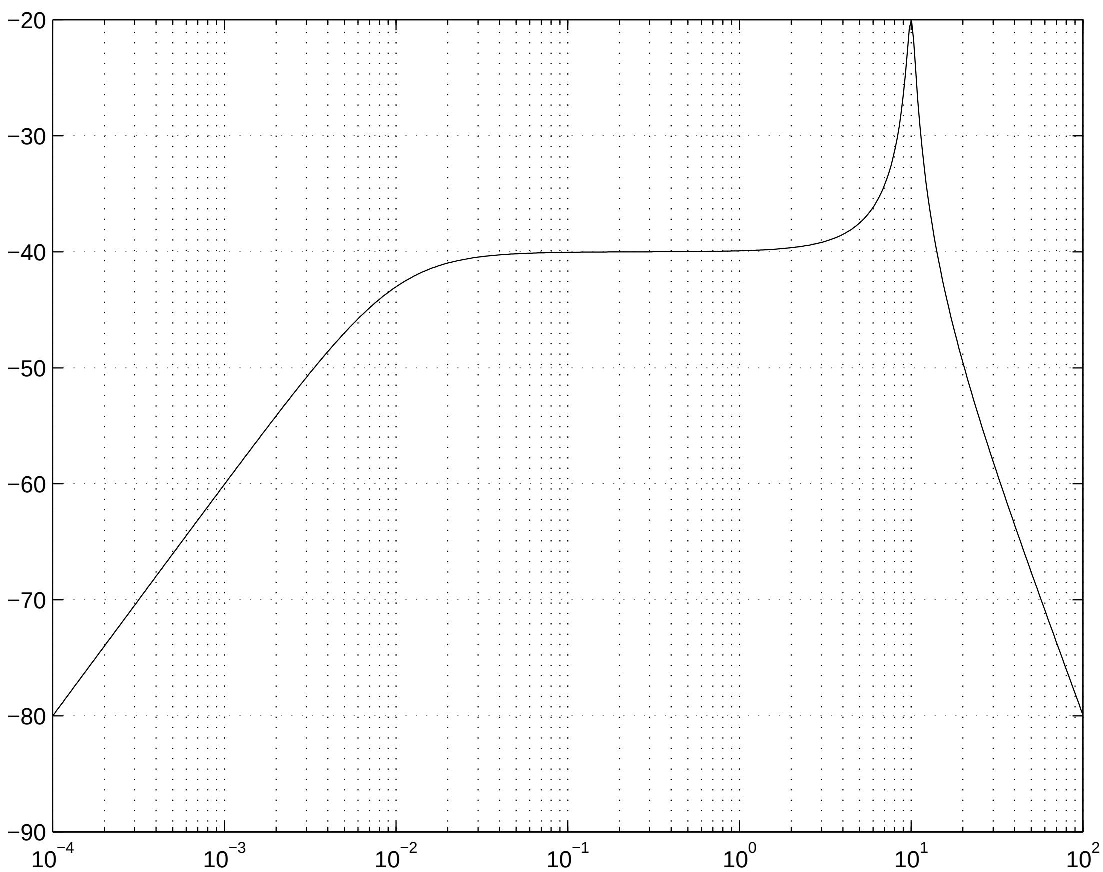
Note: Assumes $\zeta << 1$ (visible peak); else see text p.750 (ugly formulae).

EX: $H(j\omega) = (j\omega)/[(j\omega)^3 + 1.01(j\omega)^2 + 100.01(j\omega) + 1]$. Bode amplitude:

Soln: $H(j\omega) = (j\omega)/[(j\omega + 0.01)((j\omega)^2 + (j\omega) + 100)]$. 1 zero, 3 poles.

Factor: $\omega_n = \sqrt{100} = 10$. $2\zeta\omega_n = 1 \to \zeta = \frac{1}{20}$. Height=$|20\log_{10}\frac{2}{20}| = 20$ dB.

Also: Zero at origin $\omega = 0$ (20 dB/dec.); Pole at $\omega = 0.01$ (add -20 dB/dec.)



Given: Voltage source phasor V_{IN} (input) connected to R, L and C in series.
where: Measure voltage V_{OUT} (output) across resistor R (voltage divider).
Goal: Compute Bode amplitude plot for transfer function $H(j\omega) = \frac{V_{OUT}}{V_{IN}}$.

$H(j\omega) = \frac{R}{R+j\omega L+\frac{1}{j\omega C}} = \frac{j\omega\frac{R}{L}}{(j\omega)^2+j\omega\frac{R}{L}+\frac{1}{LC}}$. **Poles:** $\frac{R}{2L} \pm \frac{R}{2L}\sqrt{1-\frac{4}{R^2}\frac{L}{C}}$ (*n.b.:* sign).

$R >> \sqrt{\frac{L}{C}}$: Poles $p_1 = \frac{1}{RC}$ and $p_2 = \frac{R}{L}$ using $x = \frac{4L}{R^2C} << 1 \rightarrow \sqrt{1-x} \approx 1 - \frac{x}{2}$.
Bandpass filter: Cutoff freqs $f_L = \frac{1}{2\pi RC}$ (lower); $f_H = \frac{R}{2\pi L}$ (upper).
EX: $R = 100\Omega; L = 1H; C = 0.01F \rightarrow$Poles:1;100. Bode plot at lower left.
Note: R & C form *highpass* filter; R & L form *lowpass* filter. dB gains add.

$R << \sqrt{\frac{L}{C}}$: Complex poles at $\frac{R}{2L}(1 \pm j2Q) \Leftrightarrow \omega_n = \frac{1}{\sqrt{LC}}; \zeta = \frac{R/2}{\omega_n L} = \frac{1}{2Q}; Q = \frac{\omega_n L}{R}$.
Peak: Peak height=$20\log_{10} Q$ at (near) **resonant frequency** $\omega_n = \frac{1}{\sqrt{LC}}$.
Sharp? **Half-power freqs**: $|H(j\omega_L)| = |H(j\omega_H)| = 0.707$ where $|H(j\omega_n)| = 1$
3 dB: $|H(j\omega)| = \frac{R}{\sqrt{R^2+(\omega L - \frac{1}{\omega C})^2}} = \frac{1}{\sqrt{2}}$ when $\omega L - \frac{1}{\omega C} = \pm R \rightarrow$quadratic.
Sharp: $\frac{\omega_n}{\omega_H - \omega_L} = \frac{\omega_n L}{R}$ since $\omega_H - \omega_L = \frac{R}{L}$. For large Q, $\omega_{L,H} \approx \omega_n \pm \frac{R}{2L}$.

$\omega = \omega_n$: $V_L = j\omega_n L\frac{V_{IN}}{R} = \frac{\omega_n L}{R}e^{j90^o}V_{IN}$; $\quad V_C = \frac{-j}{\omega_n C}\frac{V_{IN}}{R} = \frac{\omega_n L}{R}e^{-j90^o}V_{IN}$.
since: $\omega_n L = \frac{1}{\omega_n C}$. Note large amplitudes and 180^o phase difference.

Q: **Quality** $Q = \frac{\omega_n L}{R} = \frac{1}{\omega_n RC} = \frac{\omega_n}{\omega_H - \omega_L}$. At $\omega = \omega_n$, $|\frac{V_L}{V_I}| = |\frac{V_C}{V_I}| = Q$.
Note: These formulae assume $Q > 10$; else see text p.720 (ugly formulae).

EX: $R = 1\Omega; L = 1H; C = 0.01F$: $\omega_n = \frac{1}{\sqrt{(1)(0.01)}} = 10$. $\zeta = \frac{1/2}{(10)(1)} = \frac{1}{20}$.
Peak: $Q = \frac{(10)(1)}{1} = 10 >> 1$. Height=$20\log_{10} 10 = 20$ dB. Plot below right.
L,C: $V_L = 10e^{j90^o}V_I$ and $V_C = 10e^{-j90^o}V_I$. Note *resonance amplification.*