HOW TO GET POWER: Read **The Prince** by N. Machiavelli.

HOW TO DEFINE IT: Power(watts) is energy(joules) per unit time(seconds).
To move charge q "upward" across potential difference v takes energy $E = vq$.
A charge q "falling" from higher to lower potential gains energy $E = vq$.
This energy has to be dissipated somewhere.

If this happens continuously in time, as with *current* (flowing charge),

Power(watts)$=\frac{dE}{dt} = \frac{d}{dt}(vq) = v\frac{dq}{dt} = vi$=(volts)(amps).

If potential difference caused by resistor for which $v = iR$, Power$=vi = i^2R = v^2/R$.
Note this is always non-negative. Makes sense: resistors dissipate power (get warm).

Instantaneous and Average Power for Sinusoidal Voltages and Currents

Now let $v(t) = V_0 \cos(\omega t + \phi)$ for any $\omega \neq 0, V_0, \phi$.
DEF: *Instantaneous power* dissipated$=p(t) = v(t)i(t) = i^2(t)R = v^2(t)/R$.

Here $p(t) = \frac{1}{R}V_0^2 \cos^2(\omega t + \phi) = \frac{1}{R}V_0^2 \frac{1}{2}(1 + \cos(2\omega t + 2\phi))$.

This is a sinusoid+constant, and the frequency of the sinusoid has doubled.

DEF: *Average power* dissipated$=\bar{p} = \frac{1}{P}\int_{t_0}^{t_0+P} p(t)dt$ where $P = \frac{2\pi}{\omega}$=period. Here

$\bar{p} = \frac{1}{P}\int_{t_0}^{t_0+P} \frac{1}{R}V_0^2 \frac{1}{2}(1 + \cos(2\omega t + 2\phi))dt = \frac{V_0^2}{2R} + \frac{V_0^2}{2R}\frac{1}{P}\int_{t_0}^{t_0+P} \cos(2\omega t + 2\phi)dt = \frac{V_0^2}{2R}$.
IN WORDS: the average value of a constant is the constant (here $\frac{V_0^2}{2R}$);
the average value of a sinusoid over an integer number of periods is zero.

RMS (Root Mean Square) Voltage and Current

Being lazy, we would like to use the same formulae $vi = i^2R = v^2/R$
for both *constant* v and i and *sinusoidal* v and i.
We can do this by defining the **rms voltage** to be $V_{rms} = \frac{V_0}{\sqrt{2}}$,
so that V_{rms} is the peak value of voltage divided by $\sqrt{2}$.

Average power dissipated is $\bar{p} = \frac{V_{rms}^2}{R} = \frac{V_0^2}{2R}$ which agrees with the correct answer.

Similarly, we define the **rms current** to be $I_{rms} = I_0/\sqrt{2}$ where $I_0 = \frac{V_0}{R}$,
so that I_{rms} is the peak value of current divided by $\sqrt{2}$.
Average power dissipated is $\bar{p} = I_{rms}^2 R = I_0^2 R/2 = \frac{V_0^2}{2R}$, again correct.

EXAMPLE: A wall socket puts out about $v(t) = 170\cos(2\pi 60t)$ volts.
$V_{rms} = \frac{170}{\sqrt{2}}$=120 volts, which sounds familiar.
But the *peak* voltage at the wall socket is 170 volts!