1a. Using the 1^{st} trig identity and assuming $C > 0$, we have
$C\cos(\omega t - D) = C(\cos D)\cos(\omega t) + C(\sin D)\sin(\omega t) = A\cos(\omega t) + B\sin(\omega t)$
where $A = C\cos D$ and $B = C\sin D \Leftrightarrow C = \sqrt{A^2+B^2}$ and $\tan D = B/A$. QED.
Use this to relate the sin-and-cosine and phase-shifted-cosine forms of Fourier series.

1b. See overleaf. Amplitude is clearly $5 = \sqrt{3^2+4^2}$ and phase is $-53.13^o = -\tan^{-1}(\frac{4}{3})$.
To estimate phase from plot, note the cosine is *delayed* by about $(\frac{1}{7})^{th}$ of a period.

2a. Using the 2^{nd} trig identity, $\frac{1}{2}\cos(2\pi 27t) + \frac{1}{2}\cos(2\pi 28t) = \cos(2\pi 0.5t)\cos(2\pi 27.5t)$,
which is a sinusoid at 27.5 Hertz whose amplitude varies sinusoidally at 0.5 Hertz.
This sounds like a *beat*: a tone that gets louder and softer sinusoidally in time.
Note the period of the amplitude is 1 sec, not 2 sec, since amplitude>0 always.

2b. See overleaf. This should have tipped you off to the answer to (a).

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3. This is $\sin(2\pi t) - \frac{1}{2}\sin(4\pi t) + \frac{1}{3}\sin(6\pi t) - \frac{1}{4}\sin(8\pi t) + ...$ Note $\sin(2\pi kt)$ is k Hertz.
For the 3 plots, see overleaf. Clearly converging to a **sawtooth** waveform.

4a. $x(t) = 1 + \frac{4}{\pi}\sin(\pi t) + \frac{4}{3\pi}\sin(3\pi t) + \frac{4}{5\pi}\sin(5\pi t) + \frac{4}{7\pi}\sin(7\pi t)$ using $\omega_o = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$
and $a_o = \frac{1}{2}\int_0^2 x(t)dt = 1$ and $a_n = \frac{2}{2}\int_0^2 x(t)\cos(2\pi nt/2)dt = \int_0^1 2\cos(\pi nt)dt = 0$ and
$b_n = \frac{2}{2}\int_0^2 x(t)\sin(2\pi nt/2)dt = \int_0^1 2\sin(\pi nt)dt = \frac{2}{n\pi}(1-\cos(n\pi)) = \begin{cases} \frac{4}{\pi n} & \text{if n odd;} \\ 0 & \text{if n even} \end{cases}$

4b. Filtered $x(t) = 1 + \frac{4}{\pi}\sin(\pi t) + \frac{4}{3\pi}\sin(3\pi t) + \frac{4}{5\pi}\sin(5\pi t)$. This is plotted overleaf.
Note $\omega = 5\pi \Leftrightarrow$f=2.5 Hz and $\omega = 7\pi \Leftrightarrow$f=3.5 Hz. The filter *smoothed* edges of $s(t)$.

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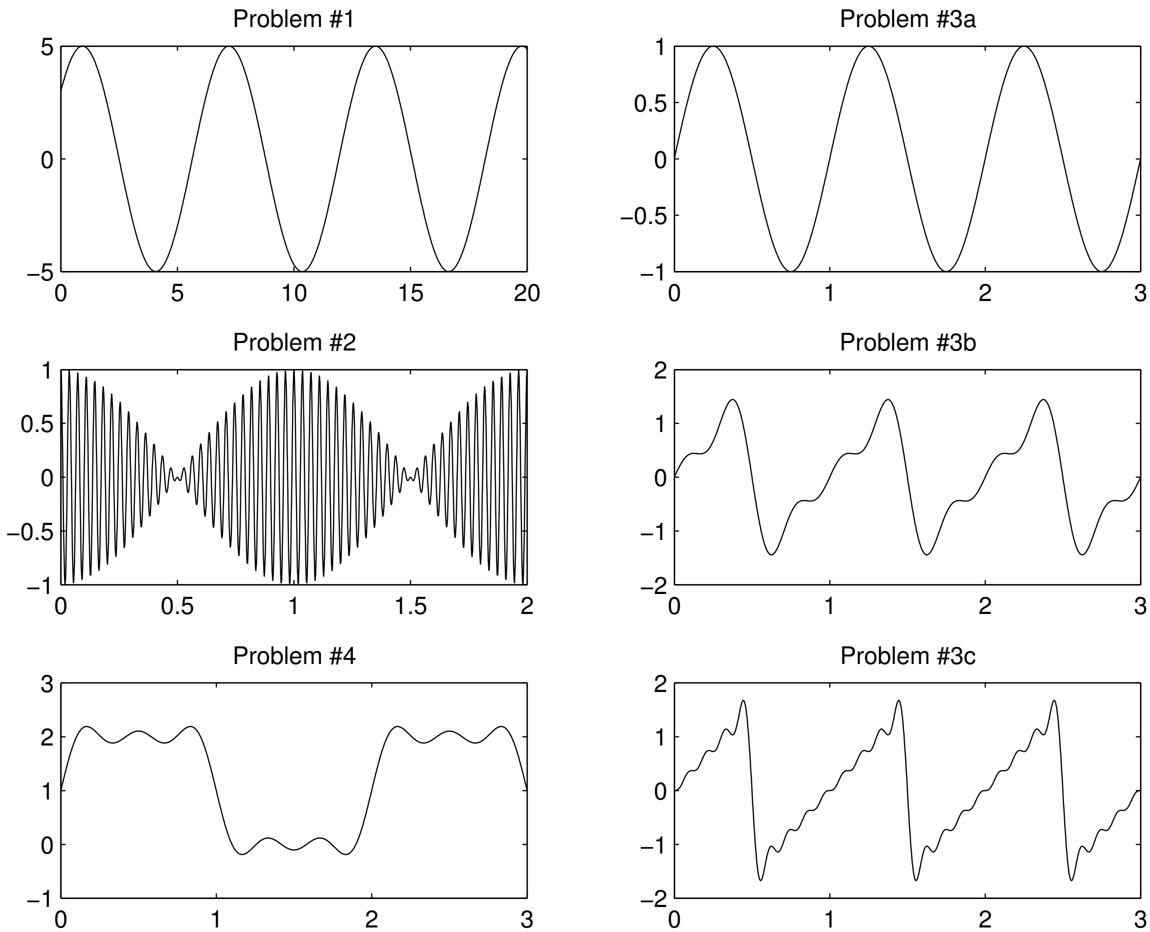
5. T is the *least common multiple* of T1=1/f1 and T2=1/f2. Let f=1/T.
T=n1/f1=n2/f2→(f)(n1)=f1 and (f)(n2)=f2→f=*greatest common divisor* of f1,f2.
Key #6: f=7=greatest common divisor of 770 and 1477→T=1/7 sec.
Key #7: f=3=greatest common divisor of 852 and 1209→T=1/3 sec.
Key #8: f=4=greatest common divisor of 852 and 1336→T=1/4 sec.


Problem #1
Problem #3a
Problem #2
Problem #3b
Problem #4
Problem #3c