

1. From the **log** plots, the gain and phase at DC ($\omega = 0$) and $\omega = 100$ are:
 $\omega=0$: Gain=(0 dB)=1; Phase=0. $\omega=100$: Gain=(-20 dB)=0.1; Phase=-45°.
 - 1a. (i) $0.1 \cos(100t - 45^\circ)$; (ii) $7 + 0.3 \cos(100t - 25^\circ)$.
 - 1b. (i) $10 \cos(100t + 45^\circ)$; (ii) $7 + 30 \cos(100t + 65^\circ)$.
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2. Plotted below **left**. Approaching an **exponential** function ($e^{3-t}, 0 < t < 3$, in fact).
 3. Gain at $\omega = 2$ is $15/3=5$; Gain at $\omega = 8$ is $10/5=2$. Solve 2 equations in 2 unknowns:
 $B/(2^2 + A^2)=5 \rightarrow B-5A^2=20$; $B/(8^2 + A^2)=2 \rightarrow B-2A^2=128$. Solving $\rightarrow \mathbf{A}=\pm\mathbf{6}, \mathbf{B}=\mathbf{200}$.
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4a. $x(t) = \begin{cases} 1-t, & \text{for } 0 < t < 1; \\ 1+t, & \text{for } -1 < t < 0 \end{cases}$. Period= $T=2 \rightarrow \omega_o = \frac{2\pi}{2} = \pi$.

Since $x(t)$ is an *even* function (symmetric about $t = 0$), we have $b_n=0$.

$$\begin{aligned} a_n &= \frac{2}{2} \int_{-1}^0 (1+t) \cos(n\pi t) dt + \frac{2}{2} \int_0^1 (1-t) \cos(n\pi t) dt = 2 \int_0^1 (1-t) \cos(n\pi t) dt \\ &= 2 \int_0^1 \cos(n\pi t) dt - 2 \int_0^1 t \cos(n\pi t) dt = \frac{2}{n\pi} \sin(n\pi t) \Big|_0^1 - 2 \frac{\cos(n\pi t)}{\pi^2 n^2} \Big|_0^1 - 2 \frac{t \sin(n\pi t)}{n\pi} \Big|_0^1 \\ &= \frac{2}{\pi^2 n^2} (1 - (-1)^n) = \begin{cases} 4/(\pi^2 n^2), & \text{for } n \text{ odd;} \\ 0, & \text{for } n \text{ even} \end{cases}. \end{aligned}$$

Only middle term above is non-zero.

$$a_o = \frac{1}{2} \int_{-1}^0 (1+t) dt + \frac{1}{2} \int_0^1 (1-t) dt = \frac{2}{2} \int_0^1 (1-t) dt = 1/2 \text{ using symmetry.}$$

$$x(t) = \frac{1}{2} + \frac{4}{\pi^2} \cos(\pi t) + \frac{4}{9\pi^2} \cos(3\pi t) + \frac{4}{25\pi^2} \cos(5\pi t) + \frac{4}{49\pi^2} \cos(7\pi t) + \dots$$

- 4b. 2 Hertz $\Leftrightarrow \omega = 4\pi \rightarrow$ keep 1st 3 terms of this series: $\frac{1}{2} + \frac{4}{\pi^2} \cos(\pi t) + \frac{4}{9\pi^2} \cos(3\pi t)$.
 Plotted below **right**. Attenuation of high freqs \rightarrow round corners: can't change quickly.

