

1a.  $v = L \frac{di}{dt} \rightarrow i(t) = i(0) + \frac{1}{L} \int_0^t v(\tau) d\tau$  for  $0 < t < 0.001, 0.001 < t < 0.002, t > 0.002$ :

$$0 < t < 0.001 : i(t) = 0 + \frac{6mv}{300\mu H} \int_0^t d\tau = (20t)A \text{ since } \frac{6mv}{300\mu H} = 20 \frac{amp}{sec}.$$

$$.001 < t < .002 : i(t) = (20 \frac{amp}{sec})(0.001sec) + \frac{6mv}{300\mu H} \int_{0.001}^t (2 - 1000 \frac{msec}{sec} \tau) d\tau = (40t - 10000t^2 - 0.01)A.$$

$$t > 0.002 : i(t) = (40(0.002) - 10000(0.002)^2 - 0.01)A + \frac{6mv}{300\mu H} \int_{0.002}^t 0\tau = 0.03A.$$

CHECK: Continuity of  $i(t)$  at  $t=0.001$ :  $(0.02A=0.02A \text{ OK})$ ;  $t=0.002$ :  $(0.03A=0.03A \text{ OK})$ . Whew!

1b. See plot below. Note continuity of  $i(t)$ .

$$2a. v = L \frac{di}{dt} = (0.02H)[-2500A_1e^{-2500t} - 7500A_2e^{-7500t}] = -(50A_1e^{-2500t} + 150A_2e^{-7500t}).$$

$$i(0) = 0.05 = A_1 + A_2 \text{ and } v(0) = 10 = -50A_1 - 150A_2 \rightarrow A_1 = 0.175, A_2 = -0.125.$$

$$v(t) = -8.75e^{-2500t} + 18.75e^{-7500t}V, t > 0; i(t) = 0.175e^{-2500t} - 0.125e^{-7500t}A, t \geq 0.$$

$$2b. p(t) = i(t)v(t) = 4.375e^{-10000t} - 1.53125e^{-5000t} - 2.34375e^{-15000t}.$$

$$p(t) = (4.375x - 1.53125x^2 - 2.34375)/x^3 = 0 \text{ where } x = e^{5000t} \text{ as suggested.}$$

$$\text{Solving quadratic} \rightarrow x = 0.7143 \rightarrow t < 0; x = 2.143 \rightarrow t = \frac{\log_e 2.143}{5000} = 152.4\mu sec.$$

$$3a. i = C \frac{dv}{dt} = (0.0000004) \frac{d(-60)}{dt} = 0, t < 0 \text{ (this wasn't the nasty derivative).}$$

$$3b. i = C \frac{dv}{dt} = (0.0000004) \frac{d}{dt} [15 - 15e^{-500t} 5 \cos(2000t) - 15e^{-500t} \sin(2000t)] = (0.0000004) \times (7500)e^{-500t} [\cos(2000t) + 21 \sin(2000t)] = 3e^{-500t} [\cos(2000t) + 21 \sin(2000t)] mA.$$

$$3c. v(0^+) = 15 - 15(e^0)(5 + 0) = -60 = v(0^-) \text{ so } v(t) \text{ doesn't jump (it better not!).}$$

$$3d. i(0^+) = 3(e^0)(1 + 0) = 3mA \neq 0 = i(0^-) \text{ so } i(t) \text{ does jump (cap. current CAN jump).}$$

$$3e. \text{Stored energy} = \frac{1}{2} C v(\infty)^2 = \frac{1}{2} (0.0000004)(15)^2 = 45\mu \text{Joules.}$$

$$4. (14||6) + 15.8 = 20. \quad (20||60) + 5 = 20. \quad (20||80) + 24 = 40. \quad (40||10) + 12 = 20H.$$

$$5a. i_o(0) = i_1(0) + i_2(0) = 10 - 5 = 5A. \quad (32||8) + 3.6 = 10H \text{ for (b).}$$

$$5b. i_o(t) = i_o(0) - \frac{1}{L} \int_0^t v(\tau) d\tau = 5 - 0.1 \int_0^t 1250e^{-25\tau} d\tau = 5e^{-25t}A, t > 0.$$

$$5c. i(t) = i(0) + \frac{1}{L} \int_0^t v(\tau) d\tau \rightarrow i_1(t) = C_1 + 4e^{-25t}, t > 0 \text{ and } i_2(t) = C_2 + e^{-25t}, t > 0.$$

Where  $C_1$  and  $C_2$  are chosen to match the given  $i_1(0) = 10$  and  $i_2 = -5$ .

$$\rightarrow i_1(t) = 4e^{-25t} + 6 \text{ and } i_2(t) = e^{-25t} - 6.$$

$$5e. \text{Stored energy} = \frac{1}{2} 8(10)^2 + \frac{1}{2} 32(-5)^2 + \frac{1}{2} 3.6(5)^2 = 845 \text{Joules.}$$

$$6. (4 + j3)(5 + j12) = (4 \cdot 5 - 3 \cdot 12) + j(3 \cdot 5 + 4 \cdot 12) = -16 + j63.$$

$$(4 + j3)(5 + j12)^* = (4 + j3)(5 - j12) = (4 \cdot 5 + 3 \cdot 12) + j(3 \cdot 5 - 4 \cdot 12) = 56 - j33.$$

$$\frac{4+j3}{5+j12} = \frac{4+j3}{5+j12} \frac{5-j12}{5-j12} = \frac{1}{169} (4 + j3)(5 - j12) = \frac{56}{169} - j \frac{33}{169} \quad (5^2 + 12^2 = 169).$$

$$1/[(4 + j3)(5 + j12)] = \frac{1}{-16+j63} \frac{-16-j63}{-16-j63} = -\frac{16}{4225} - j \frac{63}{4225} \quad (16^2 + 63^2 = 4225).$$

$$(4 + j3)^*(5 + j12) = (4 - j3)(5 + j12) = (4 \cdot 5 + 3 \cdot 12) + j(-3 \cdot 5 + 4 \cdot 12) = 56 + j33.$$

Now redo these using polar form (answers don't match perfectly due to rounding):

$$4 + j3 = 5e^{j37^\circ}; \quad 5 + j12 = 13e^{j67^\circ}. \text{ Now these are } \textit{easy}:$$

$$(4 + j3)(5 + j12) = 5e^{j37^\circ} 13e^{j67^\circ} = 65e^{j104^\circ} = -16 + j63.$$

$$(4 + j3)(5 + j12)^* = 5e^{j37^\circ} 13e^{-j67^\circ} = 65e^{-j30^\circ} = 56 - j33.$$

$$\frac{4+j3}{5+j12} = 5e^{j37^\circ} / 13e^{j67^\circ} = 0.385e^{-j30^\circ} = 0.333 - j0.192.$$

$$1/[(4 + j3)(5 + j12)] = \frac{1}{5}e^{-j37^\circ} \frac{1}{13}e^{-j67^\circ} = 0.0154e^{-j104^\circ} = -0.00372 - j0.0149$$

$$(4 + j3)^*(5 + j12) = 5e^{-j37^\circ} 13e^{j67^\circ} = 65e^{j30^\circ} = 56 + j33.$$

7. Magnitude squared =  $[(25)(65)(100)(169)(125)] / [(676)(65)(625)(125)] = 1$ .

How did I come up with these? Note the various Pythagorean triangles.

8. HARD WAY:  $5 \cos(3t) \cos(30^\circ) - 5 \sin(3t) \sin(30^\circ) + 5 \sin(3t) + 5 \cos(3t) \cos(210^\circ) + 5 \sin(3t) \sin(210^\circ) = 0 \cos(3t) + 0 \sin(3t) = 0$  *identically*. Why?

8. EASY WAY:  $5e^{j30^\circ} + 5e^{-j90^\circ} + 5e^{-j210^\circ} = 0$ . Note  $5 \sin(3t) = 5 \cos(3t - 90^\circ)$ .

See below right for a pretty picture of why these sum to zero.

9. UNGRADED: The Norton equivalent box has current flowing through a resistor.

The Thevenin equivalent box has no current flowing. The Norton equivalent is *warmer*.

