1. The system $y(t) = x(t)x(t-1)$ is:
   (a) Linear (b) Time-invariant (c) Both (d) Neither (e) Can't tell

2. An LTI system with impulse response $h(t) = e^{-|t|}$ is:
   (a) BIBO stable (b) Causal (c) Both (d) Neither (e) Can't tell

3. $\int_{-\infty}^{\infty} \sin(3t)\delta(t-\pi)dt =$:
   (a) 0 (b) 1 (c) 3 (d) π (e) ∞

4. The period of $3\cos(\frac{\pi}{3}t+1) + 5\cos(\frac{\pi}{4}t+2) + 7\cos(\frac{\pi}{6}t+3)$ is:
   (a) 6 (b) 12 (c) 24 (d) 48 (e) ∞

5. Two system are connected in series or cascade:
   $x(t) \rightarrow \boxed{h(t) = u(t)} \rightarrow \boxed{h(t) = -u(t)} \rightarrow y(t)$
   The overall impulse response of the connected systems is:
   (a) 0 (b) $-\delta(t)$ (c) $-u(t)$ (d) $-r(t)$ (e) $\frac{1}{2}t^2u(t)$

6. $y(t) = [2\delta(t) + 3\delta(t-2)] * [u(t) - u(t-1)]$. $y(2.5) =$:
   (a) 0 (b) 2 (c) 3 (d) 4 (e) 5 (here $*$ denotes convolution)

7. x(t)=triangle with support [1,4]. Sketch y(t)=x(3t-5)+x(-t+1) at right.

8. RC circuit with RC=1. Sketch the response y(t) to x(t)=u(t)-u(t-0.01).
   ALSO: Specify the peak value of y(t) on the given plot.

9. A system has frequency response $H(j\omega)=4/[(j\omega)^2 + 4(j\omega) + 4]$.
   Assume that the system is in the sinusoidal steady state.
   (05) a. Compute response to 5cos(4t). Hint: Cancel 4s.
   (05) b. Specify a differential equation that has this $H(j\omega)$.
   (15) c. Compute the response to 6+10cos(t)+10cos(2t)

10. The impulse response of an LTI system is h(t)=e^{-2t}u(t).
    The input is of the form (note the j!) x(t)=5e^{-2t}u(t)+2e^{-j5t}+s(t).
    for $-\infty < t < \infty$ where s(t) is the *unknown* part of x(t).
    The corresponding output is y(t)=f(t)+5$e^{-2(t-1)}$u(t-1)
    for $-\infty < t < \infty$ where f(t) is the *unknown* part of y(t).
    Calculate f(t) and s(t) for $-\infty < t < \infty$.

EXAM SCORE DISTRIBUTION

$90-100:$	22	*Number:*	96
$80-89:$	22	*Mean:*	78.3
$70-79:$	22	*Median:*	79
$60-69:$	20	σ:	14.7
$<60:$	10	*Distrib:*	*Flat*!

1. B NOT linear since doubling $x(t)$ quadruples $y(t)$ [TI=Time-Invariant].
IS TI since no "t" outside "x(t)": Delay input by 1 delays output by 1.

2. A NOT causal since $h(t) \neq 0$ for $t < 0$. Easiest if you *draw a sketch*: tent with pole.
IS stable since $\int_{-\infty}^{\infty} |h(t)|dt = 2\int_0^{\infty} e^{-t}dt = 2 < \infty$ (only need to know is finite).

3. A $\int x(t)\delta(t-a)dt = x(a) = \sin(3\cdot\pi) = 0$. A real gimme.

4. C Periods: $\frac{2\pi}{\pi/3} = 6$; $\frac{2\pi}{\pi/4} = 8$; $\frac{2\pi}{\pi/6} = 12$. Least Common Multiple[6,8,12]=24.

5. D $h(t) = -u(t) * u(t) = -[\int_0^t 1d\tau]u(t) = -tu(t)$.
6. C y(t)=2[u(t)-u(t-1)]+3[u(t-2)-u(t-3)]=3 (again, draw a sketch).

7. x(3t-5): Shift: [1,4]→[6,9]. Scale:[6,9]→[2,3]. Triangle with support [2,3].
x(1-t): Shift: [1,4]→[0,3]. Scale:[0,3]→[-3,0]. Reversed triangle with support [-3,0].

Grading: 5 for each triangle; minus 1-5 depending on severity of error.

8. Since pulse length=0.01<<RC=1, y(t)=0.01e^{-t}u(t) where 0.01=AREA under pulse.

Note: Writing 1–$e^{-0.01}$ instead of 0.01 worth 9/10, even though roughly equal.

Why? 1–$e^{0.01}$ means you missed point of the problem–only pulse AREA counts.
If pulse had more complicated shape, you would have gotten this wrong!
It almost was more complicated, but I chickened out at the last minute.

9. $H(j\omega)$=4/[$(j\omega)^2$+4($j\omega$)+4]. Make a table:

part	ω	$H(j\omega)$	$\lvert H(j\omega)\rvert$	$\angle H(j\omega)$	input	output	answer
(a)	4	$\frac{4}{16j-12}$	$\frac{1}{5}$	-127^o	$5\angle 0$	$1\angle -127^o$	$\cos(4t-127^o)$
(c)	0	$\frac{4}{0+0+4}$	1	0	$6\angle 0$	$6\angle 0$	6
(c)	1	$\frac{4}{-1+4j+4}$	$\frac{4}{5}$	-53^o	$10\angle 0$	$8\angle -53^o$	$8\cos(t-53^o)$
(c)	2	$\frac{4}{-4+j8+4}$	$\frac{1}{2}$	-90^o	$10\angle 0$	$5\angle -90^o$	$5\cos(2t-90^o)$

9. (a) $\cos(4t-127^o)$ (b) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 4x$ (c) 6+8cos$(t-53^o)$+5cos$(2t-90^o)$.

Note: Specific numbers $-127^o, -53^o, \tan^{-1}(2.5)$ not required, but still...
People need *badly* to do better on complex numbers! $\frac{1}{2j} \neq 2e^{j90^o}$!

Note: In (a) $\angle\frac{4}{16j-12} = \angle\frac{1}{4j-3} = -\angle(4j-3) \neq \tan^{-1}\frac{4}{3}$ (in 3^{rd} quadrant).

Grading: Roughly -1 for each amplitude or phase error (depending)

10. Recall that response to sum of inputs is sum of responses to inputs. So:
$5e^{-2t}u(t) * e^{-2t}u(t) = 5\int_0^t e^{-2\tau}e^{-2(t-\tau)}d\tau = 5e^{-2t}\int_0^t d\tau = 5te^{-2t}u(t)$.
$2e^{-j5t} * e^{-2t}u(t) = H(-j5)2e^{-j5t} = \frac{1}{2-j5}2e^{-j5t} = \frac{1}{\sqrt{29}}e^{-j\tan^{-1}(-5/2)}2e^{-j5t}$
$= \frac{2}{\sqrt{29}}e^{-j[5t-\tan^{-1}(2.5)]}$. So $f(t) = 5te^{-2t}u(t) + \frac{2}{\sqrt{29}}e^{-j[5t-\tan^{-1}(2.5)]}$.
$s(t) * e^{-2t}u(t) = 5e^{-2(t-1)}u(t-1) \rightarrow s(t) = 5\delta(t-1)$ (either saw this or didn't).

Grading: 10 for $5e^{-2t}u(t) * e^{-2t}u(t) = 5te^{-2t}u(t)$. Many students could not set up the integral.
8 for $2e^{-j5t} \rightarrow H(-5j)e^{-j5t}$. –1 for H(j5). Some *did* the convolution–hard way!
7 for $s(t) = 5\delta(t-1)$