

1. Fourier series of $x(t)$ is $\cos(t) + \frac{1}{2} \cos(2t) + \frac{1}{3} \cos(3t) + \dots$. Then $\int_{-\pi}^{\pi} x(t) \cos(3t) dt =$:
 (a) 0 (b) $\frac{1}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$ (e) $\frac{2\pi}{3}$

2. Impulse response of an LTI system with frequency response $\frac{2}{(j\omega)^2 + 4(j\omega) + 4}$ is:
 (a) Noncausal (b) $2e^{-2t} \cos(2t)u(t)$ (c) $e^{-2t} \sin(t)u(t)$ (d) $te^{-2t}u(t)$ (e) $2te^{-2t}u(t)$

3. An LTI system has impulse response $h(t) = \frac{\sin(20\pi t)}{4t} \cos(30\pi t)$. It rejects above:
 (a) 10 (b) 20 (c) 25 (d) 50 (e) ∞ Hertz.

4. Why would anyone use DSBSC modulation instead of AM radio?
 (a) Less bandwidth (b) Less power (c) Less noise (d) Simpler receiver (e) Simpler xmitter

5. The minimum sampling rate in $\frac{\text{RADIAN}}{\text{SECOND}}$ to reconstruct $[\sin(40t) \sin(60t)] / [\pi^2 t^2]$ from its samples is barely greater than: (a) 80 (b) 120 (c) 160 (d) 200 (e) 240

6. $\sin(32\pi t) + \sin(48\pi t)$ is sampled at $40 \frac{\text{SAMPLE}}{\text{SECOND}}$, then lowpass-filtered to 20 Hz. Get:
 (a) 0 (b) $\sin(16\pi t)$ (c) $2 \sin(16\pi t)$ (d) $\sin(32\pi t)$ (e) $2 \sin(32\pi t)$

7. Line spectrum $(3-4j)\delta(\omega + 2) + (1+j)\delta(\omega + 1) + (1-j)\delta(\omega - 1) + (3+4j)\delta(\omega - 2)$
 (i) Average power of the signal is: (ii) The sinusoidal component at $\omega = 1$ is:

8. A real-valued signal is modulated using single-sideband (SSB) resulting in following spectrum of the modulated signal: Right triangles on $[2,3]$ and $[-3,-2]$ kHz.

(a) Draw a block diagram of a system that will recover the signal.

(b) If we only know the modulation frequency approximately, is it better to be too low or too high? Explain why.

9. $x(t) = \sin(t) + \frac{1}{3} \sin(3t) + \frac{1}{5} \sin(5t) + \dots$ is input into:

$x(t) \rightarrow \overline{|h(t) = e^{-|t|}|} \rightarrow y(t) \rightarrow |h(t) = \frac{\sin(4t)}{\pi t}| \rightarrow z(t)$

(a) Write an explicit and simple two-term expression for $z(t)$.

(b) Compute the average value of $z(t)$.

(c) Write an explicit expression for $y(t)$. You may use Σ .

10. $x(t)$ is filtered $1/(1 + j\omega/2)$, bandpass filtered $4 < |\omega| < 8$, and sampled every T_s .

Plot the spectra at each point and determine minimum sampling rate for:

(a) $\delta(t)$ (b) $6 + \cos(6t)$

EXAM SCORE DISTRIBUTION

RANGE	#	statistic :	value	RANGE	#
91 - 100	6	AVERAGE :	64.6	51 - 60	14
81 - 90	15	STANDEV :	19.2	41 - 50	11
71 - 80	21	#TAKING :	95	31 - 40	9
61 - 70	16			< 30	3

1. D: $\frac{2}{2\pi} \int_{-\pi}^{\pi} x(t) \cos(3t) dt = a_3 = \frac{1}{3} \rightarrow f = \frac{\pi}{3}$.

2. E: $\frac{2}{(j\omega+2)^2} = -2 \frac{d}{d(j\omega)} \frac{1}{j\omega+2} \rightarrow 2te^{-2t}u(t)$.

3. C: $\frac{\sin(20\pi t)}{\pi t}$ has cutoff frequency $\frac{20\pi}{2\pi} = 10$ Hertz.

Multiplication by $\cos(2\pi 15t)$ shifts up and down by 15 Hertz $\rightarrow 10+15=25$ Hertz.

4. B: No power used for carrier. BW and noise are same as AM. AM receiver simpler.

5. D: $\mathcal{F}\left\{\frac{\sin(40t)}{\pi t} \frac{\sin(60t)}{\pi t}\right\} = \frac{1}{2\pi} \text{rect}(t/80) * \text{rect}(t/120) = \text{trap}(t/200)$.

6. A: $t = \frac{n}{40} \rightarrow \sin(32\pi \frac{n}{40}) + \sin(48\pi \frac{n}{40}) = \sin(0.8\pi n) + \sin(1.2\pi n) = 0$.

7i. $|3 - 4j|^2 + |1 + j|^2 + |1 - j|^2 + |3 + 4j|^2 = 25 + 2 + 2 + 25 = 54$.

7ii. $(1+j)e^{-jt} + (1-j)e^{jt} = 2\sqrt{2} \cos(t - \frac{\pi}{4})$.

8a. $x(t) \rightarrow \otimes \rightarrow \boxed{\text{Lowpass filter 1 kHz}} \rightarrow y(t)$
 $\cos(2\pi 2000t) \uparrow$

8b. If think too low, recovered spectrum is unaliased but at higher frequencies.

If think too high, recovered spectrum is aliased and no longer can be recovered.

9. $x(t) = \sin(t) + \frac{1}{3} \sin(3t) + \frac{1}{5} \sin(5t) + \dots$ $H_1(\omega) = \frac{2}{\omega^2+1}$ and $H_2(\omega) = 0$ for $|\omega| > 4$.

9a. $z(t) = \frac{2}{1^2+1} \sin(t) + \frac{2}{3^2+1} \frac{1}{3} \sin(3t) = \sin(t) + \frac{1}{15} \sin(3t)$.

9b. Average value = DC component = 0.

9c. $y(t) = \sum_{n \text{ odd}} \frac{2}{n^2+1} \frac{1}{n} \sin(nt)$.

10. In each case $|Y_s(\omega)|$ is $|Y(\omega)|$ repeated every $\frac{2\pi}{T_s}$.

part	$ X(\omega) $	$ Z(\omega) $	$ Y(\omega) $	T_{MIN}
10(a)	1	$\frac{1}{\sqrt{1+\omega^2/4}}$	$\begin{cases} \frac{1}{\sqrt{1+\omega^2/4}} & \text{for } 4 < \omega < 8 \\ 0 & \text{otherwise} \end{cases}$	$\pi/8$

10(b)	$\begin{matrix} 12\pi\delta(\omega) + \\ \pi\delta(\omega - 6) + \\ \pi\delta(\omega + 6) \end{matrix}$	$\begin{matrix} 12\pi\delta(\omega) + \\ \frac{\pi}{\sqrt{10}}\delta(\omega - 6) + \\ \frac{\pi}{\sqrt{10}}\delta(\omega + 6) \end{matrix}$	$\begin{matrix} \frac{\pi}{\sqrt{10}}\delta(\omega - 6) + \\ \frac{\pi}{\sqrt{10}}\delta(\omega + 6) \end{matrix}$	$\pi/6$
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