1. $H(s)=\frac{(s^2+4\pi^2)}{(s+1)[(s+2\pi)^2+9]}$. $x(t)$=100+10sin($2\pi t$)+cos($4\pi t$). Output is:
(a) Periodic with period=$\frac{1}{2}$ (b) Periodic with period=1
(c) Constant (DC signal) (d) A decaying sinusoid (e) ∞

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2. $e^{-t}u(t) \rightarrow$ [LTI] $\rightarrow e^{-2t}u(t)$. Which of these is true?
(a) Zero at –1 (b) Pole at –2 (c) It rejects no sinusoids
(d) ZIR(t)=$Ce^{-2t}u(t)$ for some constant C. (e) All are true

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3. $x(t) \rightarrow$ [$h(t)=e^{-3|t|}$] $\rightarrow$ [$H(\omega)=1/(j\omega+2)$] $\rightarrow$ [$\frac{d}{dt}$] $\rightarrow y(t)$ is described by:
(a) $\frac{d^3y}{dt^3}+2\frac{d^2y}{dt^2}+9\frac{dy}{dt}+18y=6\frac{dx}{dt}$ (b) $\frac{d^3y}{dt^3}+2\frac{d^2y}{dt^2}-9\frac{dy}{dt}-18y=6\frac{dx}{dt}$
(c) $-\frac{d^3y}{dt^3}-2\frac{d^2y}{dt^2}+9\frac{dy}{dt}+18y=6\frac{dx}{dt}$ (d) $-\frac{d^3y}{dt^3}-2\frac{d^2y}{dt^2}-9\frac{dy}{dt}-18y=6\frac{dx}{dt}$

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4. $\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=\frac{dx}{dt}+x$ with x(t)=u(t); y(0)=0; $\frac{dy}{dt}$(0)=1. Zero-input response is:
(a) $[2e^{-2t}-e^{-t}]u(t)$ (b) $[2e^{-t}-e^{-2t}]u(t)$ (c) $e^{-2t}u(t)$ (d) $[e^{-t}-e^{-2t}]u(t)$ (e) $\frac{1}{2}u(t)$

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5. $x(t)=\frac{\sin(2\pi t)}{\pi t}(1+\cos(4\pi t))$ is sampled at $\omega_s=12\pi$ ($T_s=\frac{1}{6}$).
The spectrum $X_s(\omega)$ of the sampled signal is: (a) ∞
(b) Impulse (c) Constant (d) Sinusoid (e) Non-sinusoidal periodic

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6. $X(\omega)=(2-j)\pi\delta(\omega+3)+(2+j)\pi\delta(\omega-3)$. $x(t)$=: (e) Not a real-valued function.
(a) $\cos(3t)+\frac{1}{2}\sin(3t)$ (b) $2\cos(3t)+\sin(3t)$ (c) $\cos(3t)-\frac{1}{2}\sin(3t)$ (d) $2\cos(3t)-\sin(3t)$

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7. For an LTI system, which statement(s) is/are true?
(a) Input has period=T→output has period T or less.
(b) If input=$e^{j\omega_0 t}$ then output must=$ae^{j\omega_0(t-\tau)}$ for some real a and τ,
(c) Poles on imaginary axis cannot produce unbounded output if input is bounded.
(d) Impulse response=derivative of the output to a unit step.
(e) Feedback can never make a stable system unstable.

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8. Which statement(s) about sampling is/are true?
(a) Non-impulsive signals can be reconstructed exactly from samples using a causal filter
(b) The spectrum of a sampled signal is always periodic
(c) Bandlimited signal reconstructed if sampled at exactly twice maximum frequency
(d) The bandwidth of $x(t)y(t)$ is the sum of the bandwidths of $x(t)$ and $y(t)$
(e) All AM radio uses sampling at 5 kHz to avoid aliasing

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9. An LTI system has transfer function $H(s)=s/[(s+3)(s+4)]$.
(a) Compute the zeros and poles (b) Determine diff. eqn.
(c) Compute impulse response (d) Compute step response (e) Response to cos($4t$).

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10. Compensator feedback control system with $C(s)=K_1+K_2s$ and $G(s)=\frac{1}{s^2-3s+1}$.
(a) Compute the closed loop transfer function (from $x(t)$ to $y(t)$) in terms of K_1 and K_2.
(b) Find values of K_1 and K_2 so the closed loop system: (i) Is BIBO stable
ii. $h(t)$ has damped oscillations of frequency $\frac{1}{2}$ radian/sec.
iii. Steady-state error $x(t)-y(t)$ for step input $u(t)$ is $2u(t)$.

FINAL EXAM SOLUTIONS

1. **A.** The zeros at $\pm j2\pi$ "eat" $10\sin(2\pi t)$ but leave the other two terms. Poles have real parts $< 0 \rightarrow$ stable. Period$=\frac{2\pi}{4\pi}=\frac{1}{2}$. But see #7 below.

2. **E.** $H(s)=\frac{Y(s)}{X(s)}=\frac{1/(s+2)}{1/(s+1)}=\frac{s+1}{s+2}$. Note (b)$\Leftrightarrow$(d).

3. **C.** Overall $H(j\omega)=\frac{2\cdot 3}{\omega^2+3^2}\frac{1}{j\omega+2}(j\omega)=\frac{6(j\omega)}{j(\omega)^3+2(\omega)^2+9(j\omega)+18}$. Read off diff. eqn.

4. **D.** $(s^2Y-1)+3sY+2Y=0 \rightarrow Y=\frac{1}{s^2+3s+2}=\frac{1}{(s+1)(s+2)}=\frac{1}{s+1}-\frac{1}{s+2} \rightarrow [e^{-t}-e^{-2t}]u(t)$.

5. **C.** $X(\omega)=1$ for $\omega \in [-2\pi,2\pi]\cup[2\pi,6\pi]\cup[-6\pi,-2\pi]=[-6\pi,6\pi]$. Sampling with $\omega_s=12\pi$ makes this repeat every $12\pi \rightarrow$ constant for all ω. Usually "non-sinusoidal periodic" would be OK, but not this time! (too tricky?)

6. **D.** Using table on cover page.

7. **B,D.** (a) Consider #1 with zeros at $\pm j4\pi$. (c) Sinusoid at that frequency.

8. **B,D.** (a) Ideal LPF is noncausal. (c) $\sin(\pi t)$ with $t=n$ gives 0!

9a. Zeros: $\{0\}$. Poles: $\{-3,-4\}$. (9b): $\frac{Y(s)}{X(s)}=\frac{s}{s^2+7s+12} \rightarrow \frac{d^2y}{dt^2}+7\frac{dy}{dt}+12y=\frac{dx}{dt}$.

9c. $H(s)=\frac{s}{(s+3)(s+4)}=\frac{4}{s+4}-\frac{3}{s+3} \rightarrow h(t)=[4e^{-4t}-3e^{-3t}]u(t)$.

9d. $Y(s)=\frac{s}{(s+3)(s+4)}\frac{1}{s}=\frac{1}{(s+3)(s+4)}=\frac{1}{s+3}-\frac{1}{s+4} \rightarrow s(t)=[e^{-3t}-e^{-4t}]u(t)$.

9e. $H(j4)=\frac{j4}{(j4+3)(j4+4)}=4e^{j90^o}/[5e^{j53^o}4\sqrt{2}e^{j45^o}]=\frac{1}{5\sqrt{2}}e^{-j8^o} \rightarrow \frac{\sqrt{2}}{10}\cos(4t-8^o)$.

10a. $H_{CL}(s)=\frac{C(s)G(s)}{1+C(s)G(s)}=\frac{K_1+K_2s}{s^2+(K_2-3)s+(K_1+1)}$.

10b. $y(t) \rightarrow -u(t) \rightarrow -1=H(0)=\frac{K_1}{K_1+1} \rightarrow K_1=-0.5$.

Poles: $s^2+(K_2-3)s+0.5=0 \rightarrow s=\frac{3-K_2}{2}\pm\sqrt{(\frac{3-K_2}{2})^2-\frac{1}{2}}$.

Want $\frac{1}{2}=Im[s]=\sqrt{\frac{1}{2}-(\frac{3-K_2}{2})^2} \rightarrow (\frac{3-K_2}{2})^2=\frac{1}{4} \rightarrow K_2=2$ or 4.

For stability use $K_2=4$ to make $Re[s]<0$. People had trouble with this one.

Mean=73.0. Standard deviation=12.75.