EECS 216 - Winter 2008
PRACTICE QUESTIONS FOR EXAM 1 on 02/07/08

These problems will be discussed in class and in the evening review session on Tuesday Feb. 5. Please work on them prior to that.

Problem 1: Multiple-Choice Questions

1. The system described by

$$\frac{dy(t)}{dt} + \pi^2 y(t) = \frac{dx(t)}{dt} + x^2(t)$$

is

(a) Linear and Time-Invariant;

(b) Linear and Time-Varying;

(c) Nonlinear and Time-Invariant;

(d) Nonlinear and Time-Varying.

(e) Insufficient information to answer.

Answer: This is a non-linear system (because of the $x^2(t)$) term, but it is a time-invariant system. Thus the correct answer is **C**.

2. The system described by

$$y(t) = e^t x(t - t_0)$$

is

(a) Linear and Time-Invariant;

(b) Linear and Time-Varying;

(c) Nonlinear and Time-Invariant;

(d) Nonlinear and Time-Varying.

(e) Insufficient information to answer.

Answer: The system is **linear** because $a_1 y_1(t) + a_2 y_2(t) = e^t(a_1 x_1(t - t_0) + a_2 x_2(t - t_0))$ where $y_1 = e^t x_1(t - t_0)$ and $y_2 = e^t x_2(t - t_0)$. The system is **time-varying** because of e^t term. Thus the correct answer is **B**.

3. The LTI system with Impulse Response given by

$$h(t) = t^5 e^{-0.5t} u(t - 100)$$

is

(a) BIBO stable and Causal;

(b) Causal but not BIBO stable;

(c) BIBO stable but not Causal;

(d) Neither BIBO stable nor Causal;

(e) Insufficient information to answer.

Answer: An LTI system whose impulse response is equal to zero for $t < 0$ is a causal system, and an LTI system whose impulse response is absolutely integrable is a BIBO stable system. The impulse response given in this problem fits both of those criteria, so the correct answer is **A**.

4. The Frequency Response Function of the system

$$2\frac{d^2y(t)}{dt^2} + 3y(t) = 4\frac{dx(t)}{dt}$$

is

(a) $\frac{4}{2j\omega^2+3}$

(b) $\frac{4}{2(j\omega)^2+3}$

(c) $\frac{4j\omega}{2j\omega^2+3}$

(d) $\frac{4j\omega}{2(j\omega)^2+3}$

(e) None of the above.

Answer: We can write the Frequency Response Function by inspection, as shown in the Systems supplementary notes. We find that the correct answer is **D**.

5. When the signal

$$x(t) = 5 + \cos(12t + \frac{\pi}{4})$$

is applied to a system, the output is

$$y(t) = 6\sin(12t) + 0.5\cos(24t + \frac{\pi}{4}) \quad .$$

Which of the following statements is true?

(a) The system is LTI with $H(0) = 0$ and $H(j12) = 6$.

(b) The system is LTI with $H(0) = 0$ and $|H(j12)| = 6$.

(c) The system is not LTI because the constant input 5 disappears at the output.

(d) The system is not LTI because the cos function becomes a sin function.

(e) All above statements are incorrect.

Answer: For an LTI system, the output can only contain frequencies that are in the input to the system. This system gives an output of frequency $\omega = 24$ for an input that includes frequencies of only $\omega = 0$ and $\omega = 12$, so it cannot be an LTI system. Note that choice C is wrong because a constant input can be eliminated by an LTI system if its frequency response function is zero for $\omega = 0$. Choice D is also wrong because an LTI system can cause a phase shift in a frequency component of the input, changing a cos function into a sin function. Thus the correct answer is **E**.

Problem 2

The impulse response $h(t)$ of an LTI system is given by

$$h(t) = \sqrt{1.5}[u(t) - u(t-1.5)] \quad .$$

The signal

$$x(t) = \begin{cases} 0 & t < 0 \quad \text{or} \quad t \geq 2 \\ \sqrt{1.5}t & 0 \leq t < 1 \\ \sqrt{1.5}t - 2\sqrt{1.5} & 1 < t < 2 \end{cases}$$

is applied to the system.

Calculate $y(2)$, the output at time $t = 2$.

Answer: We need to compute the value of the convolution of $x(t)$ and $y(t)$ at $t = 2$. This is best done graphically (see Fig. 1).

Multiplying and integrating, we find

$$\boxed{y(2) = -\frac{3}{16}}$$

Problem 3

Two experiments are performed on an LTI system with a real impulse response $h(t)$. In the first experiment, input

$$x_1(t) = e^{j2t}$$

results in output

$$y_1(t) = \frac{1}{2}e^{j(2t-\frac{\pi}{4})} \ .$$

In the second experiment, the input is

$$x_2(t) = 2e^{-j2t} \ .$$

What will be the output $y_2(t)$?

Answer: $x_1(t) = e^{j2t} \rightarrow$ eigenfunction, $j\omega = j.2 \Rightarrow \omega = 2$.

$$y_1(t) = H(j2).e^{j2t} = |H(j2)|e^{jarg[H(j2)]}e^{j2t}$$

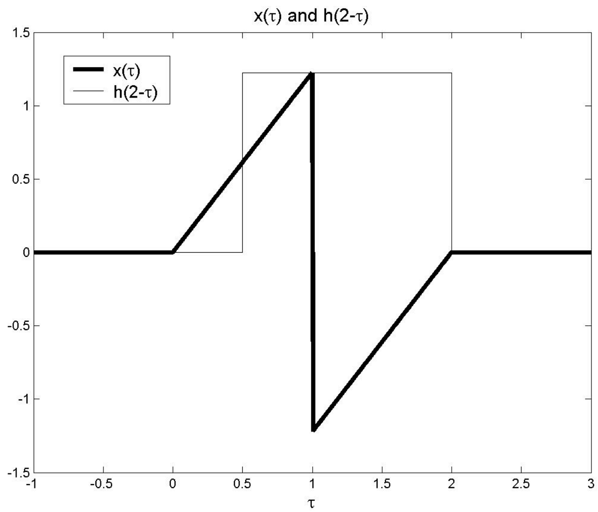

Figure 1: Problem 2

$$\Rightarrow |H(j2)| = \frac{1}{2},\ arg[H(j2)] = -\frac{\pi}{4}$$

Since e^{-j2t} is an eigenfunction with $j\omega = -j2 \Rightarrow \omega = -2$,

$$y_2(t) = H(-j2).2e^{-j2t}.$$

Since $\mathbf{h(t)}$ is **real**, $|H(j2)| = |H(-j2)|$(even) and $arg[H(-j2)] = -arg[H(j2)]$(odd).

$$\begin{aligned}
y_2(t) &= H(-j2).2e^{-j2t} \\
&= |H(-j2)|e^{jarg[H(-j2)]}2e^{-j2t} \\
&= |H(j2)|e^{-jarg[H(j2)]}2e^{-j2t} \\
&= \frac{1}{2}e^{j\pi/4}2e^{-j2t} \\
&= e^{-j(2t-\pi/4)}
\end{aligned}$$

$$\boxed{y_2(t) = e^{-j(2t-\pi/4)}}$$

Problem 4

A system is described by the first-order differential equation

$$\frac{dy(t)}{dt} + 6y(t) = 3x(t)\ , \quad t \geq 0$$

with initial condition $y(0) = 0$.

1. Calculate the output $y_1(t)$ due to the input $x_1(t) = \delta(2t-6)$.

   Answer: Since this system is described by a first-order differential equation, we can write the impulse response by inspection: $h(t) = 3e^{-6t}u(t)$. Then we simplify $x_1(t)$ as $x_1(t) = \frac{1}{2}\delta(t-3)$. Finally, using the fact that the delta function is the identity element of convolution, we find

$$\boxed{y_1(t) = \frac{3}{2}e^{-6(t-3)}u(t-3)}$$

2. Calculate the output $y_2(t)$ due to the input $x_2(t) = u(t-1) - 5u(t-10)$.

   Answer: Use the convolution integral.

$$y_2(t) = 3e^{-6t}u(t) * u(t-1) - 15e^{-6t}u(t) * u(t-10)$$

$$e^{-6t}u(t) * u(t) = \int_0^t e^{-6\tau}d\tau = \frac{-1}{6}e^{-6\tau}\bigg|_0^t = \left[\frac{1}{6} - \frac{e^{-6t}}{6}\right]u(t)$$

$$y_2(t) = \boxed{\frac{1}{2}[1-e^{-6(t-1)}]u(t-1) - \frac{5}{2}[1-e^{-6(t-10)}]u(t-10)}$$

3. Calculate the output $y_3(t)$ due to the input $x_3(t) = e^{-2t}u(t)$.

   Answer: Use the convolution integral.

$$\begin{aligned} y_3(t) &= 3e^{-6t}u(t) * e^{-2t}u(t) \\ &= 3\int_0^t e^{-6(t-\tau)}e^{-2\tau}d\tau \\ &= 3e^{-6t}\frac{1}{4}e^{4\tau}\bigg|_0^t \end{aligned}$$

$$y_3(t) = \boxed{\frac{-3}{4}e^{-6t}u(t) + \frac{3}{4}e^{-2t}]u(t)}$$

Problem 5

Consider the *series* connection of two LTI systems, S_1 and S_2; namely, the output of S_1 is the input to S_2. Let S_1 and S_2 have identical impulse responses:

$$h_1(t) = h_2(t) = u(t)\ .$$

Input $x(t) = 2[u(t-2) - u(t-4)]$ is applied to S_1. Calculate the output of S_2 at time $t = 3$, denoted by $y(3)$.

Answer: Series connection $\Rightarrow$ overall the impulse response is found as follows

$$\begin{aligned} h(t) &= h_1(\cdot) * h_2(\cdot)|_t \\ &= u(t) * u(t) \\ &= tu(t) \\ &= \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases} \end{aligned}$$

$$\begin{aligned} y(t) &= x(t) * h(t) \\ &= x(t) * tu(t) \end{aligned}$$

Graphically, for $t = 3$ (see Fig. 2)

$$y(3) = \int_0^1 \tau \cdot 2 \cdot d\tau = \tau^2|_0^1 = 1.$$

Analytically,

$$y(t) = \int_{-\infty}^{\infty} 2\underbrace{[u(\tau-2) - u(\tau-4)]}_{\neq 0 \text{ in } \tau\in(2,4)}.(t-\tau)\underbrace{u(t-\tau)}_{\neq 0 \tau < t}d\tau$$

For $t = 3$, we get nonzero integral for $(2,4) \cap (-\infty, 3) = (2,3)$:

$$y(3) = \int_2^3 2(3-\tau)d\tau = (6\tau - \tau^2)|_2^3 = 18 - 9 - 12 + 4 = 1$$

Answer: $y(3) = 1$

Problem 6

The input

$$x(t) = 6 + 7\sin(2t) + 8\cos(4t)\ \ ,\ \ -\infty < t < \infty$$

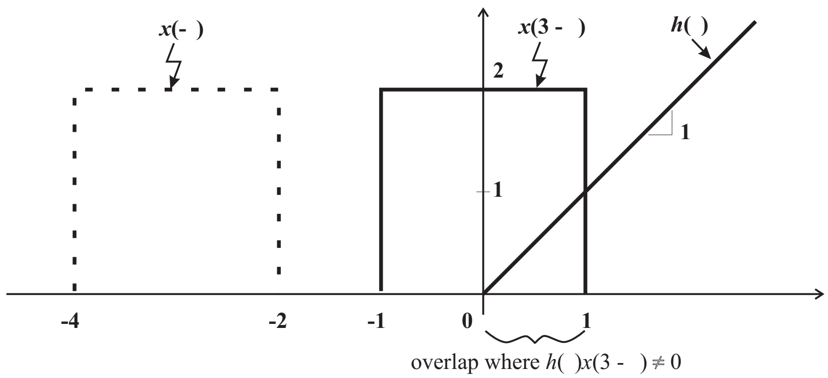

Figure 2: Problem 5

is applied to the system with Frequency Response Function

$$H(j\omega) = \frac{(j\omega)^2 + 4}{(j\omega)^2 + 25j\omega + 10} \quad .$$

The resulting output function is denoted by $y(t)$.

1. What is the fundamental period of $y(t)$?

   Answer: $y(t) = K_1 + |H(j2)| \sin(2t + arg[H(j2)]\Big) + 8|H(j4)| \cos(4t + arg[H(j4)])$. We get $|H(j2)| = 0, |H(j4)| \neq 0$.

Hence $y(t) = K_1 + 8|H(j4)|\cos(4t + arg[H(j4)])$.
Since $\omega_0 = 4, T_0 = \frac{2\pi}{4} = \frac{\pi}{2}$.
Fundamental period of $y(t) = \frac{\pi}{2}$.

2. What is the average value of $y(t)$?

   Answer: Average value of $y(t) = K_1$ where $K_1 = 6H(j0) = 6\left(\frac{4}{10}\right) = 2.4$

3. What is the maximum value of $y(t)$ for $-\infty < t < \infty$?

   Answer: From part (1), $\max[y(t)] = K_1 + 8|H(j4)|$.
   $K_1 = average[y(t)] = 2.4$.
   $|H(j4)| = \frac{12}{\sqrt{10^4+36}}$

THE END