EECS 216 - Winter 2008
PRACTICE PROBLEMS FOR EXAM 2 on 03/20/2008

THESE PROBLEMS ARE FROM PRIOR MIDTERM AND FINAL EXAMS. WE WILL DISCUSS THEM IN CLASS NEXT WEEK AND DURING THE EVENING REVIEW SESSION. PLEASE WORK ON THEM PRIOR TO THAT.

Multiple-choice questions:

Select the correct answer.

1. Consider the system with impulse reponse $h(t) = te^{-3t}u(t)$. What is the average value of the output if the input is
$$x(t) = 3 + 2\cos(t + \frac{\pi}{3}) + \sin(2t + \frac{\pi}{4})$$
   (a) 0
   (b) $\frac{1}{9}$
   (c) $\frac{1}{3}$
   (d) 1
   (e) None of the above.

2. What is the magnitude of the exponential Fourier series coefficient of the fundamental term (i.e., $k = 1$) of the periodic signal
$$x(t) = 3 + 2\cos(3t) + \sin(3t)$$
   (a) $\frac{3}{2}$
   (b) $\frac{\sqrt{5}}{2}$
   (c) $\frac{1}{2}$
   (d) 1
   (e) None of the above.

3. Recall that in AM, the so-called IF filter is centered at 455 KHz and has absolute bandwidth of 10 KHz. At what frequency (in KHz) should you set the local oscillator that precedes the IF filter at the receiver to tune in to station 1610 KHz?
   (a) 1610-455
   (b) 1610+455

(c) Either (a) or (b) would work

(d) 1610-10

(e) None of the above.

4. What is the smallest frequency (in radians/sec) in the list below at which you could sample signal

$$x(t) = [\frac{1}{t}\sin(40t) + \cos(20t)]^2$$

and still be certain that it could be exactly reconstructed from its samples?

(a) 41.

(b) 81.

(c) 121.

(d) 161.

(e) None of the above is large enough.

Problem A:

Consider two signals $x_1(t)$ and $x_2(t)$, where we have that

$$x_1(t) = \begin{cases} 5 & \text{for } |t| < 10 \\ 0 & \text{for } |t| \geq 10 \end{cases} \qquad x_2(t) = \frac{1}{\pi t}\sin(10\pi t) \quad \text{for} \quad -\infty < t < \infty$$

Form the signal $x_3(t)$ by doing the *convolution* of $x_1(t)$ and $x_2(t)$:

$$x_3(t) = x_1(t) * x_2(t) \ .$$

We want to sample $x_3(t)$ using an ideal sampler to perform some digital filtering operation. What is the minimum sampling frequency (in radians per sec.) that should be used so to that $x_3(t)$ can be reconstructed exactly from its samples?

Problem B:

Consider the causal and stable LTI system whose Frequency Response Function is

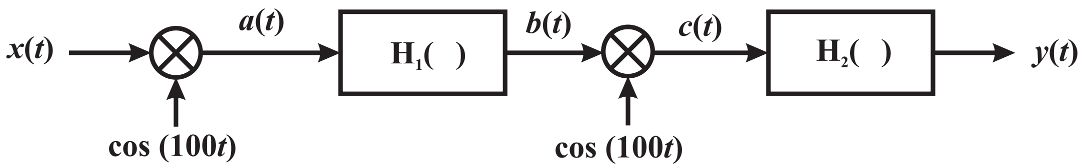
$$H(j\omega) = \frac{1}{j\omega + 1} \ .$$

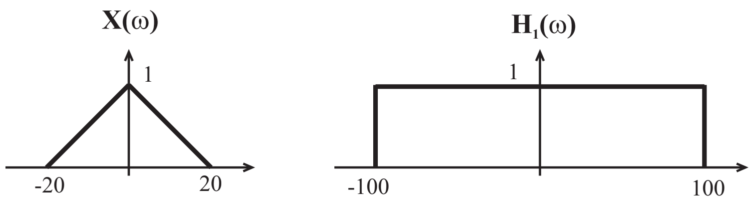
Calculate the output of this system, for $-\infty < t < \infty$, if the input is, for $-\infty < t < \infty$,

$$x(t) = e^{-j10t} + \cos(5t) + \sum_{k=-\infty}^{\infty} \delta(t - 25k) \ .$$

Problem C:

Consider the block diagram depicted below, where the spectrum of $x(t)$ and the Frequency Response Function $H_1(j\omega)$ are as indicated.



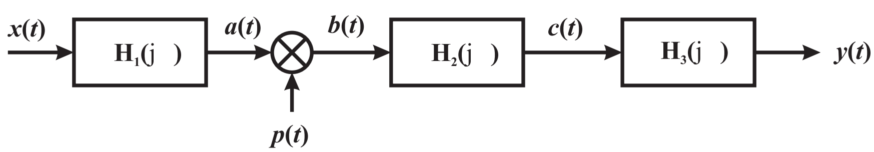


1. Draw the spectra of $a(t)$, $b(t)$, and $c(t)$.

2. We want to design $H_2(j\omega)$ such that $y(t) = -x(t)$. What $|H_2(j\omega)|$ and $arg[H_2(j\omega)]$ should be used for this purpose? Clearly indicate the analytical expressions of $|H_2(j\omega)|$ and $arg[H_2(j\omega)]$ or plot these two functions.

Problem D:

Consider the following system



where

$$x(t) = -\cos(t) \qquad p(t) = 2\sin(t)$$

$$H_1(j\omega) = j\omega$$

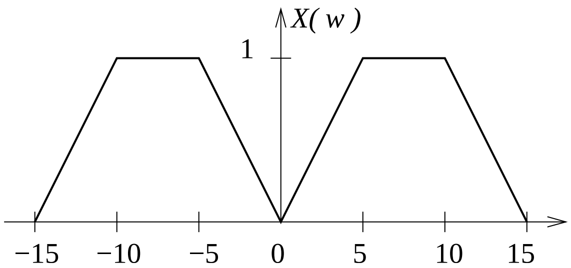
$$|H_2(j\omega)| = \begin{cases} 1 & \text{for } |\omega| \leq 1 \\ 0 & \text{for } |\omega| > 1 \end{cases} \qquad \arg[H_2(j\omega)] = -\frac{\pi}{2}\omega$$

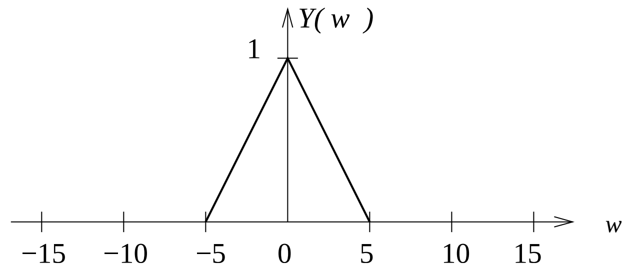
$$H_3(j\omega) = \frac{1}{3+4j\omega}$$

Calculate $B(j\omega)$, $c(t)$, and $y(t)$.

Problem E:

Design a system consisting of the interconnections of **modulator(s)** and **ideal filter(s)** that will transform $X(j\omega)$ depicted below to $Y(j\omega)$ depicted below.
(Keep your system as simple as possible!)





Problem F:

Calculate all the Fourier series coefficients $X[k]$, $k \in \mathbb{Z}$, of the periodic signal

$$x(t) = \sin(\frac{\pi}{3}t) + \cos(\frac{\pi}{4}t) \ , \quad -\infty < t < \infty \ .$$

Clearly indicate what frequency ω each coefficient $X[k]$ corresponds to.
In the case of complex coefficients, indicate their magnitude and phase.

Problem G:

Let $x_3(t)$ be a real periodic signal with fundamental period $T_0 = 2$. Its Fourier series coefficients are:

$$X_3[k] = \begin{cases} 5 & \text{if } k = 0 \\ j & \text{if } k = 2 \\ -j & \text{if } k = -2 \\ 3e^{\frac{-j\pi}{4}} & \text{if } k = 4 \\ 3e^{\frac{j\pi}{4}} & \text{if } k = -4 \\ 0 & \text{otherwise} \end{cases}$$

Find the expression of $x_3(t)$. (Write it as a real function.)

Problem H:

What is the energy in the frequency band $[-\frac{1}{2}, \frac{1}{2}]$ of the signal

$$x_2(t) = \frac{1}{\pi t}\sin(2t)[5 + \cos(10t)] \ .$$