

**Def:** *Modulation* of a message signal  $m(t)$ : Multiplies  $m(t)$  by a sinusoid.

**So:** Modulated signal:  $x(t) = m(t) \cos(2\pi f_c t)$  where  $f_c = \text{carrier frequency}$ .

**Why?** So why do we modulate signals  $m(t)$  for communications purposes?

1. *Transmission:*  $m(t)$  can't be radio broadcast;  $x(t)$  can (higher freqs).
2. *Freq. multiplexing:* Transmit several messages  $m_i(t)$  using one  $x(t)$ .

**Recall:**  $\mathcal{F}\{m(t) \cos(\omega_c t)\} = \frac{1}{2}M(\omega - \omega_c) + \frac{1}{2}M(\omega + \omega_c)$  where  $\mathcal{F}\{m(t)\} = M(\omega)$ .

**Huh?** Multiply  $m(t)$  by  $\cos(\omega_c t)$  shifts spectrum  $M(\omega)$  up and down by  $\omega_c$ .

**Below:**  $\mathcal{F}\{\text{squ}(t)\} = \text{SQU}(\omega)$ ;  $\mathcal{F}\{\text{tri}(t)\} = \text{TRI}(\omega)$ ;  $\mathcal{F}\{\text{bowl}(t)\} = \text{BOWL}(\omega)$ .

**DSBSC:** Send 3 signals at once by modulating them at different carrier freqs.

**Each:** Maximum freq.=5, bandwidth=10; total signal has bandwidth=100.

**Note:** 3 modulated signals separated by bandwidth=2(maximum frequency)!

**AM:** Amplitude Modulation; adds in *carrier*  $\cos(20t)$ :  $[(1+m(t)) \cos(20t)]$ .

**Why?** Receive AM using envelope detector ( $1+m(t) > 0$ ); easy to find signal.

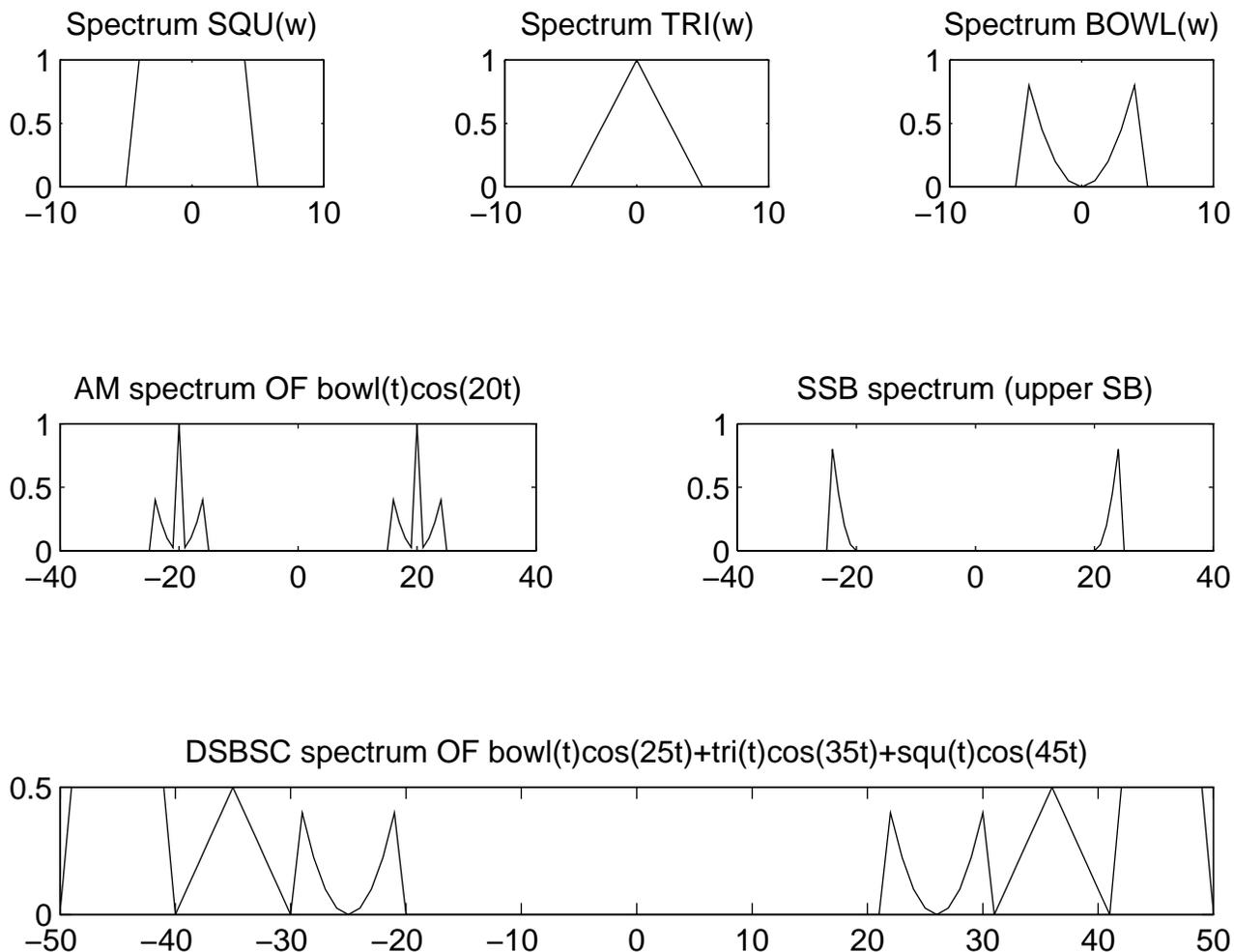
**DSBSC:** Double SideBand Suppressed Carrier (excludes carrier):  $[m(t) \cos(20t)]$ .

**Why?** Don't waste power transmitting carrier; still use envelope if  $m(t) > 0$ .

**Goal:** Send two signals left(t) and right(t) as one single signal stereo(t).

**So:** Can recover mono signal left(t)+right(t) directly from baseband.

**Soln:** stereo(t)=[left(t)+right(t)]+[left(t)-right(t)]cos(2π38000t) works.



**What:** Transmit only upper sidebands of the modulated  $m(t)$  (see overleaf).

**Why?** Half the bandwidth & half the power; has half the noise (same SNR).

**Where:** Digital TV (vestigial sideband); DSL modems; CB and ham radios.

**How?** A *really* sharp filter would clip the sidebands! Is there an easier way?

**Def:** The *Hilbert transform* of  $x(t)$  is  $\mathcal{H}\{x(t)\} = \int_{-\infty}^{\infty} \frac{x(\tau)}{\pi(t-\tau)} d\tau = x(t) * \frac{1}{\pi t}$ .

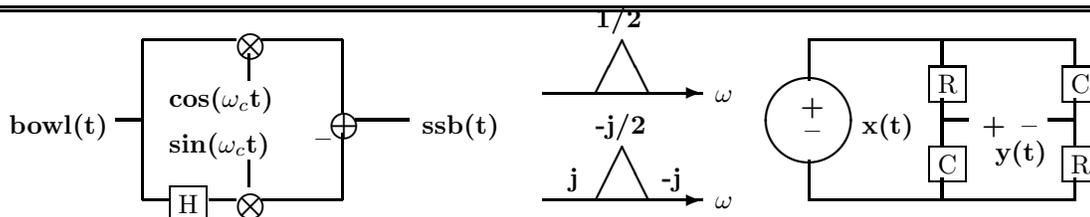
**Huh?**  $\mathcal{F}\{\frac{1}{\pi t}\} = -j\text{SGN}(\omega) = -j$  if  $\omega > 0$ ;  $j$  if  $\omega < 0$ .  $-90^\circ$  phase shift for  $\omega > 0$ .

**How?** Circuit below right:  $|H(\omega)|=1$  and  $\angle H(j\omega) = -90^\circ$  at  $\omega = 1/(RC)$ .

**So?** Let  $\text{ssb}(t) = \text{bowl}(t)\cos(\omega_c t) - \mathcal{H}\{\text{bowl}(t)\}\sin(\omega_c t)$  for  $\text{bowl}(t)$  overleaf.

**Then:**  $\text{ssb}(t)$  has the spectrum shown overleaf: Only upper sidebands appear!

**Since:** Lower sidebands of  $\text{bowl}(t)\cos(\omega_c t)$  and  $-\mathcal{H}\{\text{bowl}(t)\}\sin(\omega_c t)$  cancel.



**So?** Only use half as much bandwidth to transmit  $m(t)$ ; more efficient.

**Hence:** Can transmit twice as many messages using the same bandwidth!

**And:** Use half the power. Smaller bandwidth  $\rightarrow$  half the noise (same SNR).

## ENVELOPE RECEIVERS & HETERODYNE DEMODULATORS

**Goal:** Recover  $m(t)$  from  $(1+m(t))\cos(\omega_c t)$  (AM) or  $m(t)\cos(\omega_c t)$  (DSBSC).

**AM:** If  $|m(t)| < 1$  then  $1+m(t) > 0$  and we may use an *envelope detector*:

- 1.) Pass signal through a diode: keeps only positive parts of the signal.
- 2.) Then a capacitor smoothes out the carrier oscillations, leaving  $m(t)$ .

**AM: Superheterodyne** receiver (most transistor radios; goes back to 1920):

**Use:**  $2[(1+m(t))\cos(2\pi f_c t)] \cos(2\pi(f_c - 455000)t) = (1+m(t))\cos(2\pi 455000t) + (1+m(t))\cos(2\pi(2f_c - 455000)t)$ . Latter term is the *image*—filter out.

**Then:** Intermediate filter centered at 455 kHz and then envelope detector.

**Why?** More selective; avoids carrier phase problem noted below; history.

**DSBSC:** Can't use envelope detector:  $m(t) < 0 \rightarrow$  phase reversals at those times.

**So:** Send:  $y(t) = m(t)\cos(\omega_c t)$ . Rcvr:  $y(t)\cos(\omega_c t) = \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos(2\omega_c t)$ .

**Then:** Latter term is *image* frequency component; filtered by earphone/speaker.

**But:** Carrier *phase* unknown to receiver; received  $m(t)\cos(\Delta\text{phase})$  may=0!

**Why?** 
$$2[m(t)\cos(\omega_c t + \theta)]\cos(\omega_c t) = \underbrace{m(t)\cos(\theta)}_{\text{UNKNOWN CARRIER } \theta} + \underbrace{m(t)\cos(2\omega_c t + \theta)}_{\text{IMAGE(filtered out)}}$$

**SSB:** Rcvr  $\sim$  xmitter except sign:  $\text{bowl}(t) = \text{ssb}(t)\cos(\omega_c t) + \mathcal{H}\{\text{ssb}(t)\}\sin(\omega_c t)$ .

**Why?** Keep upper/lower sideband when shift down/up for freqs  $> / < 0$ .

**Mix:** Compute  $4x(t)y(t) = [x(t) + y(t)]^2 - [x(t) - y(t)]^2$  using square-law device.